Data Analysis and Decision Making - I
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 46
Exponential utility function

A welcome back my dear friends and students; a very good morning good afternoon good evening to all of you and as you know this is the Data Analysis and Decision Making I course under NPTEL MOOC series and the total duration of the course will be spread over 12 weeks, which is 30 hours which basically is gets converted into 60 lectures. And we are in the 46th lecture that means, we have already completed 9 weeks. And each week as you know is there five lectures of each for half an hour and my name is Raghu Nandan Sengupta from the IME department IIT Kanpur.

So, we were discussing about utility concepts utility theories and generally we started that technically what we mean by utility the net value of a decision and utility basically has a utility function, which is an function in the sense that given a W amount of of investment wealth or input. And what is the net worth which I as an individual would basically get from it. So, that is basically utility function which is U of W and technically the U of W which is the utility function would also have a corresponding a distribution function.

So, based on the distribution function which is a set of or options which are there UWS and based on that that there are probabilities outcomes are there; that means, each option which is the outcome has a probability. And when we want to compare we basically find out the multiplication between the corresponding outcome and it is corresponding probabilities sum them up if we find out the expected value of the utility.

That is e of U W. And when we could talk about the expected utility we are trying to basically consider the higher the utility better for the decision maker is. And obviously, the second moment of the utility which is the variance can also be utilized to find out what if all the utilities are the same value which decision would you take in order to basically either so, called maximize your throughput or considering your utility it needs to maximize in whatever sense it is from your point of view. Or in other sense you would basically try to minimize the overall variance of the utility. Because utility as I said you have is basically a function which has a particular distribution.

Later on we with very few very simple hypothetical examples, we consider that utilities can be linear or quadratic. And I will come to the other four utility function later on which was basically discussed in their fag end of the last few slides of the 45th lecture. And when we are considering the utilities we consider the utility and distribution functions are interrelated even though in the problem solving or trying to analyze we considered that given a decision. The probabilities are basically considering the relative chance of the outcomes of the events. And in many of the cases if the utility changes still we are considering the chance to be not dependent on that or vice versa, but which may not be true in actual sense.

So, the general thing why we would consider this simplistic sense was to basically to make you understand that as the utility change changes the ranking system can change or as the probability changes the ranking system can change or if both changes obviously, the overall ranking system based on the fact that for all the ranking system based on the fact that you want to find out the expected value. The ranking would change definitely change.

Now, later on we considered that the marginal rates of utility and we consider the two important postulates of utility. One is the concept on non cessation more I give to you more you want. And rightly so, if you basically convert that in the concept of the utility function it would mean the first derivative of the utility or the marginal rate is positive, but this positive sense can have three different connotations depending on the risk perception. So, now, we I have been repeating about the risk perception it was basically means a set of persons who are there who love to take a risk a set of persons who are there who are indifferent to the concept of risk depending on two decisions and on the third set of persons are there who would basically be averse to taking a risk.

So, based on the fact the marginal rates are basically positive. It would definitely mean that the increases of the utility function that the marginal rates can be positive with an increasing sense can be positive with the constant sense can be positive with a negative sense. It means the increase in the increase of the utility function would mean that the double derivative is positive. Constant value for the increase of the utility function would mean the double derivative is constant or is 0 sorry. And in the in the sense we will we will find it why it is 0 later on when we are coming to the concept of risk perception.

So, it would be constant. And in the sense when the increase of the marginal rate is decreasing is at as a decreasing phase we will consider the double derivative as negative. And based on that we consider the human being of the decision maker has those three properties of risk any one of them love risk indifferent to risk and hate risk. Then we further on came to the concept of the absolute risk utility absolute risk aversion property and relative risk aversion property.

In the absolute sense and the relative sense we gave just the definitions of this functional form. And we also discussed that given the first derivative of the absolute risk aversion property of the first derivative of the relative risk aversion property how we can classify a human being as a risk lover person and risk indifferent person. And also we just to mention it to substantiate or basically mention it once again. When we are considering the concept of marginal rates we the those three curves we did discuss that the marginal rate is increasing and increasing rate marginal rate increasing at constant rate. And marginal rate is basically increasing at decreasing rate and we give diagrams very simplistic diagrams to in order to make a you understand.

