Data Analysis and Decision Making - I
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 45
Utility functions

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you and as you know this is the Data Analysis and Decision Making I course under NPTEL MOOC and this total course is for 12 weeks which is total number of hours is 30 and I have which would be total 60 lectures, each week we have 5 lectures each being for half an hour and as you know this is the for 45th lecture that means. We are in the last class for the ninth week and as usual they would be assignments after the each weeks. So, they are you obviously, will get the ninth week assignments also and I am Raghu Nandan Sengupta from IMA Department, IIT Kanpur.

So, if you would remember we were discussing about Utility Theory, concept of non satiation, concept of risk aversion, risk loving, risk neutral then we discussed that giving the example of a fair gamble where there is a coin which is unbiased, probabilities it is an half enough there is an another coin.

In case 2 these probability is 1 biased coin and they have investments which you are doing per they are only of 1 unit what decisions you are taking, so, for 1 unit. And the outcomes in the unbiased case which is the fair gamble is probability is half and half the outcomes of 2 in 0 and for the case when it is biased one there is only one outcome the outcome is 1. So, the expected values between both the cases is 1 and 1.

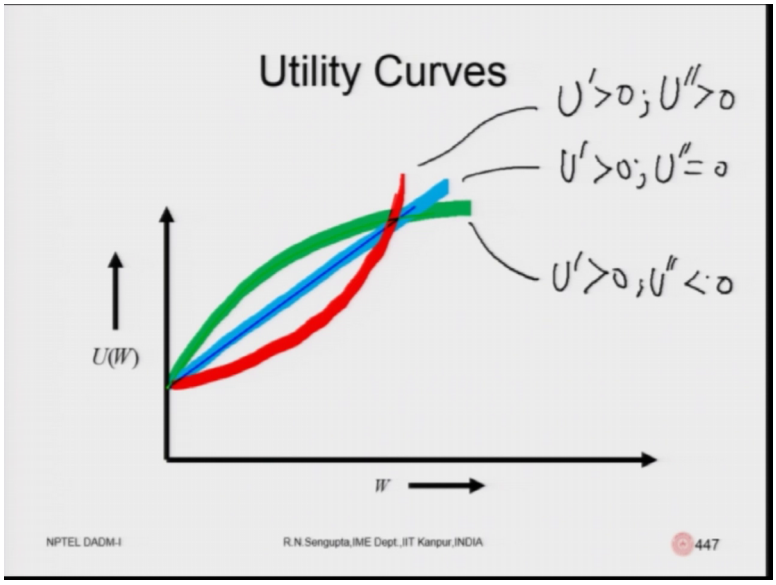
Now based on that we said that if intrinsically the person is willing to take the gamble then he is willing to he is a risk seeker. The person is indifferent between the gamble and the certainty is certain or the deterministic case is basically in different person and the person who wants to take the certain event, where the biased coin is there in that case the person is in basically trying to avoid the risk.

Now, we also said 2 and other important points is non satiation more I give more you want which means the first derivative would always being is greater than 0. But this property of whether the person wants to take the risk wants to basically in be indifferent

and wants to run away from the risk would basically depend on the second derivative. So, the second derivative being greater than 0, being equal to 0 and being less than 0 would basically classify the human being as willing to love the risk, take the risk, risk seeker.

On the second case, when it is in indifferent would be the case when the second derivative is 0 and the third and the last case would be if the person if wants to avoid the risk is a risk hater, in that case the second derivative would be less than 0 and how it is possible I will show you the graphs accordingly. So, consider the utility curves.

(Refer Slide Time: 03:20)

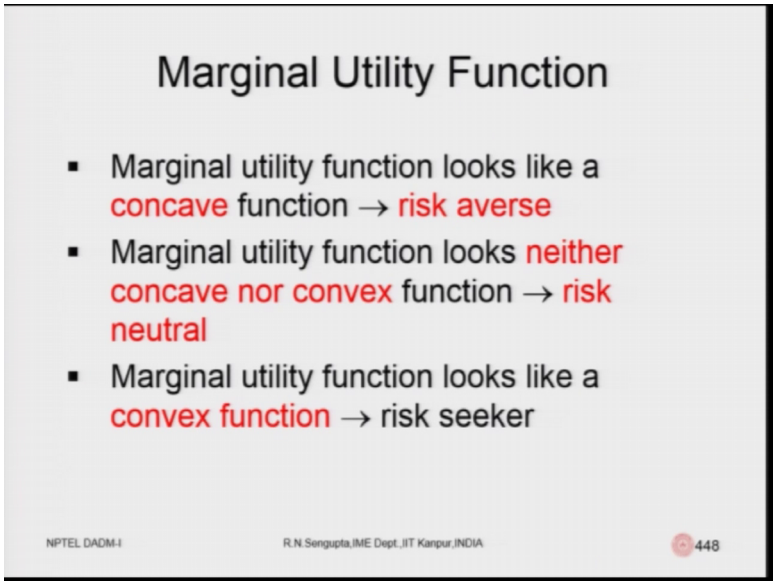


So, along the x axis we plot W which is the wealth any unit and along the y axis we plot the U of W which is the utility function. So, we have 3 graphs; the green one, the blue one and the red one. What are they? They would become very apparent and it is no such apparent reason why I have drawn from a certain value on the y axis. This is just to give you a notion that they can start at any point it could have been started at origin also.

Now, based on that, we want to analyze. So, watch the slope of the graphs I will use this different colours corresponding to the graphs. So, I will use the green one and I tell; so, it is increasing, but increasing at a slower rate. Now, consider the blue one hopefully I am able to draw it, this is the one which is increasing. So, basically the green one is for increasing under decreasing rate, blue one is increasing at a constant rate, rate one is increasing at an increasing rate.

