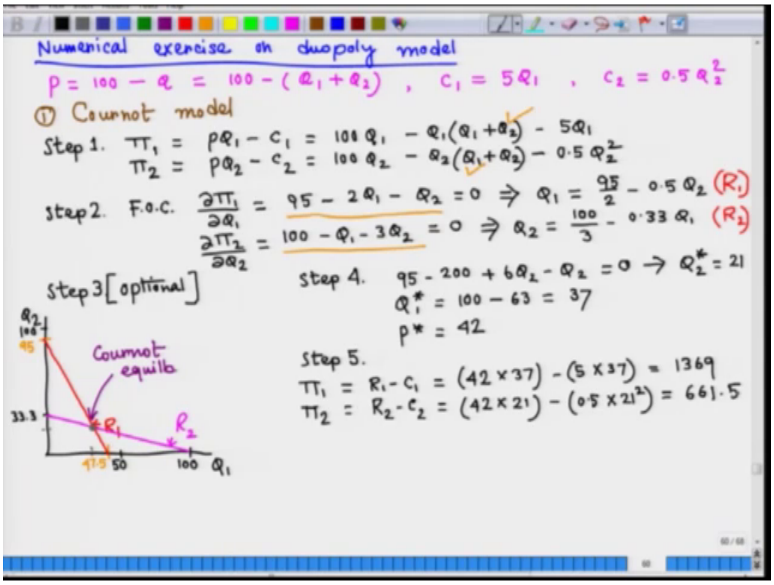
Microeconomics: Theory & Applications
Prof. Deep Mukherjee
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture - 51
Practice Session (Tutorial) on Oligopoly

Hi, welcome back to the lecture series on Microeconomics. Earlier, we have discussed the theoretical Cournot and Stackelberg models. And we have found the firms equilibriums in those cases. Now, we are going to revisit those oligopoly or duopoly models through a numerical exercise. I hope that this numerical illustration will help you to understand the working of these two models.

(Refer Slide Time: 00:45)



So, first let us write down the functions, which are common to both the models. So, we start with a market demand function, which is inverse demand function. And capital Q is basically the sum total of output being produced and supplied by these two firms 1 and 2 respectively ok. And then we also assume that the cost structure is different for these two duopolist firms. So, the cost function for the first firm or duopolist one is basically given as 5 times Q 1, and the cost function for the firm 2 or duopolist 2 is given by 0.5 times Q 2 square right. So, I must say that these are all hypothetical numbers. So, if you change these numbers, of course you expect a different solution right.

So, let us going to look at how with this common information, we can derive the Cournot equilibrium and Stackelberg equilibrium right. So, let us first study the case of the Cournot equilibrium ok, the Cournot model case. So, how to solve a Cournot model? We have to proceed step by step. So, step 1 would be writing the profit function for both firms; and if we do so, let us work start with firm 1. So, it is going to be the revenue earned by the first firm, and the cost incurred by the first firm right, and then we get 100 times Q 1 minus Q 1 times Q 1 plus Q 2 minus 5 of 5 times Q 1 right ok.

Similarly, one can write the profit expression for the duopolist 2 or firm 2, and that would be; so I hope that you can see the interdependence between these two duopolist profit levels, because note in profit function 1, Q 2 is present; and in profit function 2, Q 1 is present right ok. So, now what would be my second step? So, I hope you remember that it is reaction function, which is basically the most important concept in Cournot duopoly and Stackelberg duopoly model in order to solve this models. So, we have to derive the reaction functions from where to get the reaction functions, we have to find out the first order conditions right. And let me now find out the first order condition for firm 1 ok.

And if I now express Q 1 in terms of Q 2, then what do we get? We get basically the reaction function for firm 1, which is given by R 1 ok. So, similarly I can take derivative of the profit function with respect to my output for firm 2 and output from firm 2. And if we do so, we get 100 minus Q 1 minus 3 Q 2 and that shall be equal to 0 as well. And if now I express the Q 2 as a function of Q 1, then I get right. And if you remember the theoretical discussion that we had earlier, this is the reaction function 2 ok.

