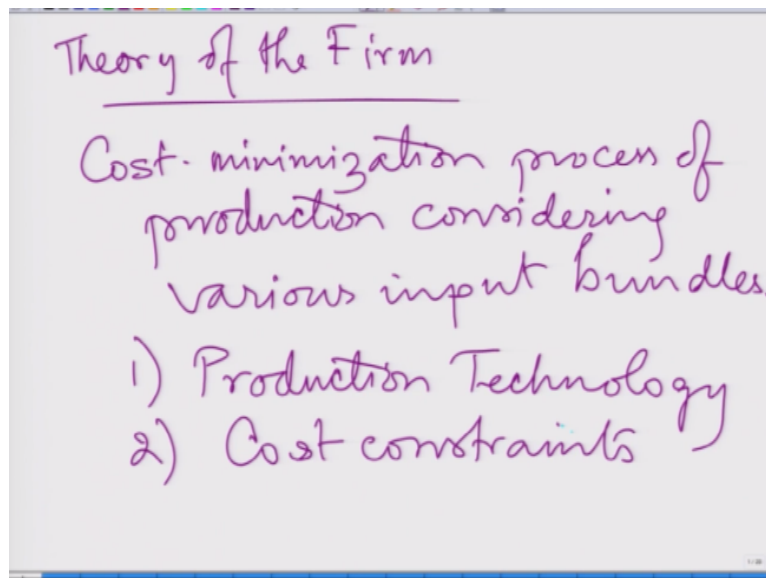


Economics of Health and HealthCare
Prof. Deep Mukherjee
Department of Economic Sciences
Indian Institute of Technology- Kanpur
Prof. Angan Sengupta
Department of Management, Amirtha Vishwa Vidyapeetham, Bangalore

Lecture - 10
Production Function

Hello everyone. Today in our session, we will discuss production function and we will largely first we will discuss about the general concepts of production function and then we will try to connect them with the healthcare or healthcare operations. Now the theory of production is basically a part of theory of the firm.

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Theory of the firm, now firm is a production unit yeah, so when a firm decides to produce something it has also a cost implication. When we talk about our demand, our resource constraint and our efficient choices, we have to connect all of them together when we discuss about theory of the firm. Let us understand that we have say we have a hospital system where we have limited number of doctors and plenty of patients.

So there is a limited supply, there is a constraint in terms of resources, in terms of doctors, nurses and other resources what we generally see in any public hospital in India but the demand is enormous. Having said that there is a cost implication that the cost can be again monetary and non-monetary. The cost of the treatment as well as the other non-monetary cost

that there should not be any high prevalence of diseases or no mistreatment as far as that death can be avoided.

So the optimization is primarily is cost minimizing. The optimization in theory of firm is basically cost minimization process of production considering various input bundles alright. These input bundles will determine that how my production will take place or given the different input bundles what can be the optimum production. Now having said that this production in a firm can be obtained by production technology number one.

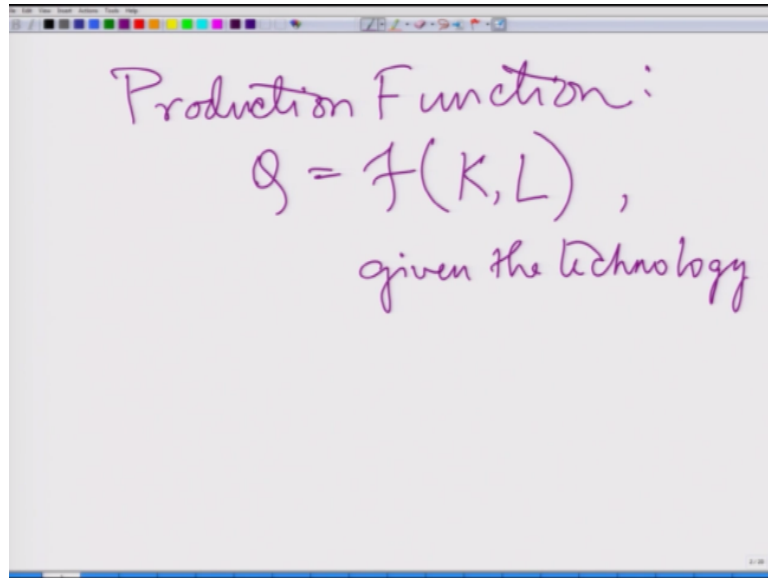
Number two, cost constraints that is basically the budgets I have been given that within this budget I have to produce whatever I want to produce you know. So even if I have the supply of input I cannot probably in a primary health center I cannot probably keep 5 doctors and 15 nurses yeah.

