

**Simulation of Business Systems**  
**Prof. Deepu Philip**  
**Department of Industrial & Management Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 04**  
**Components of Discrete Event Simulation**

Good morning guys and welcome to the yet another the lecture of Simulation of Business System and today is lecture 4 actually I have wrongfully written it as lecture 3 it is lecture 4.

(Refer Slide Time: 00:24)

**Simulation of Business Systems**  
**Introduction**

**Dr. Deepu Philip**

Learning Agenda

- I [ Nature of Simulation ✓
- I [ Brief History ✓
- I [ Systems, Models, and Simulation ✓
- I [ Applications & "Real" World Applications ✓
- I [ Why Simulation is Needed? ✓
- II [ How to Build Simulation Models? ✓
  - Simple Example

*Discrete vs Continuous  
Static vs Dynamic  
Deterministic vs Stochastic*

*Next - Event Simulation  
Timing mechanism - Simulation time*

Lecture 04

And we are trying to understand the basic concepts of premises of how do we simulate business systems? And if you look at the learning agenda we have already covered nature of simulation, we have also looked into the brief history how the simulation has already originated. So, these were covered as part of the lecture 1.

Then, systems, models and simulation we covered in the part of lecture 2 ok. Then, what are the definitions associated with this, what are the major applications in real world applications? That aspect of it we also covered and then why do we need to do simulation? This question also be answered all these were part of lecture 2. Then, how do we build simulation model? We covered in the previous lecture and what are the main aspects of simulation especially on things like various aspects of discrete, versus continuous simulation and similarly we also did with the static versus dynamic

simulation. And, we also studied deterministic versus stochastic simulation; deterministic versus stochastic simulation. And, then we also discuss the from here, why we are going to focus more towards next event simulation?

And, we also talked about what we call as the timing mechanism and we understood the concept of what we call it as simulation time? Ok. This much aspect we covered in the previous lecture. And, now today what we will try to do it will take as. So, this was our lecture 3. And, now we are going to get into a simple example. You want to see a simple simulation system and see how the principles are we are study so far gets accumulated into single concept and see how it actually works?

. So, today we will be focusing more towards developing the simulation using a pen and paper model, pen and paper approach to understand basics of this approach ok. So, let us see how we can do this today and then we will go to then we will see.

(Refer Slide Time: 03:11)

**Components of Discrete Event Simulation Model**

Countable events at instantaneous points of time. → numerical analysis.

Each simulation model must be customized to target system

Several common components do exist: ↳ There is no single model that fits all dependent on target system. → time consuming

- **System state** – Variables (collection of) that describe state of the system. ↳ # of people in system
- **Simulation clock** – Current value of the simulated time. (in between points times arise)
- **Event list** – times of future events as needed by the model. (calendar of events)
- **Statistical counters** – to accumulate various quantities for output. (X) = # of all X · time
- **Initialization routine** – to initialize the simulation model to time = 0. (simulated time)
- **Timing routine** – determine next event time and type, and then advance simulation on clock.
- **Event routines** – carry out the logic for each event. (event = arrival → demerit? → setion → server lang? → queue)
- **Library routines** – utility routines that generate random variables, pseudo events, etc. ↳ for simulating uncertainty (stochastic)
- **Report generator** – to summarize and report results at the end of simulation.
- **Main program** – ties all routines/sub-systems together and ensure their execution in the right order. ↳ tied together  
Garbage model ⇒ Garbage output (GIGO) ⇒ Estimator.

Before, we do the simulation I want to run again through the major components of discrete event simulation model. As, I mentioned earlier this is discrete means in a simple sense, discrete implies countable events at instantaneous points of time, remember we talked about the people coming into a banking system or a simple queuing system. And, today we will actually look into similar example like that ok.

So, we were talking about in discrete system. And the trick simulation every simulation model or each simulation model have to be customized to the target system. So, the most important thing that you need to realize is there is no single model that fits all. So, it is all dependent on the target system ok. So, it is dependent on target system.

