Microeconomics: Theory & Applications.

Prof. Deep Mukherjee
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture – 54
Linear Programming (Part-1)

Hello, welcome back to the lecture series on Microeconomics, we are done with our theoretical discussions and policy analysis. Now we are going to enter the last part of the course, which is based on the data analysis techniques, which are useful in applied micro economic work. So, we are going to start our discussion with Linear Programming analysis.

Now linear programming is a mathematical technique, which bridges the gap between abstract economic theory and applied managerial decision making in practical world. So, what could be a definition of linear programming problems? So, linear programming is basically special case of mathematical optimization method, which tries to find out optimal solution of mathematical economic models involving linear relationships.

It is interesting to note that 2 scholars independently worked on similar problems and developed this field one Russian mathematician Leonid Vitaliyevich Kantorovich and the other one is an American mathematician George Dantzig. So, after their fundamental work professor T C Koopmans Dutch American economist also contributed heavily in this field and that is why, professor Kantorovich and Koopmans shared Nobel prize of the year 1975. So, in this course, we are going to only look at the tip of the iceberg, we are only going to study very simple linear programming problems, but actually it is a vast field of research and it is un heavily applied field. So, we will start with standard formulation of a linear programming problem.

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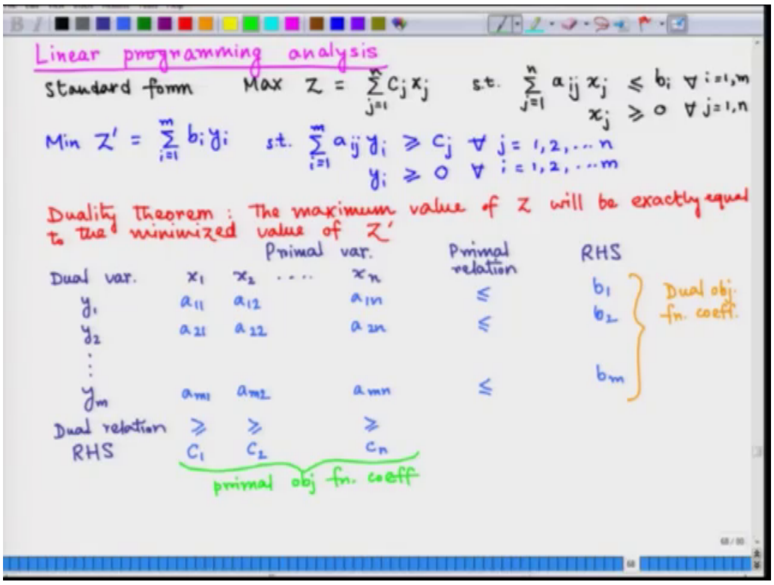

So, we start with the standard form and typically that is a maximization problem. So, we talk about a linear function Z equal to summation of C j X j, where j equal to 1 to n are basically, the decision variables and these maximization problem is a constraint 1 and here, we have the constraints, we are going to find out case specific interpretation of these objective function and the constants, but note that these are linear constraints as well. So, basically we are dealing with linear objective function to maximize with subject to linear constraints and let us assume that there are m number of constraints.

Now, this is not the end of the study, we also have to assume that our decision variables are non negative in nature. So here, we have to add the non negativity constraints, which are basically 1 to n ok. Now an optimization problem could be also in terms of a minimization exercise, hence the second standard form of a linear programming problem is the following. So, we write another linear function, this time the sum ranges from i equal to 1 to m and here b i y i right and this is the objective function. So, of course, we need the constraints.

So, we have to first write the linear constraints again, what are the interpretation of this symbols? These are k specific. So, we wait for sometime let us develop the general theory first and then we will take examples to interpret, this symbols. So, lastly the non negativity constraint right fine. So, having laid out the standard formulation of a linear programming problem both in terms of a maximization and minimization problem, now let us study, what is the relationship between them? Actually there is a very interesting strong result between this maximization problem and the minimization problem and that is known as duality.

So, basically duality says that if there is a primal problem, which could be the maximization problem, then there is a dual minimization problem to that and vice versa. So now, we are going to study duality, but before we study duality let us have duality theorems stated. So, duality theorem states that the maximum value of Z will be exactly equal to the minimized value of Z prime. So, basically that says that the optimized value of Z and Z star should be equal right.