So I have a budget constraint for each primary health center, for each hospital, for each say public health awareness campaign everything if we consider as individual production process because everything has an outcome, everything has a production process, then there is a cost constraint, there is a budget constraint that whatever I need to do there has been stated already that this is your budget, within this you have to manage the entire operation. So that is the cost constraint.

So even if I know my production technology, I can avail the technology but having the technology I have been given that this is your technology, this is your cost constraint and based on that I will decide my input choices. They are the production factors or factors of production, my input choices that which will be the how many capital, how many labour, which kind of labour, which kind of capital, which kind of machinery, what will be the other ingredients in you know in a production process.

Whatever is required I have to decide based on the budget I have been given to given my production technology.

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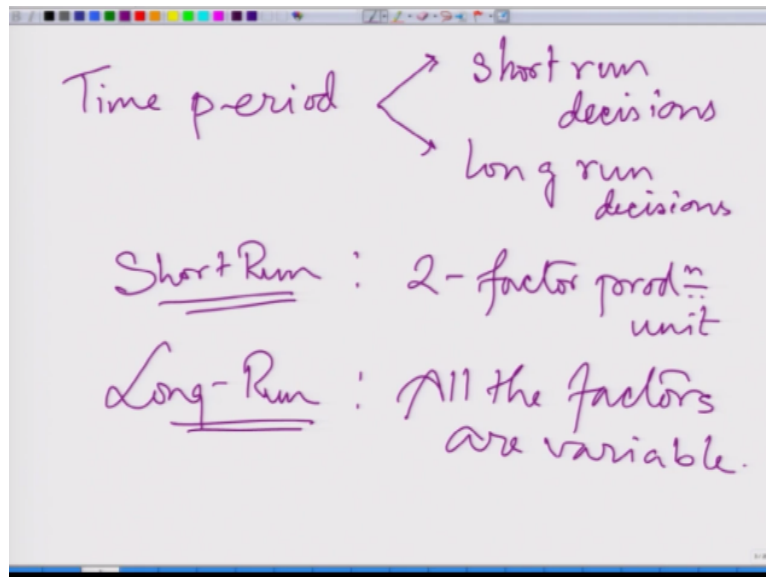
Therefore, a production function, now the theory of firm deals with both the theory of production and theory of cost. Production function can be stated as Q that is the quantity of production, amount of production as a function of K which stands for capital, L which stands for labour. K basically is non-human capital and L is the human capital you know. So L is labour, K is capital but we must state given the technology.

So technology is assumed to remain constant during a production process given the technology. So production or my output is a function of the capital and labour, the different combinations of number of capitals and number of labours and the goods and services, the production of the goods and services will depend upon that. Why goods and services? Again as I said when I take the example of a hospital service you know.

So it is not actually I am getting treated it is not actually a good, it is the services I am availing. Wherever I am going to a shop or a pharmaceutical organization manufacturer, they are manufacturing goods. So goods and services both can be you know treated as an outcome of a production process. When I (()) (06:22) an awareness campaign program say on HIV/AIDS, it is a service basically.

And the outcome is the awareness among people about HIV/AIDS yeah so and that awareness generation about HIV/AIDS depends upon how much capital I am putting in as well as how many labour forces, who are those labour forces who working towards this awareness generation program. So a production function depends upon time period right, for how long I will be producing or now given a time period I have certain constraints.

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Let us see what is time period. There are two types of time period or two types of production function given time period, one is short-run decisions, decisions in terms of production, one is long-run decisions yes. Now what is short-run? In short-run, I cannot say that it is one day, it is one minute, it is one month, it is one hour or one year, there is nothing like that you know that a short-run should be a short period of time.

No, there is nothing like that. A short-run is a time period where I cannot adjust all the processes or all the factor inputs. I cannot vary all of them which has a direct implication on cost yes. So in short-run I have a limitation. So let us take a simplified example. When we have our two factors production unit that I have just two factor inputs, one capital, one labour maybe and if in a short-run I cannot change both these factors of production.

I will change only one of them. I will only like be if it is possible that is primarily say suddenly there is a mishap in in my locality where my hospital is there and I have to treat up you know there is an inflow of patients say 50 patients suddenly come in. Now what do I do? I cannot get the number of doctors you know suddenly I cannot increase the number of doctors.

