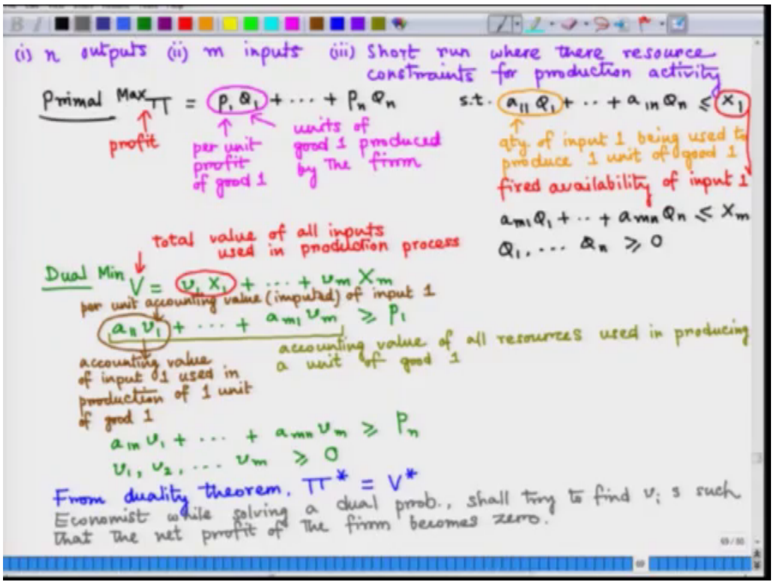
Microeconomics: Theory & Applications
Prof. Deep Mukherjee
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture – 55
Linear Programming (Part-2)

Hi, welcome back to the lecture series on Microeconomics and we have been discussing Linear Programming problems. Last time we have stated the general standard forms of maximization and minimization type problems, we have discussed duality and finally, we had started discussing linear programming problems in the context of a production setup. So, let us now go back to the previous discussion and end that first, then we will continue with the numerical exercise to understand, how these linear programming problems could be solved.

(Refer Slide Time: 00:54)

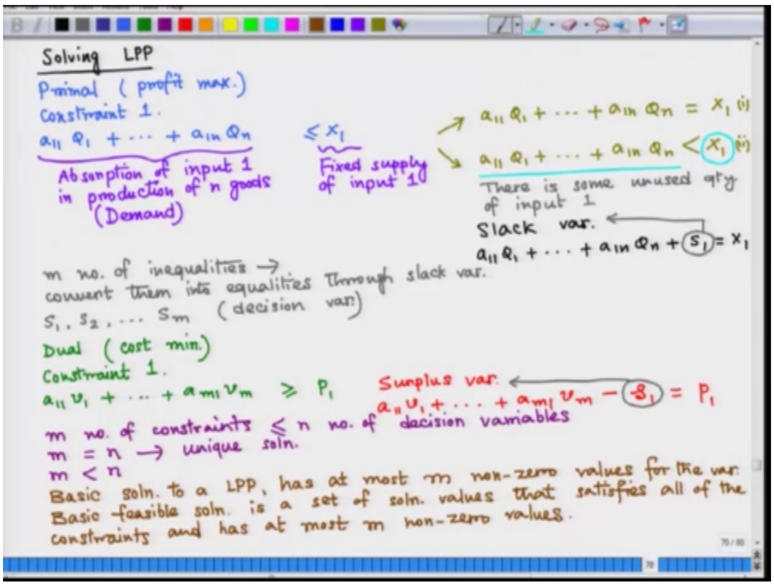


So note that, we had this discussion earlier, where we set that the optimised value from the maximization problem and the minimization problem shall be equal. So, what does that mean? So, that has a very strong implication in terms of economics. So, basically one can say that the economist, while solving a dual problem shall try to find these imputed input prices or accounting values of this resources, which are basically, find v i s such that, the net profit of the firm becomes zero.

So, what does that mean? That basically says that, suppose you have a firm where some variables, some inputs are variable and some inputs are fixed like, you know capacity of a firm, warehouse space, machine, hours, etcetera. So, there after computing all the variable costs, if you have per unit profit for a commodity, you now shall take care of your fixed costs. And, the way to capture fixed cost is through these imputed prices for inputs, which are given by v 1 to v mand you should set them as high as possible. So, that your economic profit becomes zero.