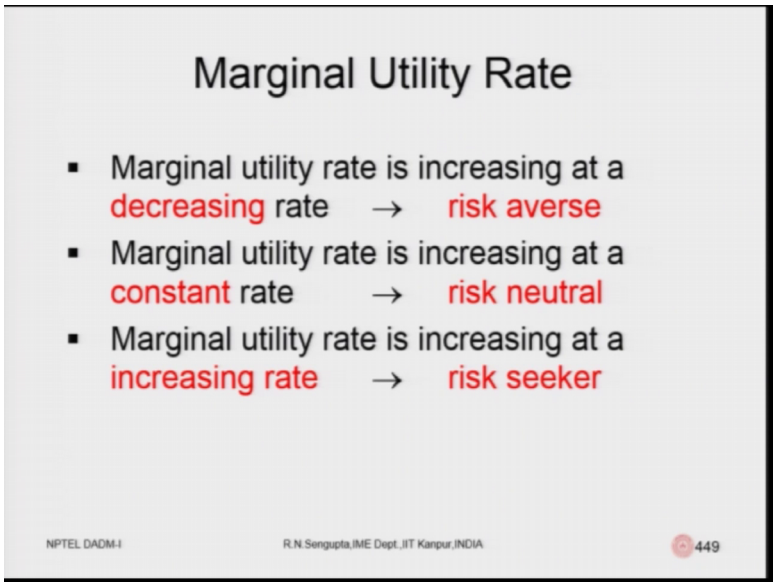
So, obviously, it would be very apparent here. For all these graphs the red one and you draw the red one first, the blue one and the green one on this case. Now, come to the second derivative in the red one, the blue one and the green one; I am not drawing W sorry, it would be easy of. So, this would become very apparent in the discussion which we have.

(Refer Slide Time: 06:06).



So, the marginal utility functions looks like a concave when it is concave is a risk covers. So, I am basically running out the risk margin utility functions which is the based on the first derivative. So, these were the first derivative which we do. The actual function and then based on that we find out the derivative. The marginal utility function looks neither concave nor convex is a risk neutral person and the marginal utility function looks like a convex function would basically be a risk seeker. So, how would you; it is like this.

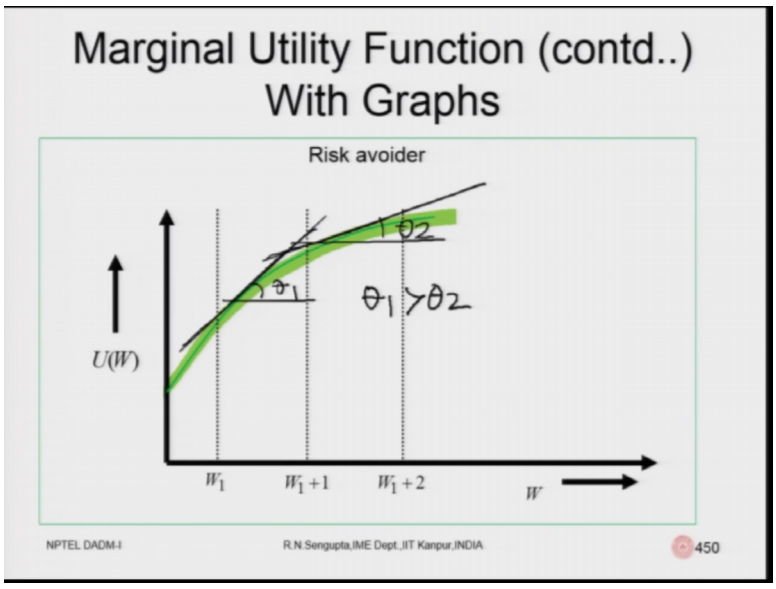
(Refer Slide Time: 06:42).



So, this is the same thing sorry there is a repetition ok, its concave not the repetition in the sense that I have basically dealt with 2 topics differently. So, the marginal rate is increasing and increasing decreasing rate would be the risk averse person. Marginal utility rate would be increasing at a constant rate is the risk neutral person and marginal utility is increasing at an increasing rate it is risk seeker.

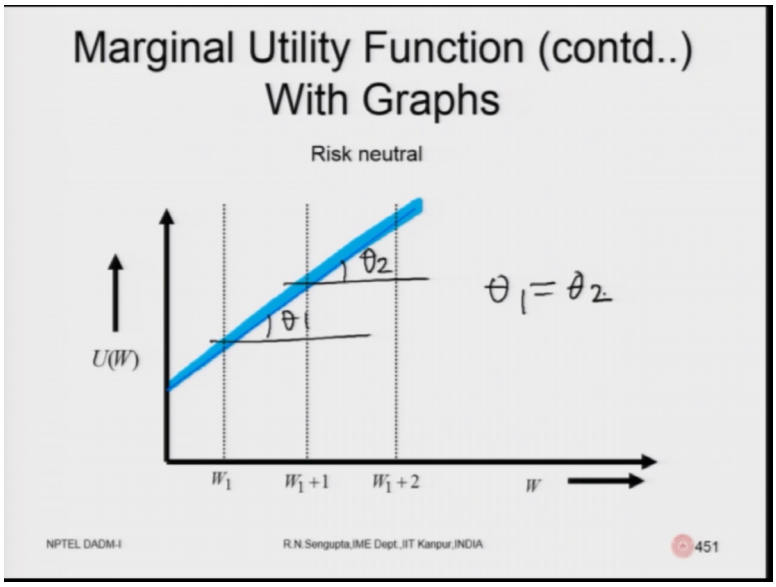
So, increasing decreasing risk averse, increasing constant a risk neutral and increasing with a risk seeker. So, let us see the graphs.

(Refer Slide Time: 07:18).