I cannot increase the number of nurses suddenly. Maybe what I can do is that I can ask for some volunteers. If I have a medical college in that hospital, I can ask some medical students to pitch in. I can take help of some blood banks or some other you know medical equipment supplying agency who can provide me at a very emergency notice that they can provide me

certain you know basic minimum or basic necessary healthcare requisitions yeah or treatment commodities.

Now so that is the short-run. It can be a one day, even a short run can be one year if it is a large production process yeah whereas in long-run so in short run I can only change the cost or I can only vary the cost based on either very few in a multifactor units, in a very few of these factors of production or else I can just change the production process that is all where in long-run I have enough time in my hand so that all the factors of productions are variable.

All the factors are variable, so I can change according to my wish I want to change in that particular hospital I want to change next year maybe 50 new beds I want to introduce, I can do that. I can sanction a budget, I can you know kind of do a predictive modeling that what has been in the next last year and what can happen in the next year whether I am seeing our increase in demand whether I can see you know the prevalence of certain ailments in that particular locality.

I can open a new discipline maybe Orthopedics, maybe Neurology, maybe Oncology, so all these things I can do and to make all these changes the basic necessary time, the minimum necessary time I will require that can be considered as the long-run and they are onwards any time period can be our long-run.

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$$Q = f(K, L), \text{ given technology}$$
$$AP_L \Rightarrow \text{Output per Labour} = \frac{Q}{L} = \frac{\text{Total Product}}{\text{Total \# of Labour}}$$
$$AP_K = \frac{Q}{K} = \frac{\text{Total Product}}{\text{Capital}}$$

So now after stating my production function Q as a function of capital and labour, given technology I do not have to mention this all the time but we need to keep it in mind yeah

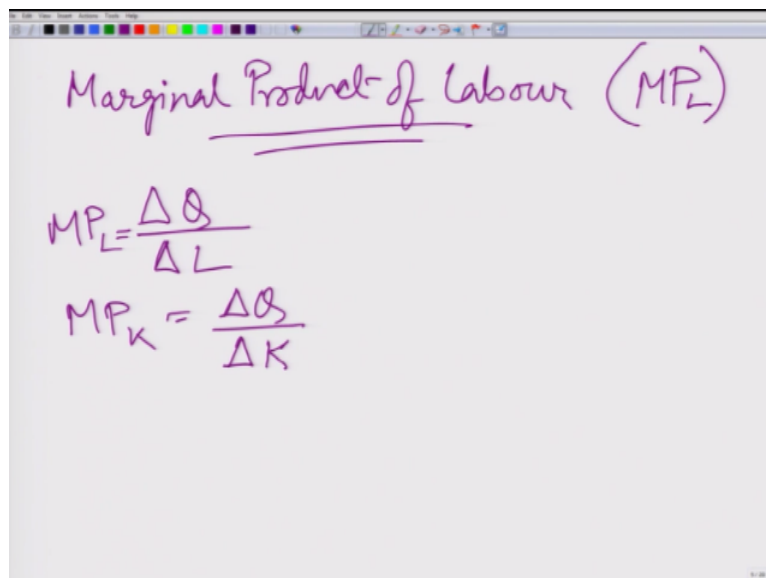
given technology. So Q stands for total output, K stands for total capital, L stands for total labour together that capital and labour is giving me an output of that amount of Q. Therefore, as we can go back to arithmetic mean or average.

My average production which we can say AP average production of labour which can be stated as output per labour or per unit labour yes. APL can be stated as total number of output/total labour. So total product/total number of labour, which is required to manufacture that many output. Similarly, average productivity of capital can be total product/total number of capital.

So total product/total number of labour required to manufacture that amount of product. Similarly, over here total product/total capital. This is the average product of labour and average product of capital. It can be you know the total number of patients being saved or treated by one doctor in a particular district, in a particular state, in a particular primary health center yes.

So total number of patients in a particular time period/total number of doctors yeah that is the average product of the doctor.

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The image shows a whiteboard with handwritten text in purple ink. At the top, it says "Marginal Product of Labour (MPL)" with "Marginal Product of Labour" underlined. Below this, two formulas are written: $MP_L = \frac{\Delta Q}{\Delta L}$ and $MP_K = \frac{\Delta Q}{\Delta K}$.

