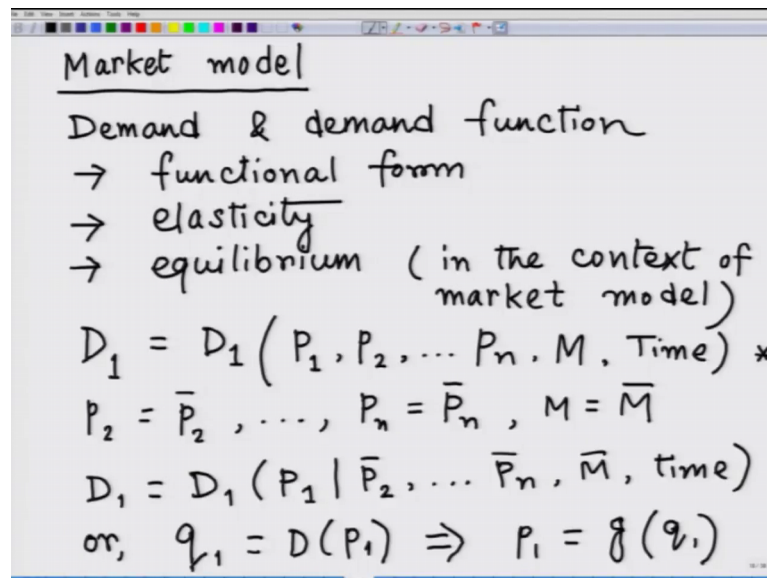


Microeconomics: Theory & Applications
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Lecture - 05
Demand Function

Welcome back to the lecture series on Microeconomics. We have already laid out a simple model to explain operations of market. Now let us get into the details of market operations and I would like to relate different mathematical concepts and you know different components of model building through this simple market model exercise.

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So, let us start with the concept of a demand and demand function. We will first look at the functional form, we will look at the concept of elasticity and how we can use this mathematical concept of elasticity in the context of demand analysis, then you know we are going to look at the concept of equilibrium.

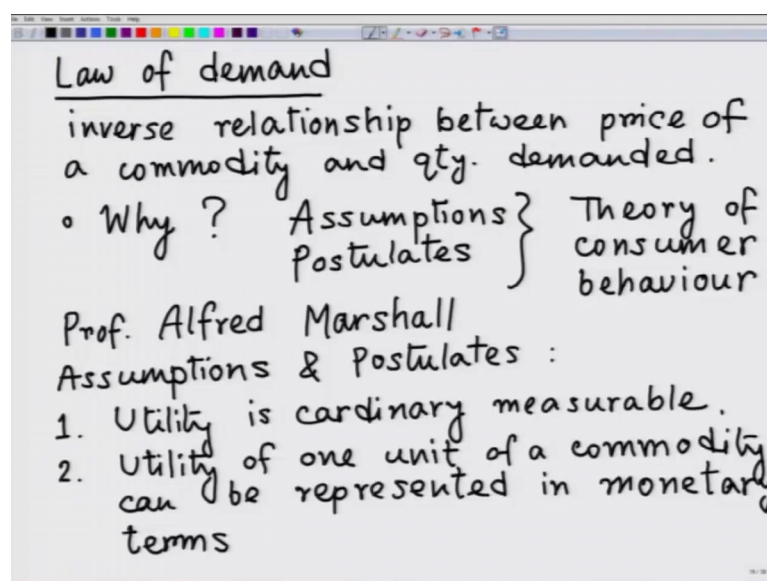
So when we say demand, we mean that at the given market price, what is the quantity that a consumer or a buyer desires to consume this particular level of quantity demanded may differ from the actual level of consumption for various reasons. Now there is also one more thing that you must note that a consumer actually consumes a wide range of consumption goods and the demand for a particular commodity or good say one is basically function of many things.