Now, we move on to step 3. And in step 3 let me tell you that this is an optional one; because, now from the step 2, you can solve these Cournot model in two ways. One is through the graphical approach, and the other through the algebra right. So, I will show you both. So, the first one is basically the graphical approach, which is kind of optional, but let us have a look at the graph of the reaction functions right.

So, we have to basically draw the reaction function for both the duopolist right. So, if we now draw graph, so two firms output levels are measured along different axis. So, let me assume that this is 100, and similarly I can also assume here this as 100. Basically, I am trying to get the intercepts properly plotted, so that these two reaction functions can intersect as they should be. So, now this is more or less half, so 50 let me mark this. And this is almost one-third of the distance. So, this is kind of this is very close to 33.3 ok.

So, now we are ready to draw the reaction functions. So, to plot the reaction functions let us first look at the reaction function of firm 1. So, that comes from this first order condition. And if i want to plot that, then you know I am going to get a Q 1 value close to 47.5, which can be here, and the corresponding Q 2 value would be 0. And if I now plug the value Q 1 equal to 0, then Q 2 takes the value 95 right. So, somewhere here right ok.

So, now if I join these two points, I get my reaction function 1 or the reaction function of duopolist 1 right ok. So, now to find the reaction function of duopolist 2, we follow the same procedure. We now have a look at the reaction function 2, which comes from this first order condition. And if I now assume Q 2 equal to 0, then I get Q 1 equal to 100. And if I assume Q 1 equal to 0, then I get Q 2 equal to 33.3.

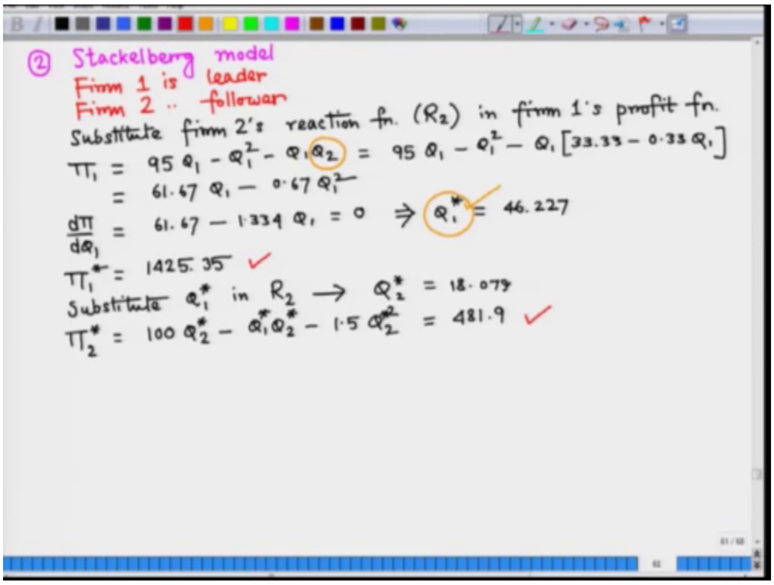
So, if I now join these two points, these are the basically intercepts of the reaction function 2 or the reaction function of duopolist 2. So, I get the second reaction function as well ok. So, we know from our theoretical discussion that it is the intersection point of these two reaction functions, where the Cournot equilibrium is going to be determined right. So, one can read from graph that it is going to be close to 20, and it is going to be close to 40, but let us find the exact numbers right.

So, then basically I move to the step number 4, which is basically finding the exact value of the coordinate of the Cournot equilibrium right. So, the step 4 would be algebraic solution of the Cournot duopoly model. And for that basically, we have to use this reaction functions only basically, we have to find the solution of these two first order conditions, which leads to the reaction functions. So, basically here after finding the intersection point of two reaction functions right.