So, whatever be the system that you want to model depending upon that you build customize the simulation model ok. So, this customization is the place where it actually this is why it is called as time consuming process consumer. So, that is why people say simulation is the time consuming purely because you have to customize into the target system. And, there are several components that do exist. So, and these components are common, but they have to be customized to a particularly.

So, the first one is what we are going to call it as the systems state ok. System state is one of the aspect we already talked about what is state of the system. And, this is in a very simple way it is a variables or another way a collection of variables collection of variables that describe state of what state of the system ok. So, the collection of variables, the variables a collection of them together that describes the state of the system is what we called as a system state.

Then we talked about simulation clock or simulation time. So, the simulation clock means specifically what it does is gives you the current value of the simulator time. So, if you remember I have yesterday draw the graph in which I tried to explain to you guys is that if you think about this is a time. And, this is number of people in the system in a queuing system and we draw a graph like this for some point of time that you people than one nobody then it goes up like this.

So, you are more in the discrete simulation you are more worried about these instantaneous points ok. You are not worried about what is happening in between. So, this area the time in between where nothing happens in the system is ignored. So, your simulation time jump from this time to this time to this time, then it jumps to this time then it is from there it jumps to there and like that.

So, it is not the real time it is actually the time. So, the main thing is in between times are ignored, you are not worried about this one these are ignore you do not care about what is happening there, because the state of the system only changes at this discrete time

points which are at which are demonstrated at one point. So, this kind of a time is called as a simulated time. And, the simulation clock keep track of the simulator time.

Then, the third one is the event list you already seen what are the event list just we know what is an event also the event list is the times of future events as needed by the model needed by the model. So, event list is mostly I can think about in the form of a calendar of events, some people call it as calendar of events. So, what are the events that are going to happen in future and when are they going to happen, the event and it is time that the future events and it is types put together in a tabular form release form for a calendar format is what we typically call as a event list. And, ask the simulation clock jumps from one event to another the event gets pulled out of the event list and the new event gets updated ok. So, that is the event list.

Then, we talked about is these statistical counters because we know that simulation as I said earlier is a numerical analysis. So, when you have a numerical analysis you need statistical counter. So, what do they do? They to accumulate various quantities for output. So, if you think about the number in the system. So, let us say  $X$  denotes the number in system. Then what we do is we keep on doing a time progresses  $X$  gets on  $X$  equal to you get the submission of  $X$  all  $X$ S. And when the simulation is over you divided by the time and then you can I get a value of how many people per whatever time (Refer Time: 09:40) ok. So, that is an that is one way of looking into this ok. So, this accumulation on this is basically what we called as a statistical counter.

Then the next one is also called as a initialization routine. Initialization  $t$  routine means to initialize the simulation model, model to time equal to 0 or how do you start the simulation at the beginning of the time? This is not the actual time; this is the simulated time, not the real time. Now, how do you initialize the simulation model to time  $t$  equal to 0 and the simulation time is what we look do in the initialization routine.

Then comes the timing routine, the timing routine is it is to determine next event time and type. What is the next event time and type? Whether it is an arrival, whether is a service completion, whether is a machine breakdown you determine they and then advance simulation clock ok. So, you determine the next the time and the type of the next event and then you advanced simulation clock. So, that the next even actually

happens. So, the timing routine ensures that the simulation model keeps on ticking as time progresses.

Then comes the next one what we call as event routine, when even routines the job is to carry out the logic for each event. So, if we assume the event is on arrival ok. So, assume event equal to arrival, then what happens is a person arriving into the queue will check whether the server free, server is free or server is busy ok. If the server is free then it immediately go to the server get served get served otherwise go to the queue. So, this logic what happens when this particular thing happens is taken care of by the event routine. So, how would the each event, when it happen, what it should do next immediately is written by the event routine?