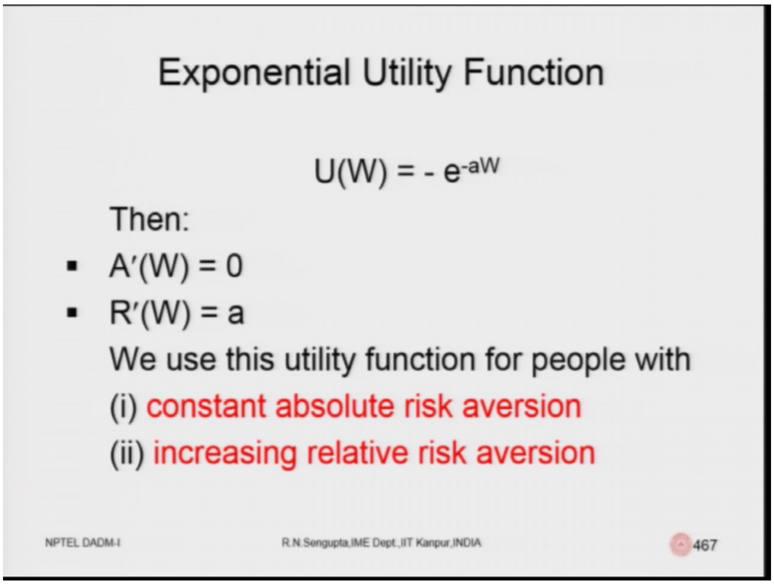
Then after defining the properties of e a prime which is absolute risk aversion and it is derivative r and r prime which is relative risk aversion property and it is derivative. We considered that what are the four important utility functions which are there for the study one is the quadratic utility function one is the power utility function what is the logarithmic utility function and one is the exponential utility functions. And for all four of them we did and we will do last part is left we will we considered that the hypothetical example we you can I I mentioned that please do it in excel sheet.

So, the first column would be the hypothetical values of w like 1 2 3 4 5 6 so, on and so, forth. In the next column you will basically have the corresponding U W values. So, for quadratic utility function considering some value of B you will find out w plus B into W square B can be negative positive whatever it is.

So, and then in the third column the fourth column you will find out the respective values of U prime and U double prime. And given U prime in a double prime which you basically calculate in the third and the fourth column you can correspondingly find out the value of A and R. And once in A R are found you can find out A prime and R prime and basically check the properties which you can double substantiate with the case of the method what mathematical formulations which are there for all the four utility functions, you can find out A A prime R R prime from the mathematical point of view also. And those who are basically substantiated both from the simple hypothetical example from the excel sheet as well as the calculation which we did.

So, let us continue the exponential utility functions.

(Refer Slide Time: 08:27)



So, in the exponential utility functions you have the utilization given as minus exponential to the that is to the power minus A W. And A is basically the constant value which you have in order to basically give the shape scale parameter for the utility for exponential utility function. If you calculate it you will find out if the given A and R A value is minus U double prime by U W U prime and R value is minus W into U double prime and U divided by U prime. You can find out the derivative of A and derivative of R. So, these values comes out to be for this exponential utility function they come out to be 0 and A.

So, 0 means that it has got a constant absolute risk aversion property. And R value R prime value being A it will basically mean it has a positive value of relative risk aversion property which is increasing relative risk aversion property. So, based on that we will again do the same type of simple diagrammatic representation using actual seat.

(Refer Slide Time: 09:38)

Exponential Utility Function (contd..)

W	U(W)	A(W)	A'(W)	R(W)	R'(W)
2.00	-1.65	-0.25	0.00	0.50	0.25
3.00	-2.12	-0.25	0.00	0.75	0.25
4.00	-2.72	-0.25	0.00	1.00	0.25
5.00	-3.49	-0.25	0.00	1.25	0.25
6.00	-4.48	-0.25	0.00	1.50	0.25
7.00	-5.75	-0.25	0.00	1.75	0.25
8.00	-7.39	-0.25	0.00	2.00	0.25
9.00	-9.49	-0.25	0.00	2.25	0.25
10.00	-12.18	-0.25	0.00	2.50	0.25
11.00	-15.64	-0.25	0.00	2.75	0.25