So, note that I can write this, and from here I get Q 2 star equal to 21 right. And as I have solved Q 2 star, I can plug the value of Q 2 star in the expression and we get Q 1 star equals. So, these actually is coming from the first order condition 2 ok. So, now, the other part of the solution is to find the price. Now, as oppose to a discriminating monopolist model, there will be only one price in the Cournot equilibrium model and how to get that price? So for that we have to use the market demand equation. So, we need to plug the values optimal values of Q 1 and Q 2 into the demand function. And if we do so, we basically get a price equal to 42 ok.

Now, next step would be to compute the profit. And this is required, if we want to compare the Cournot model outcome with Stackelberg model outcome ok. So, now let us look at the profit expressions for individual firms. So, we have R 1 minus C 1 and that is if I plug the values of price and quantities in proper places, then I get some number. And similarly, pi 2 I get revenue minus cost, and again if i write down the proper numbers in proper places, then I get this expression. And if I compute, I get a number approximately this. So, finally we have obtained the Cournot model outcome.

(Refer Slide Time: 15:08)



And now we are going to look at the Stackelberg case. Now, in the case of Stackelberg, we have to assume that one firm is a leader, and the other firm is a follower. Now, without losing anything, we can assume that our duopolist 1 or firm 1 is the leader. And the other firm 2 is a follower. If we assume that then let us see, how the model outcome is going to respond to that assumption.

So, Stackelberg model right. So, let me write down explicitly the assumption, we are making. So, we are making the assumption that our firm 1 is leader, and firm 2 is follower ok. So, we know that in that case what we have to do, the firm 1 actually knows the reaction function of the firm 2, as it is the leader in the market. So, the firm 1; we will now substitute the opponent firm's reaction function, which is R 2 that we have just derived in profit function right ok.

So, here let us start with the profit function of firm 1 or the leader itself. So, we have these profit function from our previous derivations, note that we are not changing any functions from the Cournot model, so that we can compare two model outcomes. Actually, we are going to show that the Stackelberg model produces higher profit for the leaders that is our objective to show actually.

So, now let us plug the reaction function expression R 2 in the profit expression of R 1, and that is basically the expression for Q 2 right ok. So, now let us substitute that here, so that is this expression right ok. Now, we get a final expression of profit like this ok. Now, we know what to do from our theoretical discussion, now the Stackelberg leader will behave like a monopoly, and it will try to maximize its profit. And now that now for that he can take the derivative of the profit function, and we get right. So, this first order condition for profit maximization gives us a number, which is approximately equal to this right.

So, now if I plug back that value of Q 1, I get the maximized profit and which is coming out to be around this ok. So, now we know that our firm 2 is a follower, so that poor fellow has nothing to do, but substitute this output level that the market leader has already set in the market in its reaction function, and get his optimal output level in response.

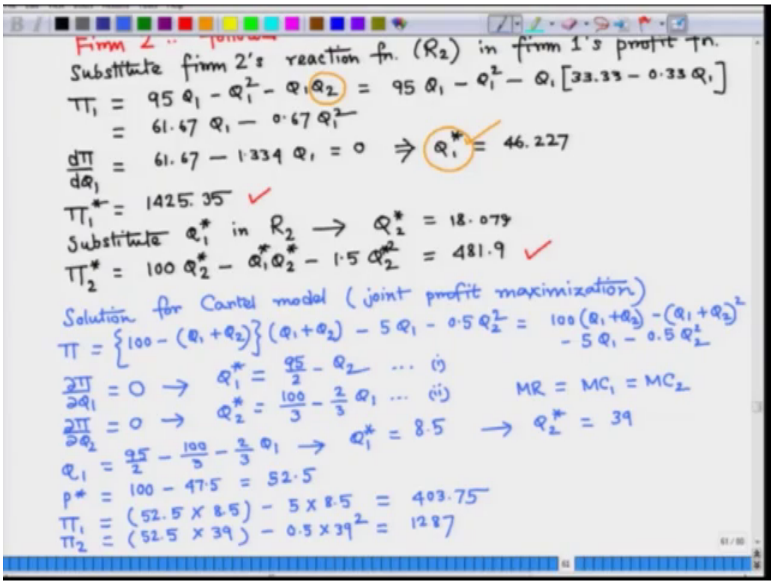
So, our follower or firm 2, we will substitute optimal output level of leader, which is Q 1 star in its reaction function, which is R 2 right. So, if it does so, then basically it gets an output level, which is approximately equal to this ok. So, correspondingly, we can look at the optimized profit for firm 2. So, firm 2 has a profit expression right. So, now you need to plug this optimal values right. And if you do so, then you get some number again approximately close to this.