Then come something called as a library routines, this is quite common and very well used these are utility routines. And what does these routines do? Utility routines that generate random variates, pseudo-events, etcetera. So, I said that we use random number. So, random variables or random variates to do the simulation, why because the random variates are used for simulating uncertainty or what we call as the stochastic part of the simulation model is done with the help of the random variates. So, the routine that generates the random variates so, that you can model uncertainty is taken care of part of this particular library routine. Similarly, the pseudo-events beginning of the simulation end of the simulation all those aspects are also taken care of by this approach.

Then comes the report generator and report generate does his job is to summarize to summarize and report results at the end of simulation ok. So, you summarize and report the results of the simulation at the summarize this at the end of the simulation, that is the a main job of the report generator and why do you need this, because again as I the simulation is a numerical analysis, analysis using a model and this data is required to get estimate of the system performance, that gives you the estimates. So, this report is important for you to calculate these various estimates.

Then comes finally, the main program, what it does is it ties or routines subsystems together and ensure their execution in the right order ok. That is what happens with the main program all these routines all these kind of proteins are we talked about all of these guys are tied together by the main program. And, then it ensures that their executers in the correct order. So, this is how a complete simulation system work.

So, you know know that the simulation software is not a simple software anymore. It is actually complicated software with various subroutines and modules sub modules and other aspects which all taken care and working together with the help of a main program tie together. So, that the logic of execution is important. And, now because it is a complex system it is also very important to ensure that the model that you are building is a right model. And, there is no logical flow in this because if you have a logical flow then you are simulation output will also be into be in trouble.

So, if you have a garbage model then you will end up getting a garbage output ok. So, this is the guy go principal garbage in garbage out. So, we want to avoid that ok. So, we our aim is to avoid this alright.

(Refer Slide Time: 17:00)

Simulation Entities

Entity have attributes  
↓  
data values

Objects that compose a simulation model

- usually they include Customers, parts, paperwork (messages), etc.
- May also include resources like Servers or machines, etc.
- They are characterized by data values ⇒ called attributes.
- For each entity that is part of the simulation model, there is a record (or a row) in the list (entity list, event list, calendar) etc.
  - ↳ The columns of such rows are the attributes of the entity.

Customer-5	Arrival time	Service start time	Service End time	Departure time
------------	--------------	--------------------	------------------	----------------

Entity ← (under Customer-5)      ↑ (under Arrival time)      ↑ (under Service start time)      ↑ (under Service End time)      ↑ (under Departure time) Attributes

Modeling approaches

- (1) Event-Scheduling → as described in the time advancement (determine the next event's time and type and then advance the simulation clock)  
↳ usually coded in a General purpose programming language.
- (2) Process — focus is on entities and their "experience" ⇒ usually requires a Special purpose Simulation Software.

So, now we will look into the next aspect of the simulation and what we talked about does the simulation entities, you are already heard me using this word entities get. So, remember there are events there are routines there are, libraries there are now entities. And, entities are very important aspect of it. And, by definition entities are those objects that compose the simulation model ok. The objects that together make the simulation model is what we called as the entities. What are they usually they include they include what customers, parts, paperwork, like messages or information etcetera may also include resources like servers or machines etcetera.

So, what we are talking about these are the objects. So, when we talk about the single server model of a bank the one of the entity is the customers that come into the system. Then also the paperwork is there might be coming to fill the long form of something then that we is also part of that also you can think about does an entity. And, also the server or the bank teller or the machines that are used in that process, also includes as the object of simulation. They are characterized by data values ok.

So, entities are characterized by data values called this data values are called attributes. So, when we say entity have attributes. When they say that an entity have attributes implies data values, that characterize that entity. So, that add data values are called as the attributes.

. So, for each entity that is part of the simulation model for each entity that is part of the simulation model, there is a record or a row for a row in the list ok. So, for each entities that is part of the simulation model, whatever entities that is part of the simulation model there is a record or there is a row one row in the list ok. In the list of whatever you can call it as the entity list, entity list some people call it as the event list ok. Some people call it as the calendar whatever you want to say whatever you want to call it etcetera. In any one of those you actually have an row a particular record a row is maintained for each entity that is part of the simulation model, the column of such a row are the attributes of the entity attributes of the entity.