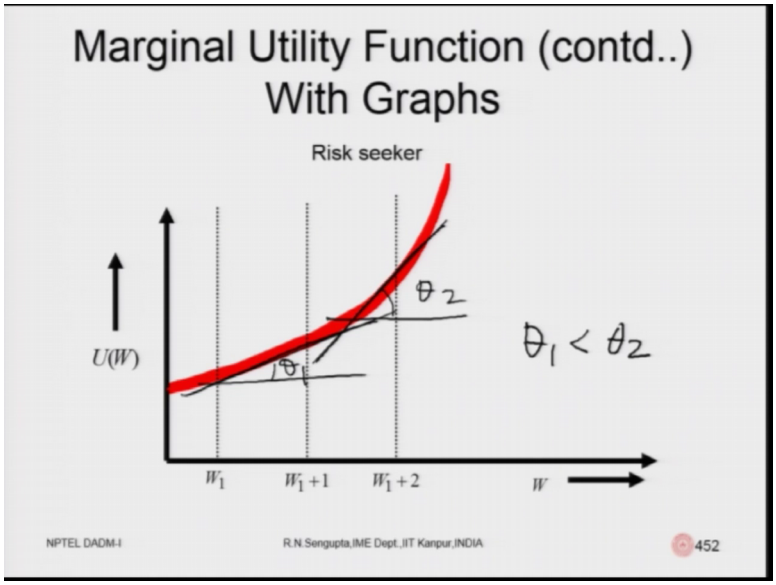
So, this is the green one if you remember. This is the risk a wider or risk hater. So, if you draw the module risk, so, this is theta 1. Again I draw, this is theta 2; in this case theta 1 is greater than theta 2 because the tan of this angle is to be considered.

(Refer Slide Time: 07:57).



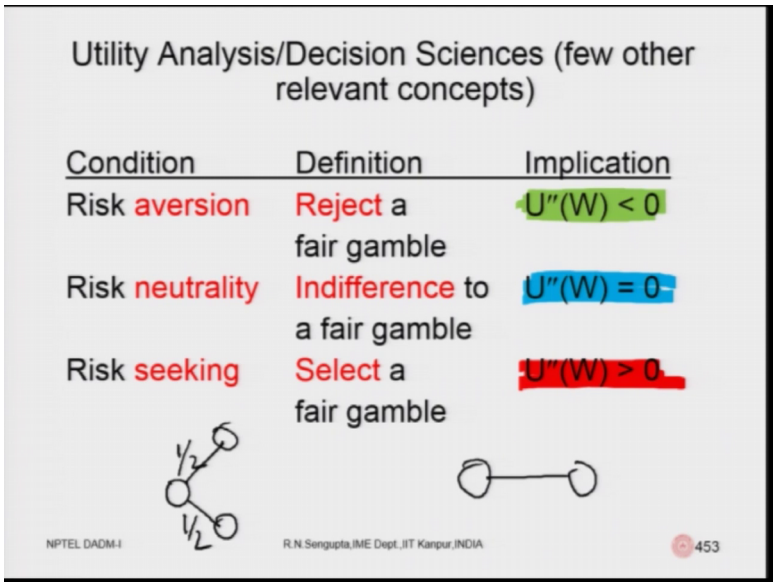
Then I come to the risk neutral person to the tan, in this case seem so, it does not matter.

(Refer Slide Time: 08:24)



Now, come to the risk lover. This is theta 1, theta 2. So, this is a risk seeker person. So, concave convex and neither concave neither convex.

(Refer Slide Time: 09:01)

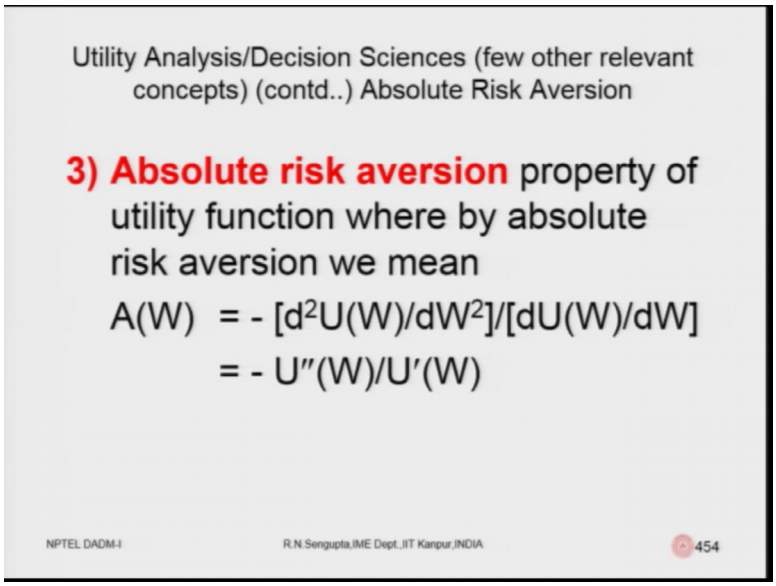


Now, few other continuing the discussion; so, risk aversion person would be reject a fair gamble. If you remember I had discussed that considering there is; so, there is gamble and in this case this is a certainty value. So, you really will reject the gamble, you are going for the sheer event because you think even with the probabilities are same probabilities the same since it is half and half you will think that you will be getting the downward trend outcome hence you are avoiding risk you are a risk aversion person.

Risk neutral person would be indifferent to the fair gamble; that means, whether you get the gamble do not get the gamble, you will basically be equally disposed in both the sense and risk seeking person would be the one who would be looking at the upward trend in their in on the gamble and see look at the fair gamble as the choice.

So, again as I mentioned, it will be let me check the colour. So, to be easy for you to understand; so, it is green one was basically so, risk avoider. So, we are right. So, this is the risk avoider person. This is the risk neutral person. The second derivative is 0 and this is the risk seeking person because the second derivative phase greater than 0.

(Refer Slide Time: 10:43).