So, you can see here that if firm 1 behaves like a Stackelberg leader, its profit is much higher than compared to the Stackelberg follower duopoly firm. Now, note that when we have derive the Stackelberg solution in the theory class, I probably forgot to mention about the conjectural variation in the case of Stackelberg model, but here we have some time to discuss that. ah

So, note that in the Stackelberg model case, actually the conjectural variation of the leader firm is equal to the slope of the reaction function of the follower firm, why because the leader firm actually is substituting the reaction function into its profit equation right. So, the leader firm actually has some guess about the behavior of the follower firm, and that is exactly given by the slope of the reaction function of the follower firm or firm 2. What does that give that gives that if there is a change in the firm 1s output, then how firm 2 is going to change its own output? So that it maximizes its own profit ok.

So, we have obtained our Stackelberg model equilibrium. Now, in the remaining time we could also explore the solution of the collusive duopoly model. What if there is no leader, there is no follower, these two duopolist firms decide to cooperate with each other. And if they decide to make a team, then they will become a monopolist in the market. And then they will set a new market output level, and how to find that that we can see through our numerical example.

(Refer Slide Time: 23:48)



So, we will now discuss the solution for cartel model, and that is basically the case of joint profit maximization ok. So, we will first write down the joint profit equation. So, we will write 100 minus Q 1 plus Q 2 that is the demand function or price, then multiply that with the industry supply, which is Q 1 plus Q 2 supply from both the duopolist firms, then cost shall be deducted 5 Q 1 for first firm, and then 0.5 Q 2 square for the second firm right.

So, finally what do we get, we get simplified expression, which has 100 Q 1 plus Q 2 minus Q 1 plus Q 2 square minus 5 Q 1 minus 0.5 Q 2 square ok. So, now from the lecture on cartel, we know how to proceed. We have to find out the individual firms profit maximizing first order condition. So, basically we have to find out the first order condition by setting this derivative expression to 0. And then if we do so, then I get the optimal value of firm 1s production, but note that here that will be through that will be a function of the output level of firm 2 right.

So, then basically I have to take another derivatives said that equal to 0, and then I get Q star 2 that is going to be a function of Q 1 fine. So, now let me name this equation. So, let me say this is my equation number 1; this is my equation number 2 ok. Now, we know that in the cartel case the equilibrium condition is basically marginal revenue equal to marginal cost equal to marginal cost of second firm. So, basically this is my first order condition ok. So, I can substitute 2 into 1 and I get Q 1 is equal to ok.

So, now I get everything in terms of Q 1. So, from here I can easily solve for Q 1 value. So, I get q 1 star equal to 8.5 right ok. Then I can plug this value of Q 1 star in equation 2, so that I can solve for Q 2 star. And if I do so, I get Q 2 star equal to 39 ok. So, then basically I need to plug this Q 1 star and Q 2 star values in my demand function. And if I do so, I get my price to be set in the market, and I get 100 minus 47.5, which is equal to 52.5 in terms of either rupee or dollar, whatever monitory unit I am assuming for price ok.

So, then from here the calculation of profit becomes quite straight forward. So, and we can also find out pi 2, similarly ok. So, these are basically the individual firm's profits. So, with this illustration, we come to an end of our discussions on oligopoly models. So, now we are going to enter into the discussions on externality. All of us are aware of environmental issues around us. And now we are going to see how microeconomic theory on externality can be helpful to model environmental problems that we see around us. So, next we are going to discuss the applications of microeconomic theory to model environmental problems.