The next concept is marginal product of input. I can keep it as labour, marginal product of labour. I can write it like MPL marginal product of labour. Now this marginal product again if we go back to our marginal utility concept, marginal is incremental yeah. How to remember

it? When we talk about marginal utility, we talk about change in utility. Then, we think why utility is changing because of consumption of one extra unit of a particular commodity.

Similarly, here marginal product of labour again marginal is associated with which word product. So that is incremental change or the total change in the total product or total production total output by why output changes because of the inputs either because of capital or because of labour, so total change in capital or total change in product by one unit change in either in labour or in capital.

So we can state it like ΔQ , marginal product of labour is $\Delta Q/\Delta L$ whereas marginal product of capital can be stated as $\Delta Q/\Delta K$. Is not it? So again I will repeat it that marginal product change in product of when I say labour that means change in product because of one unit extra labour. Similarly, marginal product of capital is change in total product because of one-unit extra capital.

So that is my marginal product of labour and marginal product of capital. So it says us when we hire or recruit one extra input be it labour or capital what is its implication, how much is it returning towards my production total product yes and then I can have an idea that whether we will go on increasing more and more labour or capital or we will stop at a particular point of time, yes because my budget has a limitation.

And I will have to make an efficient choice. I cannot take you know too many labours or too many capitals. We will learn it now. Say let us take a very simple table yes and then we will go how this average product and marginal product behaves.

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Ant. of (L) Labour	Ant. of (K) Capital	Total Product (Q)	Avg. Prod. (AP _L)	MP _L
0	10	0	-	-
1	10	100	$100/1=100$	$\frac{\Delta Q}{\Delta L} = \frac{100-0}{1-0}=100$
2	10	250	$250/2=125$	150
3	10	350	$350/3=116.7$	100
4	10	400	$400/4=100$	50
5	10	400	80	0
6	10	380	63.3	-20

The simple table, here will take amount of labour, let us take it as 0 1 2 3 4 5 6, amount of capital, amount of capital will keep it fixed, it is the short-run production function and two input production function, two factor, total product which is basically our Q, this is our L, this is our K, total product is say 0 100 250 350 400 400 380 yes. So what we see? Let us first calculate the average product or APL average product of labour APL.

I would not do average product of capital because nothing is changing essentially. Everything is similar but we still can calculate 0/10, 100/10, 250/10 but here capital does not really make sense that is primarily because labour is a variable factor. When I try to compare with different scenarios with different number of labour, I will you know look at the labour because that is where I am changing my decisions.

And over here I will keep marginal product of labour yes. Now so average product over here is 0 because anything you know it does not really make sense 0/0. The amount of labour over here is average product is 10/1 I mean sorry $100/1=100$ right, over here $250/2=125$, $350/3$ it should be 116.7, $400/4=100$ and so on 80 and it should be something like 63.3 yeah. So the marginal product again here it really does not matter.

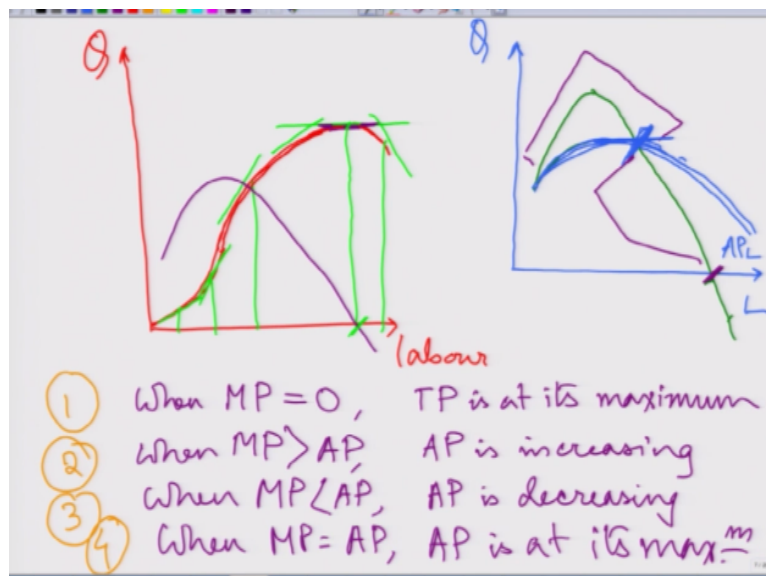
So when I do not have any labour, I incorporate the first labour. So again if you go back to the marginal utility class, here I will do $\Delta Q/\Delta L$, it is basically $\Delta Q/\Delta L$ and this $\Delta Q/\Delta L$ is basically you know $100-0/1-0$ is not it? Because I am incorporating one extra unit of labour and my total product changes from 0 to 100 when my labour increases from 0 to 1 keeping the number of capital same.