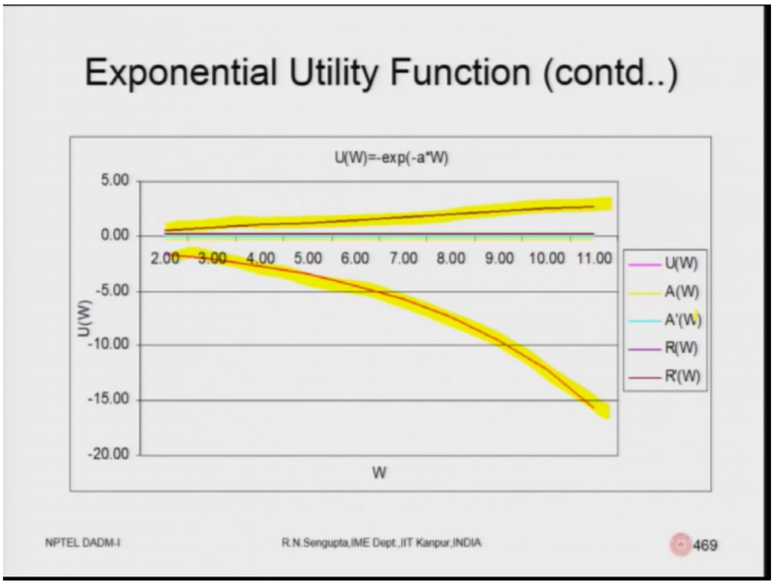
NPTEL DADM-I R.N.Sengupta,IME Dept.,IIT Kanpur,INDIA 468

So, we basically plot first write down the W values on the first column it can be 2 3 4 5 whatever it is. So, this value will be changed which are there. So, these values of 2 3 4 5 6 7 they could have been saved for example, 20 30 40 50 whatever it is, but I took simple values in order to make it much more simplistic for me to explain.

The next are column was basically the utility function which is minus e to the power minus A W with the certain value of A and we write down these values of U U. And then in the third and the fourth column which are not there third and the fourth columns which are not there we will basically consider the values of A and R and given this of sorry of of U prime and U double prime. So, those values will be given these values of U prime and U double prime U can find out the values of A here and R.

So, given A and R you can further on calculate the values of A prime and R prime. So, as rightly found out in the mathematic calculation A prime is coming in to 0 and R prime is coming to constant which is the a value which is 0.25.

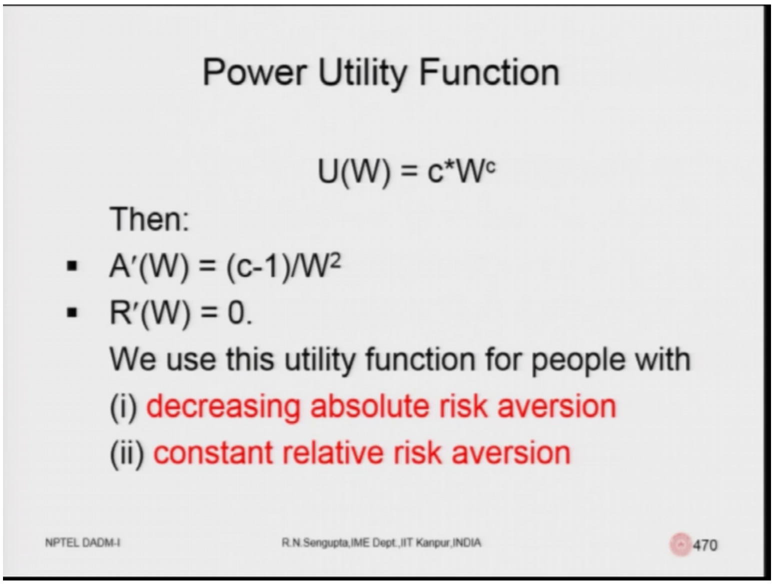
(Refer Slide Time: 11:24)



So, plot these values in the excel sheet and then draw the diagram. So, the pink yellow. So, I would I if you remember that I have mentioned very explicitly in 4 45th lecture please take these values in a simple excel sheet and draw them that will basically clear all the doubts you may have in order to understand the utility function.