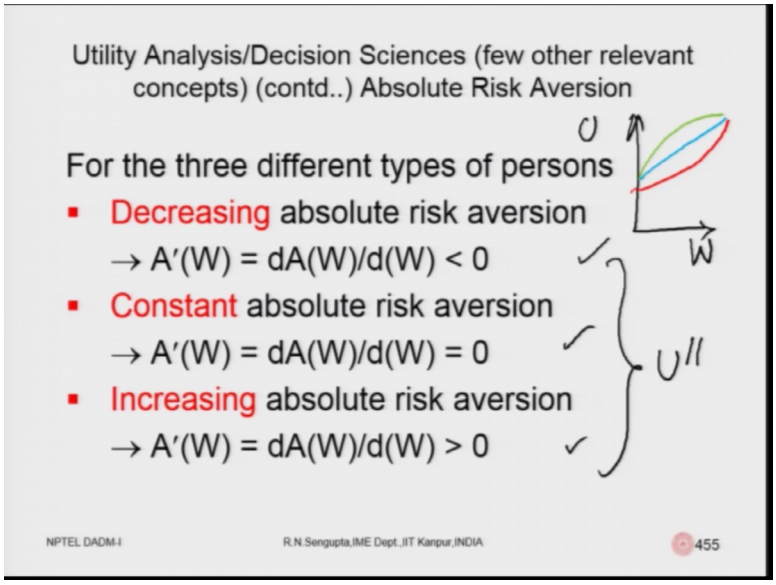


Now, we will discuss 2 important properties some of the risk perspective. One is absolute risk aversion property and one is the relative risk aversion property. So, absolute risk aversion property what are the things I will come to that later. So, in the absolute risk aversion property of a utility well my absolute risk aversion we technically mean that we want to find out the negative of the ratio of the second differential to the first differential.

So, now the first differential is obvious which is in the numerator is always I am not marking it is always greater than 0. So, your property of absolute risk aversion would depend on the derivative of the second property; whether its positive 0 and or negative and that would basically dictate that what would be the absolute risk aversion property and the derivative of the same which is important for us to understand.

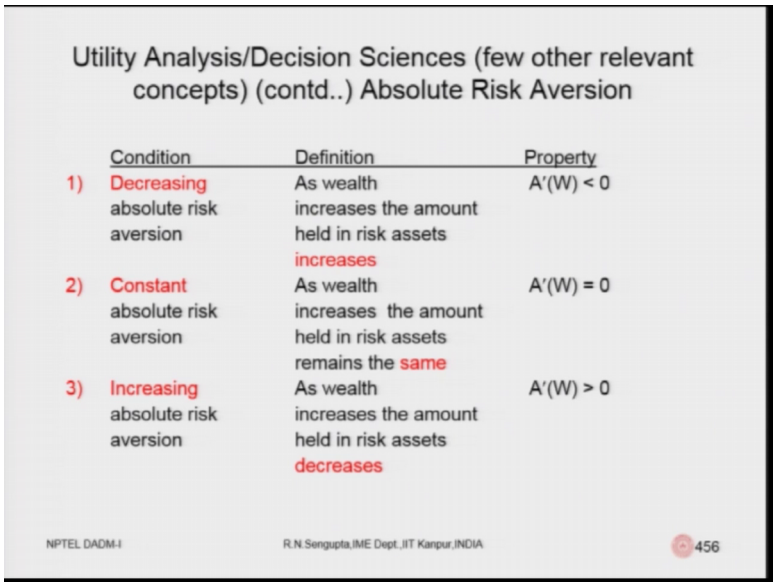
So, what we need to find out is basically first fine or the absolute risk aversion property and then use there on ratio sorry the derivative of the absolute risk aversion property based on which you would basically be able to club. A decision maker as a person who wants to take a risk, wants to basically be indifferent to risk and wants to basically avoid the risk. And obviously, that would come from the second derivative with the first derivative would always be is greater than 0. So, obviously, the second derivative will dictate that what is the A and A prime property.

(Refer Slide Time: 12:12).



Continuing absolute risk aversion, for the 3 different types of persons you will basically have a decreasing absolute risk aversion property would mean that the person has in the absolute sense his or her level of risk is absolute is decreasing. Constant absolute risk aversion property would be mean that the first derivative of A which is A prime would be 0 and increasing absolute risk aversion property would be the case where the A prime value is basically greater than 0 and then you will basically come to the. So, in all these cases what would basically be dictating is the U double prime which is the derivative of the marginal rates and if you remember that graphs. This is for the constant one and this is the increasing one. So, you are trying to basically major W and here U W in the x and y direction respectively.

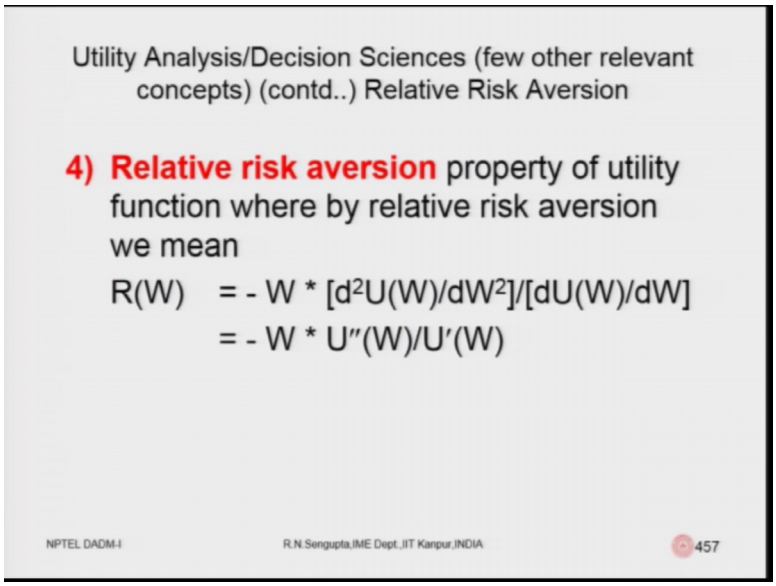
(Refer Slide Time: 13:40).



Now, in the implication from the qualitative point of view how would you state it? So, decreasing absolute risk aversion property would mean that as wealth increases, the amount helped in risky assets increases. So, this is decreasing. So, if you are decreasing so; obviously, it would mean that you are A prime is less than 0 and as wealth increases the amount or held in risk assets increases; that means, you are trying to basically more and more what towards risk run towards risk.