Definitely, it is going to be a function of its own price, then it is going to be function of the other commodities that the consumer or buyer potentially can consume suppose there are n number of such commodities. Then of course, when the buyer enters the market to demand something and finally, purchase something there is some money income that is given to him or her and then finally, time is also held constant.

So, we can see that demand function is a function of many independent variables to simplify we can assume that this other goods prices are held constant like this. So, they do not change in our model consumer cannot also control this, the consumer also is under some kind of a budget constraint. It implies that consumer works with fixed amount of money income and as I said the time is also given. So, if I set these parameters in the model the demand function this multi variable demand function gets simplified to or as it is the demand for commodity 1, I am denoting this by q_1 as right.

So, this is basically the demand function as the independent variable price of that particular commodity changes, how the demand for commodity one is going to change, but as we get to know the explicit functional form of this demand function, we can also write the inverse of this function and this is known as the inverse demand function. Actually, we plot an inverse demand function when we are doing micro economic analysis.

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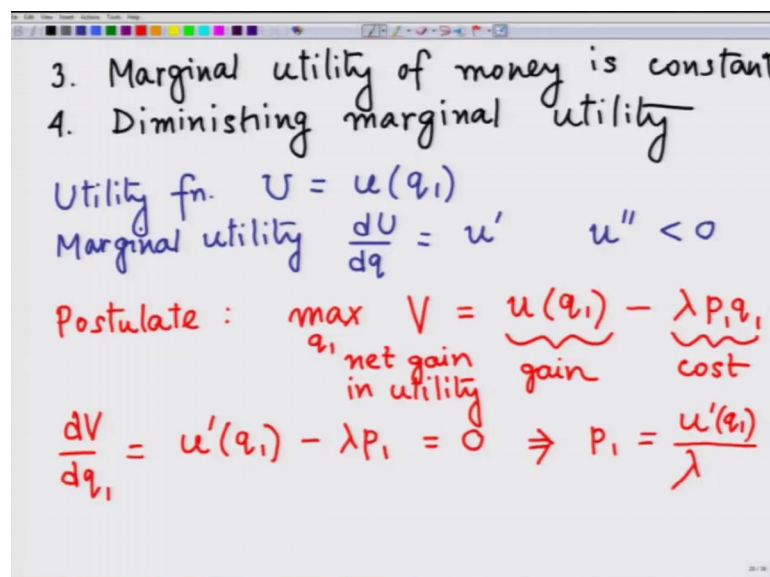
Now, last time we have seen this concept called law of demand, right what does it say? It says other things remaining the same if price of a commodity increases, then the quantity demanded of that particular commodity decreases and vice versa. So, we have seen that there is an inverse relationship between price of a commodity and quantity demanded and that is why we get downward sloping demand functions, right the question is why.

And then we will see to answer this particular question we need some assumptions and postulates which will together build what is called theory of consumer behavior. So, now, we see we are going to work with different components of theoretical economic model to derive the downward sloping demand function of course, there are different ways to do it, we are going to look at you know the most simplest case and this is related to Professor Alfred Marshall.

So, what are these assumptions that are required to establish the assumptions and postulates of course, you can add that as well. So, first assumption would be utility is cardinally measurable. So, what do I mean by that? I mean when you consume a commodity, you can express your happiness or your satisfaction through numbers and these numbers are the utilities of consumption and as these are you know measured in cardinal sales, you know addition and subtraction is possible.

Number 2 utility of one unit of a commodity can be represented in monetary terms, this is a very strong assumption later in the course we will rewrite this assumption.

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3. Marginal utility of money is constant
 4. Diminishing marginal utility

Utility fn. $U = u(q_1)$
 Marginal utility $\frac{dU}{dq} = u' \quad u'' < 0$

Postulate: $\max_{q_1} V = \underbrace{u(q_1)}_{\text{net gain in utility gain}} - \underbrace{\lambda p_1 q_1}_{\text{cost}}$

$\frac{dV}{dq_1} = u'(q_1) - \lambda p_1 = 0 \Rightarrow p_1 = \frac{u'(q_1)}{\lambda}$

Number 3; assumption will be the marginal utility of money is constant. Now with all this there is another important one which is known as and I have already listed this assumption before diminishing marginal utility.