So now, we are going to continue our discussion on this production problem through a simple numerical exercise. Because, it is quite difficult to understand, how to solve a linear programming problem using this m cross n production data setup. But, before we go to solve a numerical problem, let me spend couple of minutes to discuss the process, how LPP should be solved in general m cross n setup.

(Refer Slide Time: 03:56)



So, basically the linear programming problems involve inequalities, but it is difficult to deal with inequalities. So first, we have to convert these inequalities into equalities, let us see how one could do this. So here, let us go back to our previous problem, which is a general problem of profit maximization the primal one. We are saving time by not writing the objective function, I hope we all know what was there. So, just let me write it was a profit maximization problem ok. So now we will focus on the constraint number 1 in that case because, the method that we are going to discuss is general enough. So, what is valid for constraint number 1 is also valid for constraint number m ok.

So, let us now look at the constraint. So, we had a 1 1 times Q 1 plus a 1 n times Q n right and that is basically less than equal to X m right. So, how did we interpret that previously? So, this is basically absorption of fixed input 1 in production of n goods right. So, you can say that, this is the demand side of the story. So, this is basically the input demand right. Now let us look at the right hand side here, what is this? So, these basically talks about the fixed supply of input 1 right.

Now, as there is inequality of less than type then basically there could be 2 possibilities that could have right. And what are these possibilities? So, of course, possibility number 1 would be that a 1 1 Q 1 plus a 1 n Q n is equal to X n X 1 and the other possibility, which could arises is the following a 1 1 Q 1 plus a 1 n Q n is less than X 1. So now, note that in the first case, which is basically this and this is the second possibility. Now in possibility number 1, there is no problem we get a linear constraint which actually we would love to get right.

Now, the problem you are just when we have the possibility 2, where we have strict inequality right. So, basically we have to convert the strict inequality to an equality, so that it is easier for us to handle. How to get this done? So, of course, you know we can see that this right hand side of this inequality is basically, higher, compared to the left hand side of the inequality right. So basically, there is a deficit right. So in terms of this production problem, we can say that there is some unused quantity of input 1 ok.

So, we can use a concept called slack variable and the slack variable could be used to convert this inequality, strict inequality into an equality. So, basically one can write or rewrite this expression as right. So here, this is one these, newly introduced variable is basically the slack variable. So, to convert m number of inequalities convert them, each of them into equality, strict equality through slack variables right. Now note that, the slack variables, which are given by capital S 1, capital S 2 for the second constraint and then finally, for the mth constraint S m these are also the decision variables right.

Because, we do not know their values approprie; so, by adopting these concept of slack variable, we get what is known as the augmented or slack surplus form of the linear programming problem ok. Now let us move to the dual problem. So, dual was basically a kind of cost minimization problem right ok. So again, we have linear objective function, but inequality constraints, we have to convert these inequalities into equalities. How? So, we are going to show that only for constraint number 1 and what is valid for constraint number 1 is also valid for constraint number n as well; there are n number of constraints remember ok.

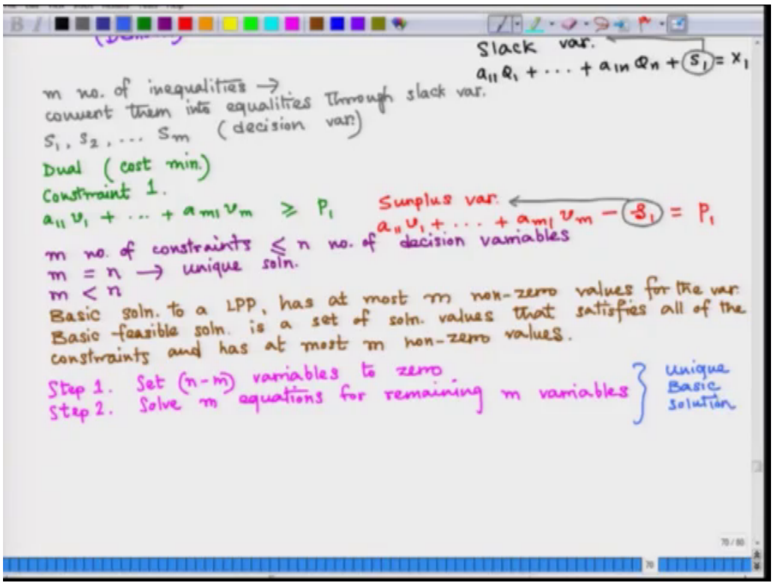
So, what is the first constraint that was there in the dual problem that we had seen earlier? So, a 1 1 v 1 plus a m 1 v m greater than equal to the per unit profit from commodity 1 ok. So, now again, there could be 2 possibilities, one strict equality and the other one is strict greater than inequality. We are not going to write the same, but note that we can now introduce another variable concept, which is known as surplus variable and follow the same procedure to convert this inequality into equality and this is the way, we are going to do that. So, note that here, we have higher value in the left hand side. So basically, we need to deduct something from the left hand side, such that it becomes equal to the right hand side right.