You have basically the pink green greenish blue violet and brown corresponding to U U A A prime R and R prime. So, once you have that you can basically understand. So, this is basically the functional form of U. This is basically the functional form of R and all the values of C for example, the yellow one which is almost closer to 0 the greenish blue in blue one whichever almost closer to 0 and the brown one would basically give us the values of A prime A A A prime and R prime we can calculate them accordingly.

(Refer Slide Time: 12:36)

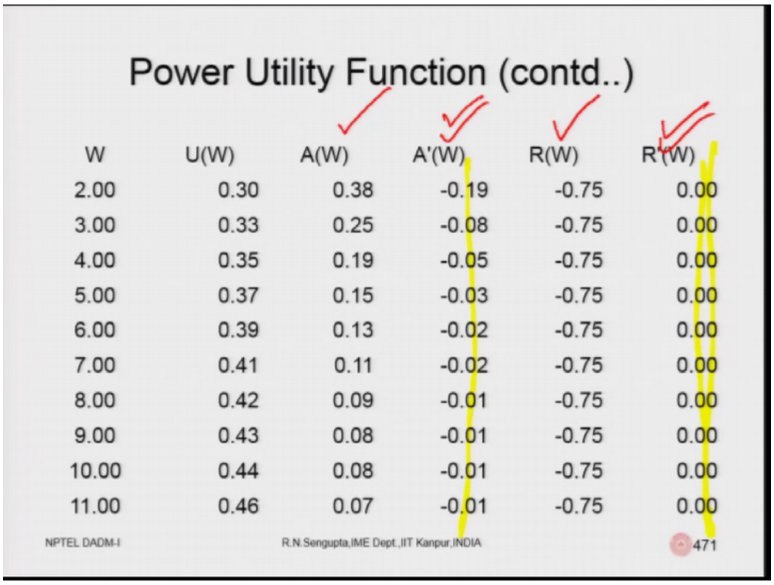


Finally we come to the power utility function. Power utility function is given by a constant value c multiplied by W to the power c. And then depending on how we are able to find out U, U prime and U double prime which can be calculated very easily we can basically and then in the next step calculate A and R. And then given A and R we can find out A prime and R prime which comes out to be as given.

So, A prime comes out to be c minus 1 divided by W square, W square is obviously, a positive. And c minus 1 would basically be the negative value or positive value depending on how what values of c would you have. So obviously, c would be less than one. And R prime is basically 0.

So, obviously, if the c minus 1 value is negative. So, you can immediately arrive at the conclusion. That this power utility function has decreasing absolute risk aversion property and as R prime is 0 obviously, it would have a constant relative risk aversion property and based on that you can comment.

(Refer Slide Time: 13:47)



Power Utility Function (contd..)

W	U(W)	A(W)	A'(W)	R(W)	R'(W)
2.00	0.30	0.38	-0.19	-0.75	0.00
3.00	0.33	0.25	-0.08	-0.75	0.00
4.00	0.35	0.19	-0.05	-0.75	0.00
5.00	0.37	0.15	-0.03	-0.75	0.00
6.00	0.39	0.13	-0.02	-0.75	0.00
7.00	0.41	0.11	-0.02	-0.75	0.00
8.00	0.42	0.09	-0.01	-0.75	0.00
9.00	0.43	0.08	-0.01	-0.75	0.00
10.00	0.44	0.08	-0.01	-0.75	0.00
11.00	0.46	0.07	-0.01	-0.75	0.00