So that means, that as a consumer continues to consume more and more unit of the same commodity for each incremental unit of the commodity he or she derives less amount of utility or satisfaction. So now, with this assumption set let us try to model this mathematically. So, if I mean Marshallian world let me write down my utility function as u is a function of q_1 . Remember, we are discussing the derivation of the demand function for commodity one. So, you know I am expressing utility as a function of the consumption of commodity one.

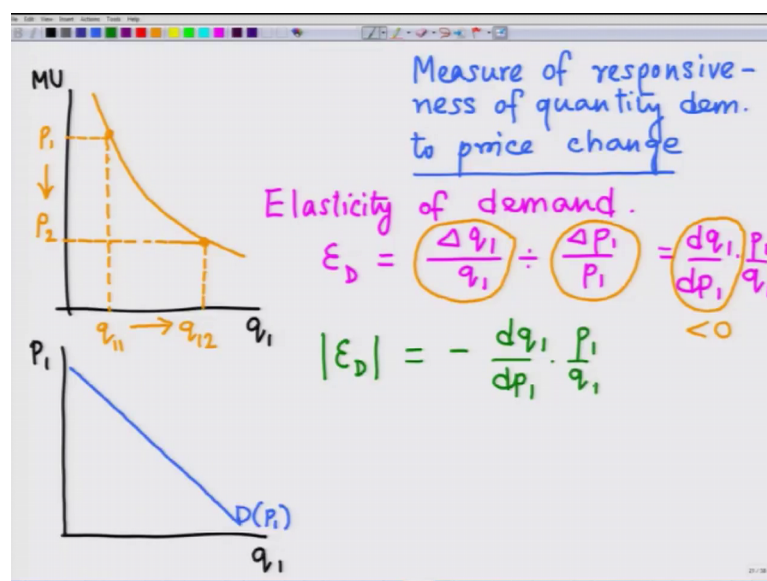
So, marginal utility will be defined as right and as it is diminishing it means that right. So, these are the assumptions that are required. Now if we assume let us see what happens to the consumer behavior. So, remember we also need some postulates to have a theoretical model.

So, here comes the postulate consumer wants to maximize the net utility from consumption. So, the direct utility comes from this utility function that we have already written and there is a loss of utility, because as more and more units of commodity one are purchased money is lost. So, the expenditure on the consumption item commodity 1 is P_1 times q_1 , you multiply that with a constant marginal utility of income denoted by λ and that is basically a cost to the consumer and this is basically the gain or the benefit to the consumer and this V is basically the net gain in terms of utility.

In a model, we also should have some decision variables or control variables. So, in this case as you can see P the market price is given λ is constant the marginal utility of money is constant. So, which is left the only variable left is basically q_1 . So, the units of consumption of commodity one will be the decision variable in this model. So, we need to maximize this with respect to q_1 .

So, if we know we are maximizing or minimizing we know we have to find out the first order condition and the second order condition using differential calculus right. So, let us take the first order derivative and for maximization we need to set this first order derivative to 0 and from here. We get this first order condition which says that P_1 equals u' divided by λ right and note that this u' is a function of q_1 , right.

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So, now, let us move to a diagram, here we are going to plot q_1 , the quantity demanded or consumed of commodity one and here we are going to plot the marginal utility. Now note from the previous diagram that you know we had sorry the previous expression that we had note λ is a constant he do not change as q changes, but u' will change as q_1 changes and we have already assumed that u'' is negative so; that means, that u' as a graph will be a downward sloping graph, right.

And each point on this graph actually gives you the if you assume that λ to be one you can you know you know for simplicity sake you can assume that λ takes value 1. So, if λ takes value 1, then basically that first order condition for net utility maximization gives you the quantity demanded by the consumer at each price. So, price is P_1 the consumer is going to demand q_1 suppose price falls to P_2 and the consumer goes back to the curve and he demands q_2 .