Constant absolute risk aversion property; obviously, would mean that A prime is 0 which means that at wealth increases the amount held in risk asset remains the same in absolute sense. And increasing absolute risk aversion property would mean that A prime is greater than 0 which means that as wealth increases the amount of. So, called wealth held in risk asset decreases; so, will come to this in more details.

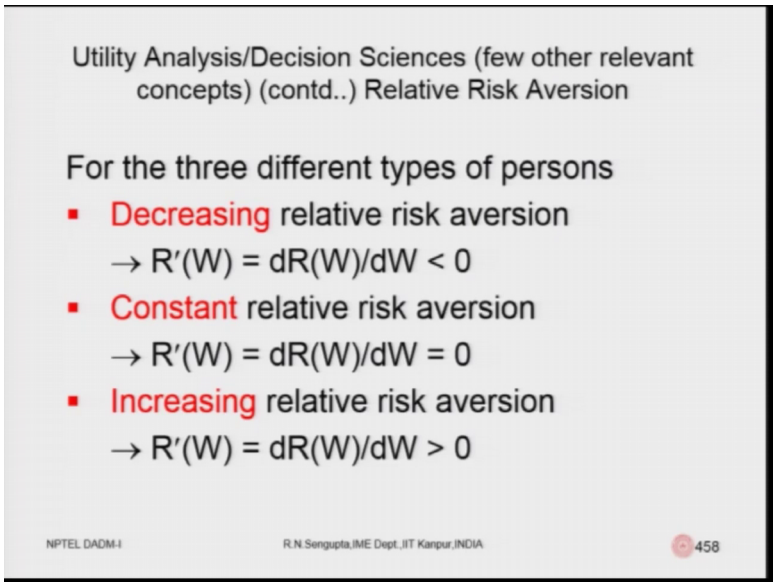
(Refer Slide Time: 14:39)



Now, we will consider the fourth properties from risk relative risk aversion property which means, that is the property of the utility function whereby the relative risk aversion would mean by minus of W multi not minus W multiplied by the concept of A; which means, that if you are multiplying my A with W then; obviously, the property of A or property of R would be in the way because W would be related in the sense very simple sense way. Because U prime is always positive and what is dictating the sign of A prime with a greater than 0 less than 0 equal to 0 would only depend on U double prime and; obviously, the minus sign is therefore, both R and A that is immaterial.

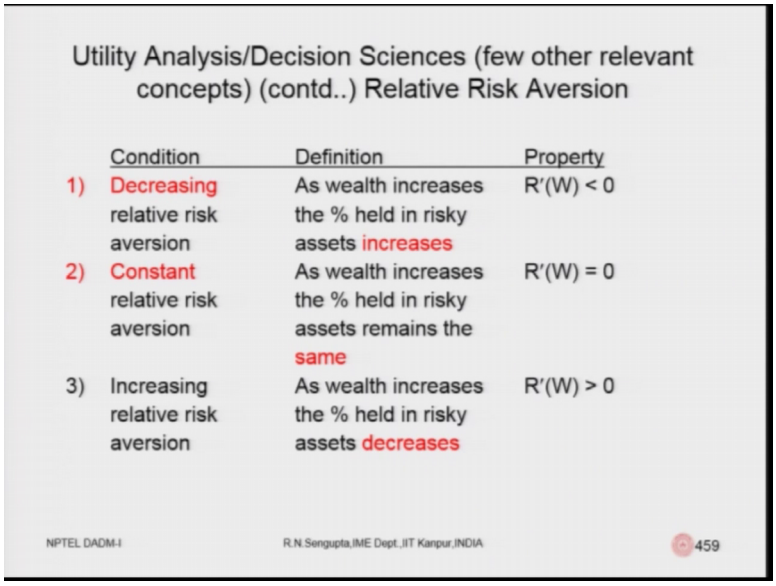
But also remember W is positive because that is the way have wealth. So, any properties coming from A prime and A would also be reflected in R and R prime. So, because the concept of derivative when you are taking, W would not have an effect on the derivative sense because it is positive.

(Refer Slide Time: 15:59).



So, for with this 3 is any person as you know we define him or her as risk loving risk neutral and risk heater. So, from the point of view these 3 different types of persons would be clubbed as decreasing relative risk aversion property constant relative risk aversion property and increasing relative risk aversion property. So, in that case you will have basically R prime as less than 0 for decreasing relative risk aversion property constant relative risk aversion property would means R prime is equal to 0 and increasing relative risk aversion property would be the case when R prime is greater than 0. Based on this we can basically find it out accordingly.

(Refer Slide Time: 16:46).

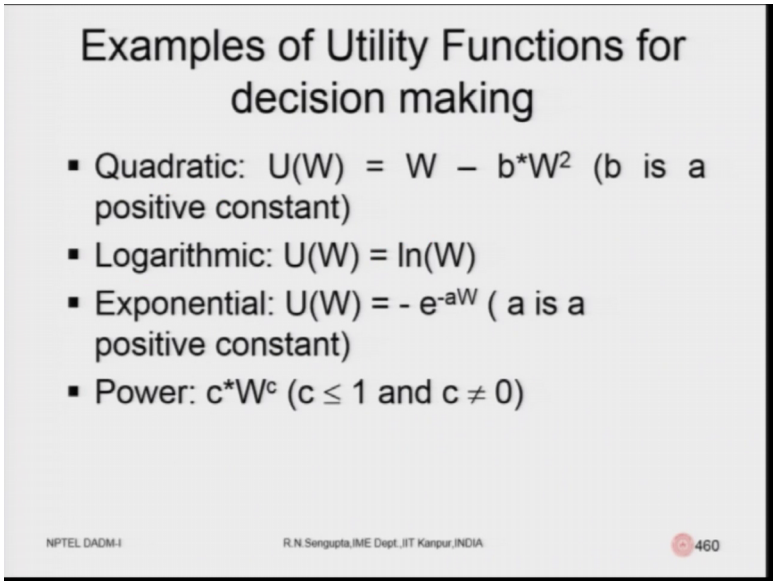


Now, when we basically go in trying to basic understand in the qualitative sense the conditions would be as it is the same thing. The first the conditions on left column the middle column with the definition and basically the properties would be discussed in the numerical way how you analyze would be the last column.