NPTEL DADM-I R.N.Sengupta,IME Dept.,IIT Kanpur,INDIA 471

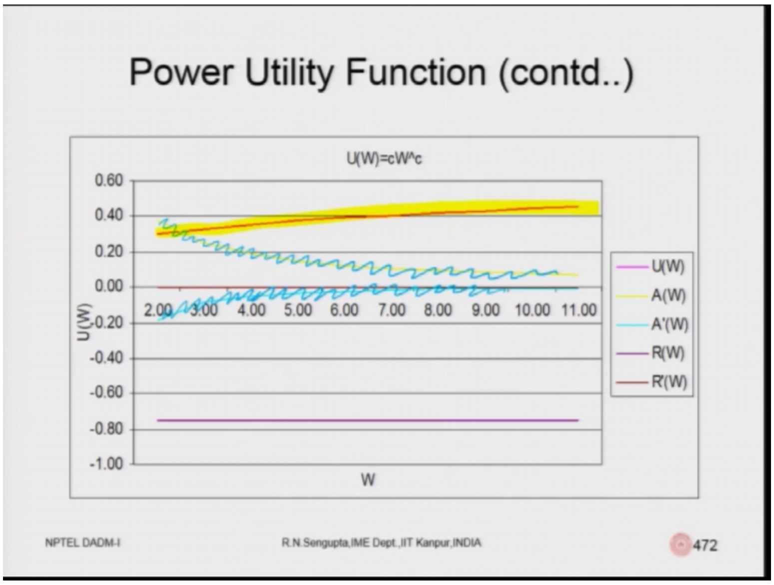
Again we use excel simple excel to find out the double checking of the theoretical results which we did given the utility function you want to double check that for different values of utility this. So, these values of R prime and A prime would be as they have been stated. So, in the first column again we find out write the values or W, in the next column you write down the values of U U W depending on the value of c.

Then in the third and the fourth column which I have omitted here you will basically find out the corresponding values of U prime and U double prime given this U prime and U double prime you can immediately calculate R A. And then you can fairly find out R and given A and E you can find out a double prime and R double prime.

So, let us double check the R double prime as per the calculation of the utility function which is power utility function they were 0. So, rightly so, they are 0. And the values of A bar prime was basically negative.

So, here it is negative. So, technically that can conclusions we arrived at using the utility function or the part utility function case we double check that, it really comes out to be true as we substantion that using a very simple hypothetical calculation in a experiment.

(Refer Slide Time: 15:21)

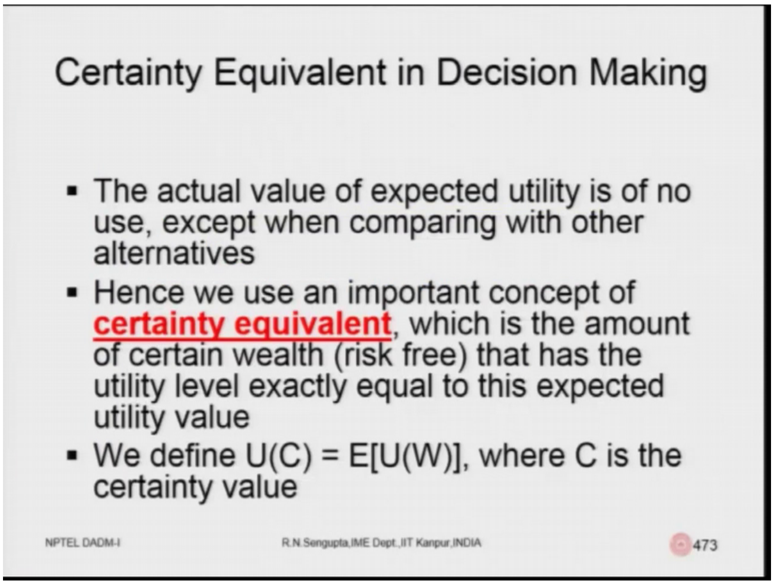


And in using these values of U then U prime U double prime we calculate further A A prime R R prime and we plot them.

Similarly, we follow the same nomenclature of color. The pink yellow bluish green violet and brown are the respective graphs for you A A prime R R prime. So, this is the U function. This is this it is this difficult to view.

So, basically market. So, this is the value. The yellow one which is basically A and the corresponding reddish blue one is basically the sorry the greenish blue one is basically for e prime. And the corresponding values brown and this violet would basically be for R and R prime. So, we can change the values over the left one column which is basically 2 3 4 5 change them and you can get different values.

(Refer Slide Time: 16:46)



Now we will consider the concept of certainty equivalent in decision making and what significance does it have for our calculation.