So, let me introduce the small s as my surplus variable that I introduce for my constraint 1 and now, if I take that extra part out, it shall equal to my per unit profit from commodity 1 right ok. So here, the small s 1 is basically my surplus variable. So similarly, there will be some changes in the dual problem. So, from the standard form or dual problem after converting the inequalities into equalities, we get what is known as an augmented or surplus form of the dual minimization problem. And, as we are introducing new variables, these surplus variables we do not know their values approprie.

So, they would be the decision variables as well. So, the number of decision variables in the model will go up proportionately. So now, we are going to finally, discuss the general procedure to solve general a m cross n LPP linear programming problem. So now, suppose we deal with m number of constraints, which is less than or equal to n, which is basically the number of decision variables; sometimes, they are also called control variables ok. So, this is basically the general LP problem. So now, we discuss a special case, where m is exactly equal to n, see there is actually no problem then the system of equations has only one solution right.

So, we get unique solution ok, but now we are going to discuss, what if m is less than n? Right ok so, to see how in this case, where n is greater than m to solve the linear programming problem, we are going to now introduce 2 more new concepts, which are known as basic solution and basic feasible solution. So, let us first define, what is a basic solution to a LPP? Ok so, basic solution to LPP has at most m non-zero values for the variables. And what is a basic feasible solution? Basic feasible solution is a set of solution values that satisfies all of the constraints and has at most m non-zero values.

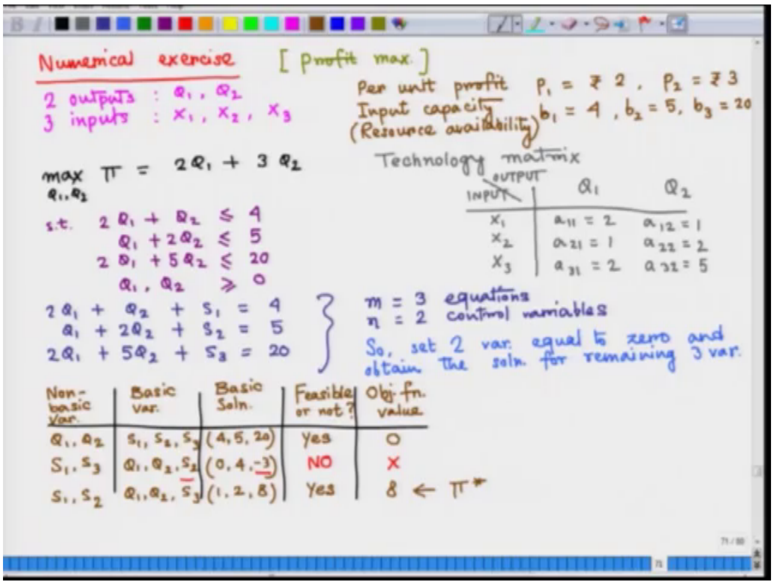
(Refer Slide Time: 18:26)



So, basically for solution of LPP in step 1, you set n minus m decision variables to zero. And then in step 2 solve m equation, a set n minus m variables to zero then basically, we get n equations in n variables. Then of course, we can solve the system of equations and if that is done then the resulting solution, if it is a unique one then it becomes a basic solution.