So, decreasing relative risk aversion property would basically mean as the wealth increases the percentage held. In risk asset would increase, which means that R prime would be less than 0, for the constant relative risk aversion property would be as wealth increases the percentage held risk assets remain the same and; obviously, in that case R prime would be equal to 0 because constant is constant value.

I am just giving the definitions I am going to come to the solutions problems in later on, please have patience. And increasing relative risk aversion property would mean that as wealth increases the percentage held in risky assets decreases which means, R prime would be greater than 0 and we will utilize this properties accordingly.

(Refer Slide Time: 17:48).



Now, were here is where we utilize. So, we will consider four different examples of utility function very simple one. One is the quadratic one, one is the logarithm one, third is the exponential one and the fourth one is the power utility function. So, in the quadratic utility function what we have is basically is a, of a quadratic form. So, it will basically be a x squared plus bx plus c, in general x being W. So, but the equation of

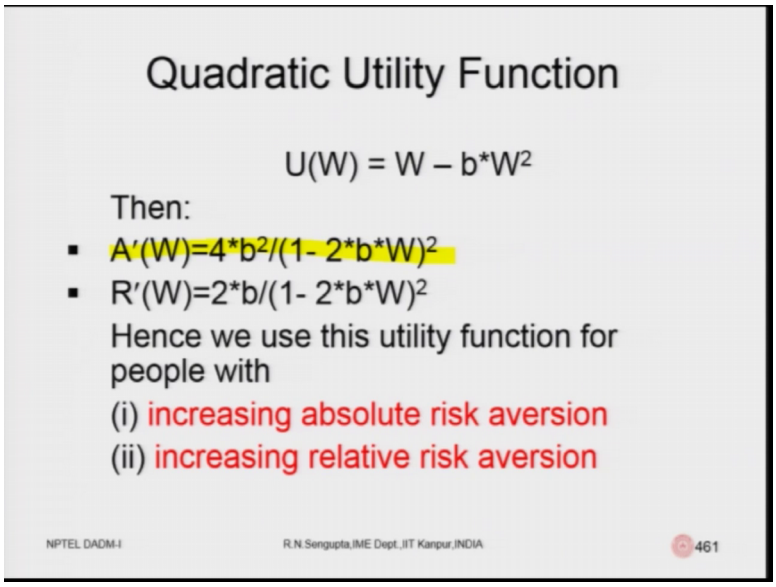
form will be W minus b star W square or W plus b of W square where that value of b can be positive and make you depending on how you want to basically express it.

So, b is a positive quantity in this case so; obviously, it would mean that minus b of b W square as b is positive would be minus because the minus sign would dictate in the logarithmic case it is the nepean log. So, U of W is ln of W and in the exponential case you will basically have the exponential function which is minus e to the power minus a dot aw where the a is basically a positive quantity and in the power sense it will be c which is a constant into W to the power c I will come to these definitions more, but I am just stating them in the in mathematical format.

So, first let us concentrate on the quadratic and then obviously, the properties of relative risk aversion, then R prime, absolute risk aversion A prime, the property is what is the derivative means, first derivative second derivative of U, all these things would now become clear for all these four different utility functions and we will solve them very simply.

So, first we will going to consider the quadratic utility function.

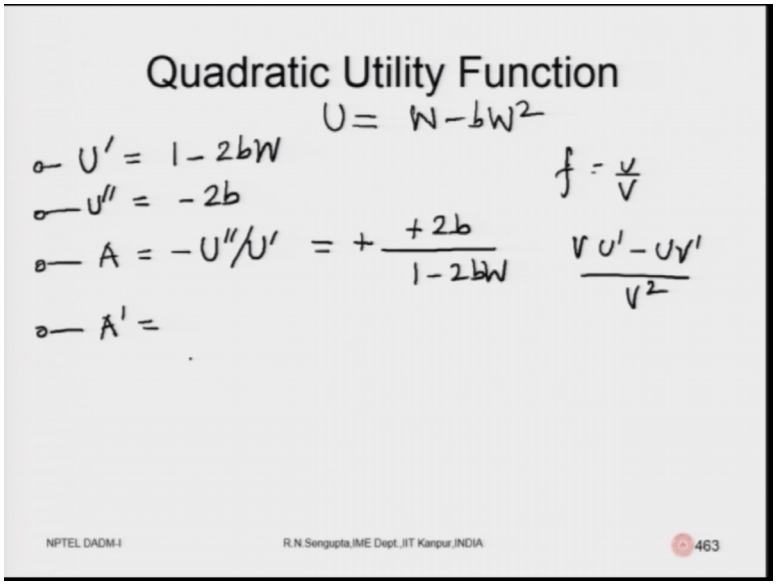
(Refer Slide Time: 19:37)



So, the quadratic utility function is given by W minus into b into W square. So, so let us basically make us blank slides and solve it.

So, we have made the 2 blank slides. Now, I will basically go through the derivation. So, first let me write down the utility function quadratic one.

(Refer Slide Time: 20:01)



So, your are to I will only write U not the W 1 is going to W minus b W square. So, first U prime is equal to 1 minus 2 b W, U double prime is equal to minus 2 b. So, it becomes 1 minus 2 b W and this.