So, now the actual value of expected utility which you want to utilize is basically of no use to us because that is a very theoretical notion and technicaally when I am were I as a decision maker I am taking a decision, it does not mean that I sit down and calculate the expected utility value and then take a decision. But it has and some very great help in trying to understand that the how the utility function can be utilized and the expected concept can be utilized to draw conclusions and basically understand what the utility function says.

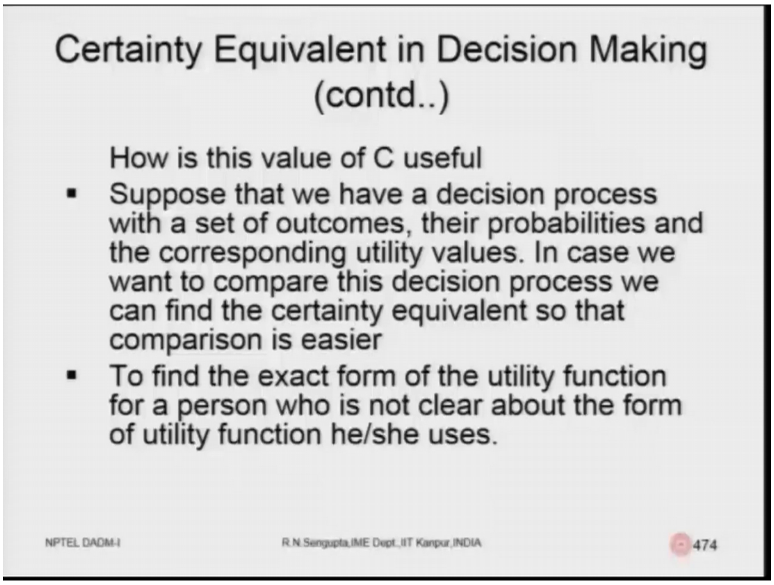
So, let me continue reading it the actual value of the expected utility is of no use except when comparing with other alternatives. Hence we use an important concept of certainty equivalent which is the amount of certain wealth or the risk free wealth that as the utility have a exactly equal to this expected utility value.

So, we will define that value of c as such that the expected value of utility which you see when it becomes exactly equal to the expected value of utility is that value based on which it can be sure to take the decision accordingly.

So, c would be that value such that the expected value of c would exactly match the expected value of the utilities, which means that that certainty value technically would have a probability of 1 because that is certain. Now that expected value which you will gain from the certain event should exactly match the overall outcome on their expected sense for the probabilistic outcome also.

So, if theoretically. If that value of c which is a theoretical concept is kept in front of us or kept in front of me or in front of any of one of you. And on the right hand side on the left hand side now on the right hand side there is an probabilistic decision. And if the expected value of the probabilistic decision exactly matches the expected value of the certainty event then that value of c would be the certainty equivalent which will basically balance my decision both for the probabilistic sense and the and the deterministic sense.

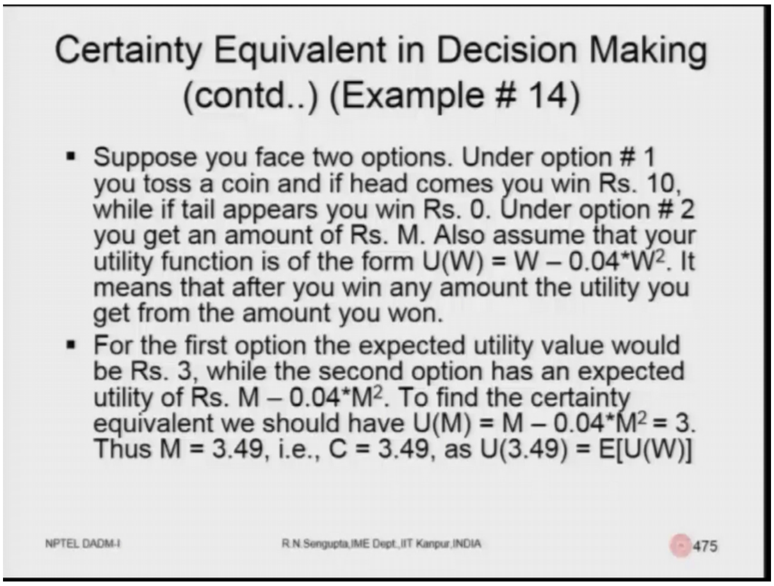
(Refer Slide Time: 19:08)



So, how is the value of C is important. Suppose that we have a decision process with a set of outcomes. Their probabilities and the corresponding utility values are known to me. In case if you want to compare this decision process we can find the certain equivalent so, that the comparison is easy for us to make and we can understand it.