Now, we start looking at the duality and that is through a table. So here, let us list down the dual decision variables, which are basically y 1, y 2 to y m and now let us list the primal decision variables, which are n number of them ok. So, they are n variables x 1, x 2 to x n and they are we also write the primal relation and finally, let us also write the right hand side constant parameter values ok. So, here at the bottom we write dual relation and again, we list down the right hand side parameter values in the dual problem fine.

So, now you are going to write down these coefficients and note that from my general problem I can write this. So, this a i j are basically the technical coefficients, we are going to find proper interpretation of this symbols shortly and then if we continue like this we get a m 1, a m 2, a m n less than equal to b m ok. So now, note that my right hand side parameter values of the constraints in a primal problem are basically, the objective function coefficients in a dual problem right ok.

So now, let us go back to complete the table. So, here in the case of dual problem the direction of the inequalities reverse and there is a change in the parameter values in the constraint also and here, we note that the c 1, c 2, c n, this basically where the primal objective function coefficients, they have become now the right hand side parameter constraint values in a dual problem ok.

So, after having this brief discussion on duality lets going to, let us find out some economic problem through which we can understand the deeper details of linear programming problem. So, we will start with a standard production problem.

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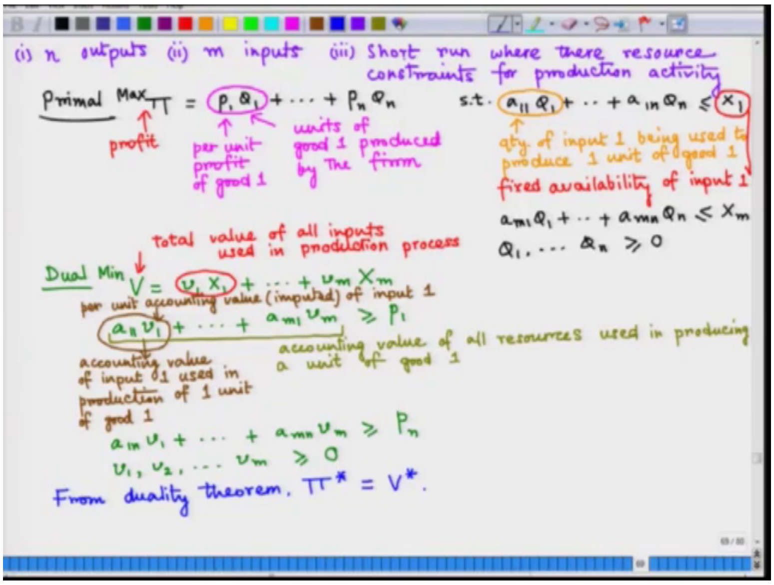

So, in a standard production problem, let us assume we are talking about n outputs and m inputs and let us assume that, we are talking about short run. So, basically in short run. So, we can just write this is like you know features of our LPP, linear programming problem. So, we have the short run, where there are resource constraints for production activity fine. So now, let us write down primal problem of this multi output multi input firm.

So, we are going to write a profit expression of this multi output firm because, we assume the objective of the firm is to maximize profit of it is operation. So, let us assume that P 1, P 2 etcetera are basically unit profit of commodity 1, commodity 2 respectively. So, we are basically going to have n number of commodities right. So, this is what we have ok. So now, let me write down this variables in detail. So, that there is no confusion later on. So, this is basically per unit profit, this is not price note, the difference from a previous discussions. So, this is per unit profit of good 1 or commodity 1 there are n number of commodities ok.

And now this Q 1 is basically what? Units of good 1 or community 1 produced by the firm. So, in total PQ gives basically the profit generated by 1 output and the firm basically, wishes to maximize the sum total of profits coming from all types of outputs it produces right ok. But of course, there is some constraints and there are resource constraints right. So, that we are going to write next, let us assume that, we are working with m number of inputs. So, basically there will be m number of resource constraints as well right ok. So, let us write down this constraints, one by one. So, first concentrate on input number 1 and we have this ok. So, what does it say? Now it involve these symbols like a 1 1, a n, etcetera.

So, let us find out the interpretation of this ok. So, this is basically, this a i js are basically my input output coefficients, this is like you know the technical coefficient of production model. So, that basically shows quantity of input 1 being used to produce 1 unit of good 1 ok. So, this is basically giving us the absorption of input 1 in production of commodity 1. So, similarly we get n components showing the absorption of input 1 in n different commodity production. So, the sum total of absorption of input 1 shall be less than the total quantity available of input 1.