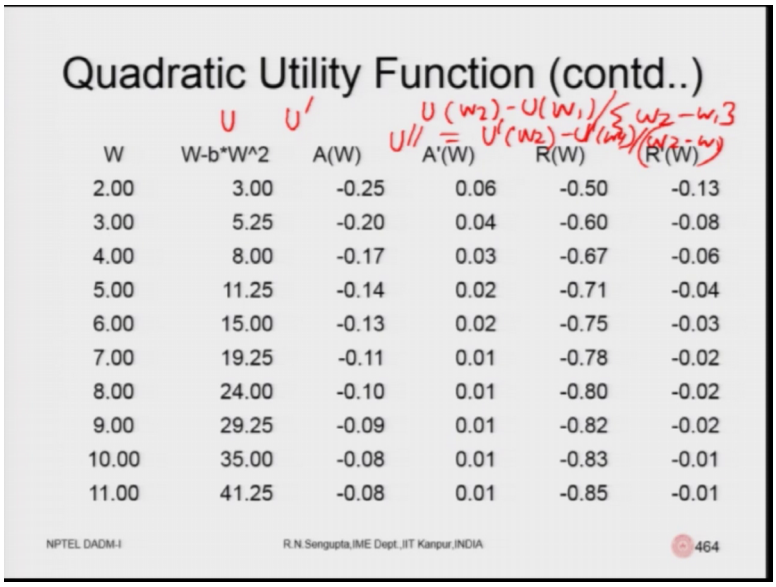
Now, I need to find out A; is equal to minus U double prime by U equal to minus of I will write it is as minus 2 b and U prime is 1 minus 2 b W. So, this become plus. Now, basically I will solve it for A prime. So, once you solve it from pay prime. So, we will use the function as of f as U by V. So, V into U prime minus the actual calculations U into V prime by V square. So, once you solve it, the actual values which you will get is U prime is this.

So, let us concentrate. So, the numerator is always positive because W b square and the denominator is always positive because the square term. So, whether b is positive or negative is not going to matter. So, A prime would basically be positive which means, it does not increasing absolute risk aversion property.

Now, when we find out R in the R case when the relative one; R means, relative risk aversion basically mine that A value multiplied by W, W is always positive. So, finding an R prime would give me in the value which is now in the numerator view of to be

divided by again a square term square term would not matter. So, the b value if it is positive we will have basically positive R prime. Hence, again it is increasing relative risk aversion property.

(Refer Slide Time: 22:16).



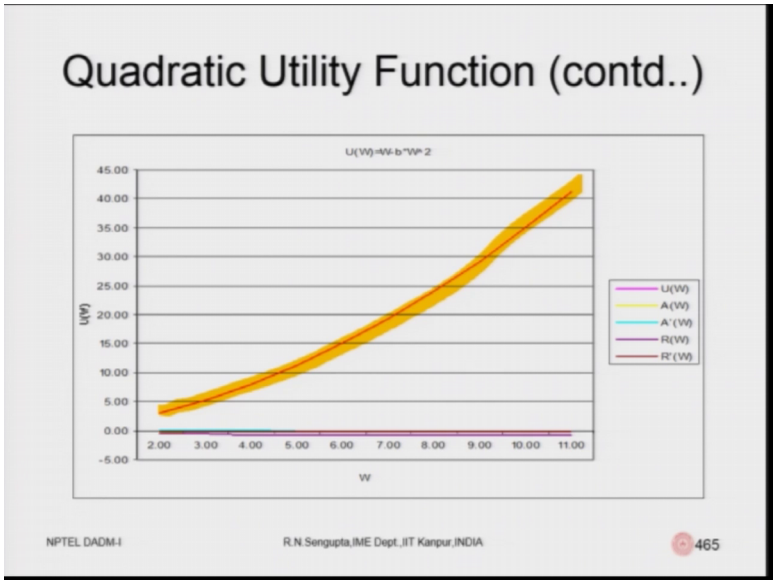
Now, this is what I do in order to basically make things understand and anybody can solve it in excel sheet. So, what we have is basically we write down the values of W on the first column, the second column we have basically the values of U for the utility function and the corresponding third and fourth column I basically a not give in here, but let me concentrate on how we can make it.

So; obviously, this is yes this is U. So, once you U have you will basically have U prime which is not this. So, U prime would be technically you know the prime value would be U could respect to W 2 minus U with respect to W 1; the whole thing divide by W 2 minus W 1. So, this would be 5.25 minus 3 divided by 3 minus 2 that will give you the first derivative for the first case.

The next would be 8 minus 5.25 divided by 4 minus 3 that would be the second value of the first derivative. Then it will 11.25 minus 8 divided by 5 minus 4 which is the third value the first derivative. So, based on that you can find out the U prime; again I want to basically find out U double prime, it would basically be the U prime for W 2 minus U prime for W 1 divided by W 2 minus W 1. Find out these values and you could proceed accordingly, again the same thing.

Then I use the formulas for A, A is minus U double prime by U prime, put those values find out A. Then find out the derivative of A which is A prime, again the same thing. The A value of W 2 minus A value of W 2 1 divided by W 2 minus W 1, you will give you the A prime value. Similar, I write down the R and the R prime values. So, you have the first column W, second column U, third and fourth column are not given here which is U and U prime, then the third column or technically the fifth column which is there in front of you where I am hovering my electronic pen. So, this is A prime A, sorry A prime are R prime the values would be given and you can plot them. So, let us plot in excel.

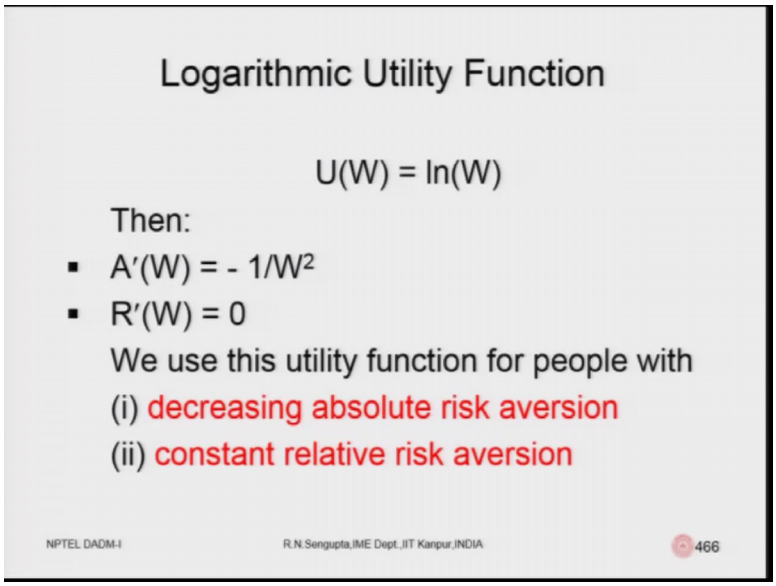
(Refer Slide Time: 24:43)



Even though it is very small, but I am sure you can do it in order to make the, yourself very clear. So, the pink color is basically give function. So, I do not have this. So, you share the one for the time you know I do not have the do not have the luxury to use the yellow one and use the orange one. Do I have? Yes, orange one, I will highlight it.