Therefore, my marginal product of labour is 100. You can also do it 100-0 if the changes is one unit you know every time. Over here it is 150, 100, 50, 0 -20 yes. So 100, 150, 100, 50, 0 and -20. Let us see how it really matters you know. So if I plot them graphically, I can see that the total product generally I am not plotting this particular graph but you know it is universally same.

So when I plot a total product curve, over here you can see it increases right, 0 to 100, 100 to 250, 250 to 350, 350 to 400, 400 it remains constant and then it comes down to 380. That means it is increasing, initially it is increasing at a very fast rate and then it you know when we look at the marginal productivity we can see if it has increased by 100 units 0 to 100, then 100 to 250, again at a faster rate and then 250 to 350 the rate slows down.

But it is increasing, 350 to 400 it is increasing, the rate is slower, 400 it does not increase anymore and then 380. We have a proverb in probably all the languages, I will just take the English one. Too many cooks spoil the food yes. So here we have seen that when we kept on increasing the number of labour, the production finally went down. So we have to decide till which point we will continue our production process, very nice.

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So when we plot a total product curve over here output in a given total product in a given period of time and over here you will mention your input labour or capital whatever. So initially it increases at an increasing rate so dy/dx is you know the first order derivative is

positive, the second order derivative is also positive, d^2y/dx^2 , so it is increasing at an increasing rate.

Next, it continues increasing but at a decreasing rate. You can see it is increasing at this phase but at a decreasing rate. At a certain point, it is being stagnant and then it comes down right. That is what we saw here. It is increasing at an increasing rate, first 100 then 150 then it started falling down 100, 50 increasing 0 and then it falls, decreased to 400 to 380 alright. If we take a very simple example you know say the production of a bike I have got a new bike.

So the mileage when I buy a new bike it is not really high you know but it increases at an increasing rate but after a certain year with its depreciation the mileage probably increases but then it increases at a decreasing rate and then after a certain while when it is old the mileage starts falling yeah. So it can be visible in all production process almost yeah across the world.

Now therefore this is my production process where you can see by the slope, the slopes are steeper and it starts getting flatter, at this point it is stagnant and then it is decreasing yes. So yeah it is decreasing yeah anyways. Now if we want to plot the average product curve how does it look like? An average product curve if you see here, it again increases 100 to 125 and then it falls right 125 to 116.7, 100, 80, 63.3.

But even if the marginal product goes to negative, average product does not go to negative because it would not because the total product is never negative yeah. Marginal product can be negative because it is showing the change. Change can be negative but the total product together even if it is falling down it can maximum be 0 if they you know stop operating but it cannot be negative right.

So a total product is an absolute number, so it cannot be negative. It is a non-negative form. Now therefore average product will never go to a negative point and hence as we observe here it is increasing and it is then decreasing, you can keep your average product curve something like inversely u-shaped curve average product and inversely u-shaped curve. You can write here as APL right.

Over here it is L, over here it is Q right. Now if we put this is labour yeah L, if we now plot marginal product, what we see in marginal product? Again it increases, see both of them are at 100 and then average product is at 125 and it goes higher 150 and it falls very fast you know. So the marginal product at the very initial stage will grow faster than the average product, will grow faster so it can start from the same point but it grows faster than the average product.

And then it comes down maybe till the negative point where it is like 400 to 380 that -20, it goes down to -20. So if my average product you know is inversely sloping, my marginal product is also inversely sloping but the slope is steeper for this marginal product curve. If I want to keep them together side by side and try to analyze what happens, I can observe few things.

Number 1, when marginal product=0, TP total product is at its maximum. Yes, at this point, marginal product if I plot it here maybe you know if I plot it here maybe it is something like this at this point. At this point, my marginal product goes to 0 and the total product is at its maximum. Number 2, when my marginal product is > average product, at this point when my marginal product you can see at this area when my marginal product is > the average product, average product is increasing.

When my marginal product is greater than the average product, average product is increasing. So I will say average product is increasing. Third, when my marginal product is < the average product, my average product is decreasing. At this point right my marginal product is < the average product and we can see my average product is decreasing anyways and when marginal product = average product, my average product is at its maximum.

So you can keep it like this, you know something like this, at this point your average product is maximum, yes at this point when marginal product is equals to your average product.