So, basically by following these 2 steps, we can get unique basic solution ok. So now, let us concentrate on a simplified numerical exercise and we will try to solve it graphically.

(Refer Slide Time: 20:01)



So, we are going to work with a numerical exercise now and in that numerical exercise, let us simplify the dimension of the problem. So now, we are going to assume that we have 2 outputs to be produced by the firm and they are basically, denoted by symbols Q 1 and Q 2. Then these 2 outputs are produced by 3 inputs and they are basically given by X 1, X 1 and X 3 right and now we are also going to assume some value. So, we have to assume the per unit profits, if we want to solve a profit maximization problem.

So, let us assume that we want to first study the profit maximization problem. So the firms problem is to maximize the profit from its production of 2 outputs right. So, we have to now introduce per unit profits and we know, we have introduced 2 such symbols before. So, P 1 let us assume is rupees 2 or dollar 2 whatever and P 2 is rupees or dollar 3, whatever and then we have to assume the fixed input capacities right. So, there we assume so, input capacity or in other words resource constraints.

So, they are basically given as this terms b 1 for input 1 and that is basically number 4, 4 units of input 1 is available and then we assume b 2 ok. So now, we are in a position to state our primal profit maximization problem. So, the firm wishes to maximize profit, which is given by 2 Q 1 plus 3 Q 2 and it maximizes profit with respect to the decision variables Q 1 and Q 2 of course. After writing the linear objective function, which is a profit equation, let us now write down the technical constraints and non negativity constraints.

For that first of all we have to look at the technology matrix. So now, if I write down the technical constraints, I will get 3 of them because, there are 3 inputs. This is for input 1, this is for input 2 and this is for input 3 and there will be 2 non-negativity constraints. So, if we now introduce slack variable S 1, S 2 and S 3 then we get the following system of equations ok. So, then basically what do we get? We get 3 equations and 2 control or decision variables ok, we have to set 2 variables; then obtain the solution for remaining 3 variables right and that can be done in the following manner.

So, there will be different combinations of variables to get different basic solutions, let us look at a simple table to describe the procedure. We will not enumerate all possibilities, but I will show you at least couple of them so, that you understand the process. So now, here look at this table here, I am going to list down the non basic variables that, I choose. So, by non basic variables I mean the variables, which assumes zero value right and the basic variables are basically for those variables, which are basically non-zero value, which assumes non-zero values right. So here, I first assume the simplest possible case where, I have Q 1 and Q 2 set to 0 right. So, in that case the basic variable becomes, basic variables become S 1, S 2 and S 3 from this slack form of the LP problem right.

Then what will be the basic solution from the system of equations? So, of course, we have 3 variables in 3 equations, now because Q 2 and Q 1, they take 0 values right. So, then the basic solution will become 4, 5 and 20 ok. So, is this are feasible solution? Yes of course, because this basically gives me the origin point of the solution space. So here, at the origin all technical constraints and non negativity constraints are satisfied. Now, what will be the value of objective function, if I evaluate this solution then I get 0 right ok.

So, the next case that we are going to study is S 1 and S 3 being the non basic variables means that we assume 0 values for these 2 variables. And if we assume so, then basically, we have the basic variables as Q 1, Q 2 and S 2 right ok. So, if that is the case then what is my basic solution? Basic solution would be 0 4 minus 3. Now, note is this a feasible solution or not? No because, for the slack variable S 2, we obtain a negative number here which is not permissible right. So, this solution is not feasible solution right.

So, as this is not a feasible solution, there is no need to compute the objective function value. So, we will not compute it ok. So, this is the way our journey we will continue and you know we can get a combination of non basic variables as S 1 and S 2 and if we choose that then the basic variables become Q 1, Q 2 and S 3 and then we find solution 1 2 and 8. And is this a feasible solution? Yes of course, because at this point basically, all the constraints are met right. So, we have yes here and what is the objective function value that we can compute to be 8.

Now, we can show there are many other possibilities, but we can show that this gives the highest value of my profit. So, this is basically the optimized value or the maximized value of profit expression that I started with ok. So now, we are going to prove our point that the last basic feasible solution that, we have obtained indeed is the profit maximizing point through a graphical exercise.

So, we will continue our discussion on this numerical exercise in the next lecture.