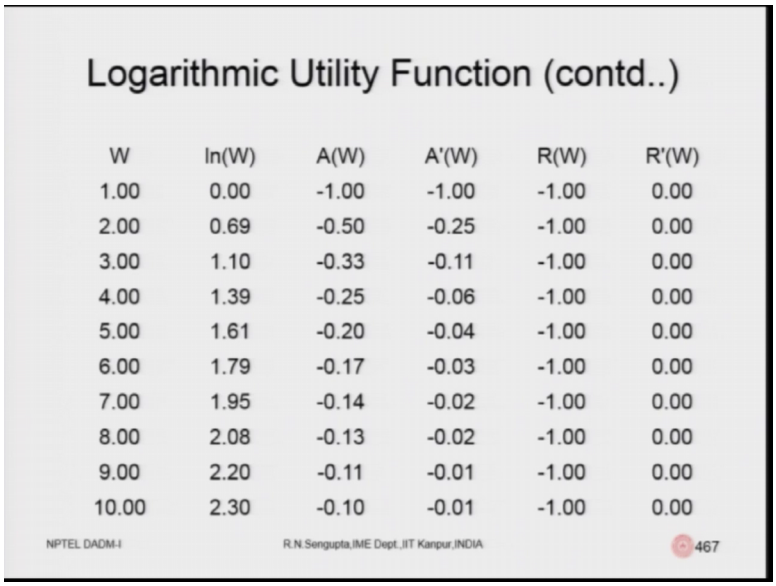
So, this is basically U is the quadratic equation. The yellow one where it would to be here would basically give you A, the pink one and the light grayish blue would be the A prime, the violet will be R and the brown would be R prime. So, you can basic plot them from the data which you have and basically can have on have an understanding, how the graphs for the quadratic utility function look like and how A and A prime are calculated and how R and R prime are calculated.

(Refer Slide Time: 25:57).



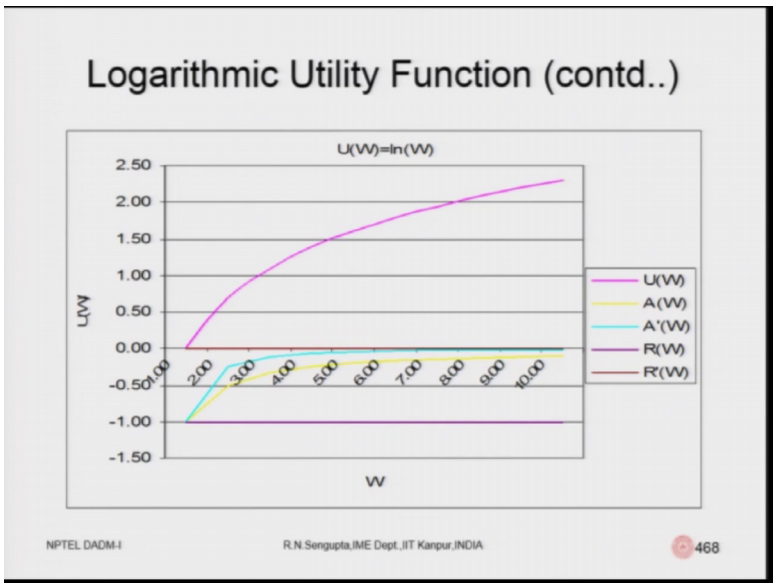
Now, we go to the logarithmic utility function. So, in logarithmic utility function is ln of W. So, you want to find out the differentiation of that ln of W differentiation is 1 by W and double differentiation would be minus 1 by W square. You put these values based on that you can calculate A and then you can calculate A prime, similarly you can calculate R and R prime. The values are given as it is A prime is given by minus 1 by W square and R prime is given by 0. Hence, from this you can understand it will have a utility function for the people would be decreasing absolute risk aversion property because it is minus 1 by W square, W square is positive. So, it is negative and it is 0 that; obviously, it would mean is a constant relative risk aversion property.

(Refer Slide Time: 26:46).



Then again I do the same thing, I plot the values of W in the first column, U of W in the second column, third and fourth columns are I am not drawn it, you can simply draw it using excel where you are plotting U and U prime, U prime U is basically and W you can plot U prime and U double prime plot it and based on that you can get A A prime R R prime and you can see the values of R prime which we just calculated using mathematics is very simply matching if you do the calculation. So, is 0, another case the A values which is more important also is also negative you can plot them.

(Refer Slide Time: 27:27).



And once you plot these values again you will have the pink one for U then the yellow one for A, the A prime R and R prime values are given respectively as I am just mentioning the colours. If you plot it, it will be much better. Take that I might trust me and follow the instructions just utilize the values which are given for the quadratic one and the logarithm one I will repeat them for the other logarithmic utility function also and you will plot them you will get the equations as it is given. So, it will be much easier for you to understand it. So, the greenish blue, the violet and the brown are respectively for the A prime, R and R prime.

So, with this I will end with the forty fifth lecture which is the end of the ninth week and I will continue discussing more about the utility functions in the beginning of the tenth week and then basically go into the applications in the regression and the other fields.

Thank you very much and have a nice day.