So, here like if you say that a customer 5 arrived, if this is a entity then the attribute. So, this will be the row of the entity, you will have a row on this. And, one of them will be the arrival time, then you will have another called service start time, service end time, departure time. Let us say these are the attributes of this particular case, then you can say that the customer 5 arrived the, what is arrival time? When did the service begin? When the service end? When did the person left the system? So, these once these becomes the attributes and this becomes the entities. Hopefully, now you understand the concept of entities in the simulation model or when we talk about simulation entity what they are.

So, how do we model? Ok. Typically, what are the major modeling approaches in doing this ok? There is the first approach the first and the most importantly used one is the event scheduling. So, what happens is described as described, in the time advancement in

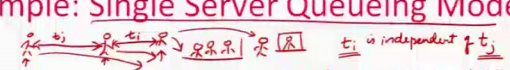
the time advancement ok. So, what it be described in the time advancement. The time advancement we actually said that determine the next event events time and type and then advance the clock, advance the simulation clock. This is what we mentioned in the time advancement.

So, the event scheduling is pretty much this as described in the time advancement next event time and type and then advance the simulation clock ok. And, this is usually coded in a general purpose programming language. Many a times you have a general purpose programming language that actually would implement this concept of event scheduling n ok, c plus python people use different programming languages to do that.

The second approach the modeling approach is what we called as the process is where the focus is on entities and their experience. So, what we try to do it is it is the one approach is do the event scheduling which is quite popular the other one is one of the process approach, where you focus on the entities and their experience ok. And, usually requires special purpose simulation software. So, the software if you are focusing on entity and their experience if you look the take the entity approach. Then, what you are going to how we say you require a special purpose simulation software to really deal with that like, Siman Sanskrit all those kind of things are taken are part of such kind of an approach.

(Refer Slide Time: 25:42)

**Simple Example: Single Server Queueing Model**

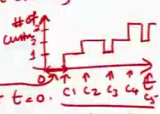


Assume inter-arrival times — they are independent and are identically distributed (IID) random variables

Assume service times — are also IID, and are also independent of the inter-arrival times.  
 time spent by the server with the customer.  
 $t_i, t_j$  do not influence  $s_i, s_j$

Queue discipline  
 ↳ The way in which the next person is called from the queue.  
 FIFO ⇒ First In First out (FIFS) // other discipline ⇒ LIFO (Last In First out)  
 SIRO (Service In Random Order)

Start empty and idle (Server)  
 ↳ at the beginning of the simulation (time = 0)  
 ↓ no customers



First customer arrives → after an inter-arrival time, not at  $t=0$ .  
 First customer will arrive some time later (following inter-arrival times) and not at  $t=0$ .

Stopping rule: → when the  $n^{\text{th}}$  customer has completed arrival into the system, then we stop.  $n=5$  (after 5 arrivals) ↳ Change to Completed Service  
 Simulation duration depends upon the stopping condition.



Now, in the a simple example that we are promising about the single server queuing model we discussed earlier that we will be working on this and we will do a hand simulation of this, so that we can understand how the system works. So, a single server model as I drew earlier ok. It is like there is a server one individual serving here, there entities waiting to get served and there are people in the queue they are all waiting to get served. And, ask when a entities comes in, when a new arrival happens, if the server is free it will go directly or it will join the queue depending upon the status of the server.

So, before doing this we have to make some assumptions. So let us start with the first assumption assume inter arrival time ok. So, they are independent turntables are independent and are identically distributed ok. Typically, we call it as IID, Identically and Independent and Identically Distributed Random Variable. We will soon have a lecture on what we call as probability which is specific to the simulation systems, but as of now assume that all the inter arrival times the time between arrivals.