To find the exact form of the utility for a person who is not clear about the form of the utility function that is also much easier way to make his him or her understand that means, rather than trying to basically make one understand what is the utility function what is the concept of utility if we have a certainty value we will see that look. If you are indifferent between the certainty value it is outcome and the expected value. Then you can consider that certainty value is a certain sort of 1 to 1 correspondence between the non deterministic event and the certainty value. And based on that you can always think the certain value is the one which you are taken notionally even if your actual decision is basically probabilistic.

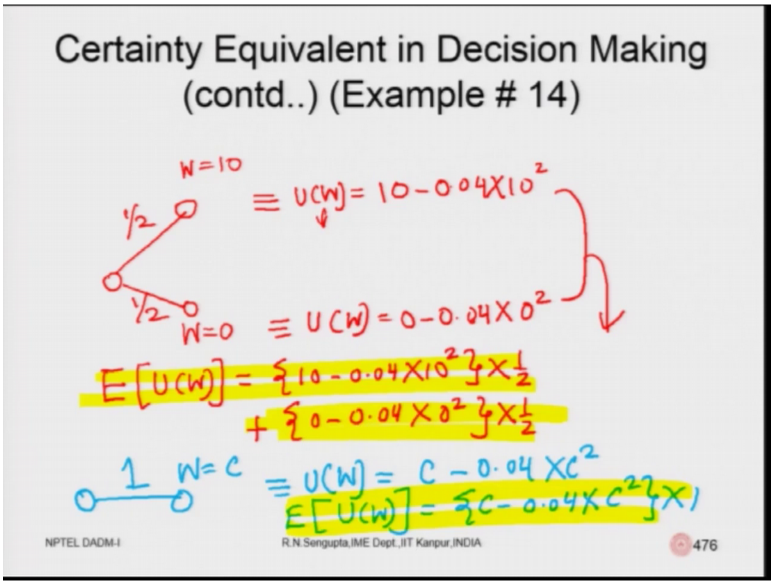
(Refer Slide Time: 20:16)



So, let us consider an example suppose you face two options under option one you toss a coin and if a head comes you win rupees 10. While if a tail appears you win 0. Under option two you get an amount of rupees M also assume that your utility function is the is of the form of quadratic which is U W minus 0.04 which is the value of B into W square. It it means then after you win any amount the utility you get from the amount you would be given by the utility.

So, say for example, you come you basically are winning 10 and 0 respectively for the case of a unbiased coin being tossed and a head at a tail. So, when I basically win 10 the 10 value which I get for myself is the and not the actual value based on which I will accrue the decision. So, obviously, 10 is there on the table and when I see the value of 10 a 10 rupee note. It would mean that the overall value of the 10 would basically be accruing to me and utility based on the fact that what is the utility function.

(Refer Slide Time: 21:27)



So, let me check the value. So, it is 10 and 0 and the utility is w minus 0.04 double square. So, this is the W is 10, W value which is coming out to be 0. This is probability half this is probability half. And the utility function which you will get from this case is 10 minus 0.04 into 10 square. And in this case is utility is equal to 0 minus 0.04 into 0 square.

So, when I basically find out the utility for this expected value for the utility would be consider this I mention as say for example, I mentioned it as one minute let me write it. So, it will be easy. So, it will be 10 minus 0.04 into 10 square multiplied by half plus 0 minus 0.04 into 0 square into half.

So, once I find it out I find out the utility function. Now on the other hand let me change the color and basically come to the certainty value. Now certainty value would basically have a decision like this. So, this probability is 1 and in W value would basically equal to c. Now when I want to find out the utility function for W in this case it will be C minus 0.04 into C square I want to find out the expected value.

So, expected value in this case you would be C minus 0.04 multiplied by C square into 1.