So, basically this X 1 here is giving the availability fixed of course, fixed availability of input 1 ok. So now, let us go to other constraint. So, there will be m number of them because, there are n number of inputs. So, we straight go to the mth and the final constraint right and of course, we have to add the non negativity constraint. So here, we have to add this non negativity constraints. So, these are basically the decision variables, which are basically output levels produced by the firm ok. So now, we are going to write down the dual of this profit maximization problem.

So, in dual we write there is something like, there is some linear objective function to start with say V and we write v 1 times X 1 plus note that here, the constraint values of the fixed inputs, which are basically X 1 X n which are basically the constraint coefficients have now entered in the objective function and I hope you have remember this discussion and duality, it is basically following that discussion ok. So, we write dual linear objective function and this now has to be minimized ok. So, I forgot to write this here. So, this is basically a maximization problem ok. So here, we are going to have a minimization problem minimize V fine.

So now, we are going to have the constraints. So, let us write them one by one and then you know we will discuss the interpretations, etcetera. There will be m number of constraints, but before that we shall interpret what is happening here. So, let us start with this term here ok. So, first we have to understand, what is this v 1? Now, I will take you to the discussion that, we had long back on economic profit and accounting profit and what was the difference between them? We have said that there are many inputs for which readymade cost numbers or input prices are not available because, some of them are of that nature only think about the input warehouse space right. So, it is very difficult to find unit cost of that input, but it is a very important input in manufacturing type production processes.

So, how do one accommodate these type of inputs in the profit calculation? So, it certainly does not appear in the accounting profit, but if you hire an economist, economist will suggest a way to accommodate them in the economic profit and that is basically through the imputed cost route. So, although there is no unit input price for this type of input variables, which are given in fixed quantities, but one can think about a shadow input price for this type of inputs and how to find the shadow input price? That is through this technique that, we are currently discussing linear programming ok. So, basically here, the objective of an economist who is trying to solve a dual minimization problem of a firm is to find out these v 1, v 2, v n these are basically, the imputed values of inputs or accounting values of inputs with something in mind.

So, what is that something in mind, we will discuss soon, but at least you know note that these v 1 to v m are basically the unit input prices and mostly they are accounting values, they may not be equal to the unit price, unit input price that one observes in reality. So, v 1 is basically my per unit, accounting value which is imputed of course, of input 1. So, if say my first input is warehouse space. So, then in that case v 1 would be the accounting value of 1 unit warehouse space. So, if my input number 2 is machine hours then v 2 would be the imputed price or accounting value of 1 unit machine hour ok. So, then altogether what does a 1 1 v 1 gives us?

So, this is basically the accounting value of input 1 used in production of 1 unit of good 1 right ok. So, if we now concentrate on this entire expression in the left hand side of this inequality then, what is the interpretation that we get? So, this interpretation is basically is the following; accounting value or imputed value of all resources used in producing a unit of good 1 ok. So now, let us complete the other constraint. So, there will be n number of constraints here, in this case because, there were n number of decision variables in the primal maximization problem right.

So, keeping that in mind let us go straight to the nth constraint, in this case and complete this linear programming problem, do not forget to write the non negativity constraints. So here, the decision variables are actually these imputed values, which the firm or economists want to find to minimize this expression. So, we have to write this non negativity constraint as well right ok. So, they are all positive. So now, let us look at the objective function that we started with what is basically, the interpretation of this objective function.

So earlier, in the primal problem, we understood that the firm's objective was to maximize the profit from all outputs that it produces right. So here, these says something different. So now, we have proper understanding of this x's and this v's. So basically, what does this individual component give? v 1 x 1. So, that is basically, the cost in imputed sense that the firm incurs on input 1 right. So basically, this V which is sum total of v 1 x 1 to v m x m actually gives, total value of all inputs used in production process ok.

Now, let us apply duality theorem that, we have stated earlier and see what is the implication in this case? So, from duality theorem, we can write the maximized values, maximized value of the profit denoted by pi star shall equal to the minimized value of the resource value, which can be denoted as V star shall be equal right. So, this is the very interesting result. So, we will continue our discussion on linear programming problems in the next lecture.