So, there is one entity that is arrived right now and here is another entity who just arrived, second entity this time in between them what we call as inter arrival time or time between arrivals this time are identical and independence. So, the time for this the  $t_1$  if you take this or  $t_i$  and let us take another arrival ok. This is another arrival take this as  $t_j$  the  $t_i$  is independent of  $t_j$ , which means the value of  $t_i$  does not influence the value of  $t_j$ , neither the value of  $t_j$  does not influence the value of  $t_i$ . They are independent to each other. So, you might have studied in probability you about independent events where the intersection is 0 on those kind of things we will study that later.

But, assume that their independent means the occurrence of  $t_i$  has no influence of  $t_j$  or visa versa. So, all the times are independent, but they are identically distributed means they all part of the same probability distribution ok, we will see an example for the time b.

Then again service times service times are also IID ok, and also independent of the inter arrival times. So, we know that these are the IIDs the service times are also IID services time is the time spend by the server with the customer. So, this is what we call as the service time ok. The time spend by the server with the customer, they are also IID, Independent Identical and Independently Distributor, but they also independent to that of

the inter arrival. So, these does not have any influence on the as the  $t_i$   $t_j$  do not influence  $s_i$ ,  $s_j$  let us say  $s_i$  and  $s_j$  are the service times  $t_i$  and  $t_j$  are the inter arrival times ok.

Queuing discipline so, this is the way in which the next person is called from the queue where people waiting in the queue whom will you call the next person. The main node we are assuming here is FIFO ok, the stands for first in first out ok. For first come first serve it is also called as FCFS, First Come First Serve ok. What we called as FIFO, First In First Out. So, for assuming that whoever comes in the first gets served first so, the people who come later will join the back of the queue so, they will be kind of joining the back of the queue like this alright.

There are other queue in disciplines the other disciplines are other discipline queue discipline are one is called LIFO; it stars for last in first out ok, LIFO ok. Then there is another service called SIRO, Service In Random Order ok. Then there is HVF LVF order of queue disciplines, there are many other disciplined. So, I will just give 2 examples of this, but this we are looking at the FIFO is the queue discipline queue discipline for this example ok.

You start the system empty nobodies in the system and idle at the beginning of the simulation, beginning of the simulation or we call as time equal to 0, time  $t$  is equal to 0 this is semi started empty and idle. So, there is nobody in this system and syste and the server is idle. So, idle is the server empty means no customer so, but is the initial starting condition of the system. Then when would the first customer arrive we have to make that happen.

So, first customer arrives after an inter arrival time, not at  $t$  equal to 0. So, the simulation clock starts if you think about it graphically this is the time, this is 0 and here is the number of customer. So, the system will remain 0 for some time and after some point the time the arrival will happen. So, this is where the  $C_1$  the first customer will arrive and this is the number of people be 1 2 3 like this, and this will continue for one until some point of time and then maybe somebody else will also come then it will become 2, and then the first person service got over then will come back to one and like this. So, this is  $C_1$  and this is where the  $C_2$  will happen.

. So, this time this is the time is the inter arrival time after  $t$  equal to 0. So, the first customer will arrive after the beginning of the simulation ok. So, first customer will

arrive some time later following inter arrival times and not at time  $t$  equal 0 ok. So, when you go on open a bank people will come in little late that is what you are trying to simulate? Stopping rule then when how do we stop the simulation we started the simulation it is running. So, how do we stop? Ok.

Stopping rule is when the  $n$ th customer half completed arrival into the system then we stop. So, if you say  $n$  is equal to 5 which means after 5 arrivals, you can change this condition, you can change set of this completed arrival, you can change this to change to completed service. Then; that means, 5 people are finished service. So, as soon as 5 people join the system.

So, then here in this case we can see that you will have a this is a C 3 C 4 comes down goes up again this is your C 5, this means 5 people are arrived into the system, but if you say 5 people are complete service this will continue further ok. So, depending upon what is the stopping condition that you are used the simulation will run for different durations say. So, the duration the simulation duration sorry the simulation duration depends upon the stopping condition ok. So, that is what we will be doing in this world.

Now, once you do the simulation. So, what?