Now, what should I do I should equate them. What do I equate? I would basically equate the value which is there everything is known here. I equate with this here everything is known here except the value of C. So, you basically a quadratic find out C value and that

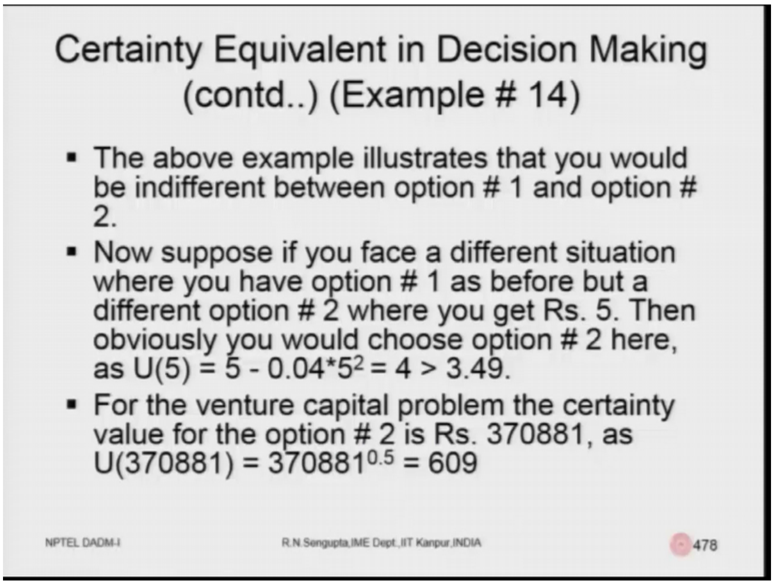
C value is the certainty equivalent. It means that if this non deterministic event is given on the right on the left hand side I have a value of C, and if I am able to calculate one to one correspondence and do the calculation exactly which means I will be indifferent between the value of C which is given and the overall expected value which is there for that gamble which we want to analyze. And obviously, you would be interested to know if the values of C increases or decreases what happens I will come to that later.

So, suppose you face two options under option one you toss a coin if head comes you win 10. And while if a tail appears you win 0 also on an option 2 you get an amount of rupees M. M is the value of C which we are talking about. Also assume that your utilities of the U W is equal to W minus 0.04 into W square it would means that after you win any amount the utility you get from the amount you won is given by that utility function.

For the first option the expected utility value would be 3 because you are basically trying to analyze the utility function based on the calculation. That as I showed it will be 10 minus 0.4 into 10 square in the bracket multiplied by half this is the probability. Plus the other option is 0. So, it would be 0 minus 0.04 into 0 square bracket that whole thing into half that will be equivalent to the value of M, it is equivalent terms in the utility which will be M minus 0.04 into M square.

To find out the certainty equivalent we should have the U M value utility value would be equal to 3.49. So, 3 rupees 49 paisa or 3.49 units if I place on the left hand side on the right hand side of that gamble where half and half probabilities are there for the tossing a coin. And the values I am getting is 10 and 0 then I will basically be indifferent between them this value of 3.49 is the certainty value which I have.

(Refer Slide Time: 26:25)



The above example illustrates that you would be indifferent between option 1 and option 2 means considering is 3.49. Now suppose if you face at different situations, this is what I mentioned that if it is greater or less than C where you have option one as before, but a different option number two where you get 5. Then obviously, you would choose option 2 because in that case the value which you basically get for the certainty value if you calculate it comes out to be 4.

So, now for would basically be greater than 3.49. So, when you are trying to find out the expected value of that gamble with respect to the certainty value so, called value I will not use the word certainty value is greater than 3.49. I will be more inclined to take a decision where in the long run the expected value is basically comes out in the long run, the value which accrues to me based on the utility function which is there for me would be higher.

Now, if you remember the venture capital example where you had one government bond and they were basically three other options with probabilities of 0.2 0.4 0.4 based on that if you do that example the utility function the utility function was W to the power half. And based on that you find at 3 the certain value comes out to be 609 rupees or whatever units. So, with this I will end this first class of the 10th week and continue discussion moreover the utility function and then go into the multivariable statistics later on.

Thank you very much and have a nice day.