(Refer Slide Time: 36:20)

**Simple Example: Quantities to be Estimated**

*How long does the customer have to wait before getting served?*  
Expected average delay in queue (excluding service time) of the  $n$  customers completing their delays (waiting in queue).

*How long will be the average queue length?  $\rightarrow$  if you reach the system; how many will be waiting ahead of you!*  
Expected average number of customers in queue (excluding any in service)

*How much of the time the server is busy serving the customer?*  
 $\frac{72 \text{ min}}{100 \text{ min}}$   
Expected utilization (proportion of time busy) of the server  $0.72$

Many others are possible (maxima, minima, time or number in system, proportions, quantiles, variances, ...)  $\rightarrow$  measures of dispersion.  
*max. of queue length*  
*No body who comes to the system leaves without getting served.*

*Time in the system = Time in queue + Time to be served.*  
*Number in the system = Number in the queue + Number that is served.*

Important: Discrete-time vs. continuous-time statistics  
*Statistics of events that happens at specific instances.*  
*time for which the server is busy.*  
*Balking.*

After that what you do? The, you have to do lot of estimate of quantities another aspects. So, one of them that you will end up estimating is the expected average delay in the

queue, which means how long does the customer have to wait before getting served. How long will you end up waiting in the queue, ok before you starting to get served excluding the time? So, you just focus only on the waiting time of n customers how many our customers completing their delays or completing there this delay means you know waiting in queue delay implies waiting in the queue.

So, how long does an customer have to wait on an average ok, that is what you are trying to find out here. Expected average number of customers in the queue, how long will be the average queue length? Ok. So, we go to a so, it like if you reach the system, how many will be waiting for you will be waiting ahead of you this is an important questions. So, people decide when to go for service depending upon answering this question.

Then, the third one is expected utilization or what we called as proportion of time busy of the server. So, it basically is the question is how much of the time the server is busy he is busy serving the customer? So, if you say that the time simulation time is 400 minutes and the server was busy for 72 minutes, then it is 0.72 or 72 percentage of the time the server is busy. In the rest of the time the server is idle waiting, because there is nobody there to get this service.

Then comes the other possible options you have many other possible options those options include the maxima. So, this is the maximum of queue length can be maximum waiting time that kind of thing then minimum. So, how quickly ok? Then time or number in the system this is another example, what is the time in the system? Time in the system means time in the system is equal to time in queue plus time to be served ok, that is the time in the system, number in the system. The number system is equal to k number in the queue in the queue plus number that is served ok. So, that is this the number in the system. So, time in system and the number in the system can be taken care of that.

Then, proportions is the quantile what proportions of them a quarter quantiles and then variances, variance is how what is the dispersion. So, these are all measures of measures of dispersion, how much is a data vary in the system? That is also part of this analysis ok. And, you should remember that we are talking about discrete time versus continuous time statistics, discrete time is the like statistics of events that happens at specific time instances. So, like arrival ok. That is happens in the discrete time. Whereas, continuous times statistics for example, is the you know time for which the server is busy, that is a

continuous time statistic. Because, whenever it is busy the server you continuous the remain busy is not an instantaneous thing it actually happens when the customer is being served by the server.

So, now, with this much of basic preparation, what we will try to do is we will try to create a simple simulation model of a system where single are server are we discussed one server and people coming to get served and waiting in the queue. And, also remember that in this model we are not talked about you know and one aspect we should also remember here is, nobody who comes to the system lease without getting served.

So; that means, a person who coming in will come and join with queue. So, we think about it, if this is the server s one and it is serving an individual and here is a queue people are waiting in the queue ok. And, if a new person coming in he will join the queue he will not leave if you leave this is called balking you see the queue is long and then you leave. So, there is no balking, no renegading, not joking everybody who comes in come to the queue sets and way it is properly as civilized individuals.

With this what we will now do is we will try to create a simulation model of the system as required. And, then we will try to do a hand simulation for few number of entities let us say 5 entities and see how it works ok.

Thank you.