So, as we can see at price falls we can see that the quantity demanded increases. So, that the consumer maximizes the net gain in terms of utility now this discussion that we have gives us the shape of the demand function which is downward slopping function, we do not know the curvature property of this function it may be a straight line, it may be a curve you know convex to origin as I have drawn in the upper panel, but let us for the sake of simplicity we assume that we have a straight line demand function, right.

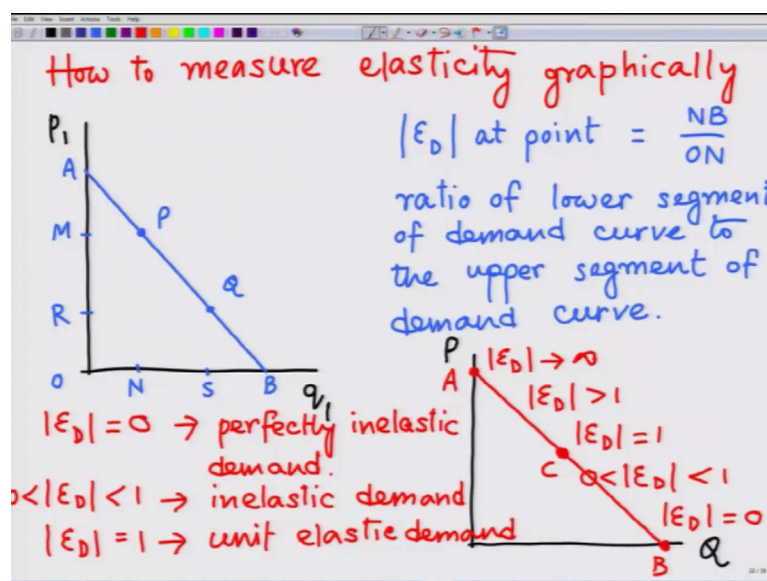
Now in the demand function, we can see that everything remaining the same if price falls the quantity demanded of that good increases. Now let us have a measure of responsiveness measure of responsiveness of quantity demanded to price change and that is given by this concept called elasticity that we have already seen previously. And as we are discussing the demand function this is called elasticity of demand.

So, in the case of elasticity of demand; how to express this; so, in words elasticity of demand means that what is the percentage change in quantity consumption of good 1 due to 1 percent change in price of good one keeping every other thing the same. So, let us express this concept in terms of symbols ϵ_D is this change in quantity divided by the original quantity divided by change in price divided by the original price. So, this is the proportional change in quantity demanded due to some percentage change in the price.

Now, this can be also rewritten as right and as you can see that this component is basically the slope component of the demand function and we have seen that although we do not know the curvature, but demand function or demand curves always downward sloping. So, we know for sure that this is negative. So, ϵ_D elasticity of demand is basically a negative number, but you know it is somewhat difficult to handle with negative numbers.

So, we want to express this negative number in a positive one and that is why we use the mod value of the elasticity and then we express this as the final formula for elasticity of demand and this is how we can transform a negative number in a positive number.

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Now, we start with a simple case with a straight line demand function now a straight line demand function; of course, we have all seen how to draw the straight line demand function is a negative relationship between the price of a commodity, and the quantity demanded of that commodity and here we draw. So, q_1 the quantity of commodity one and price of that commodity P_1 and let us have a straight line with intercepts both on the price axis and the quantity axis, right. Now let us have two different points P and Q.

So, suppose there is a journey from point P to point Q right and that by that I mean that in a price has fallen from level say OM to level OR and we can see as a result the quantity demanded has increased from ON to OS, let us name this demand function as AB. Now how do we measure, I am not getting into the details of you know the geometry of this, but one can measure elasticity of demand at point P, I am giving you the final result not getting into detail.

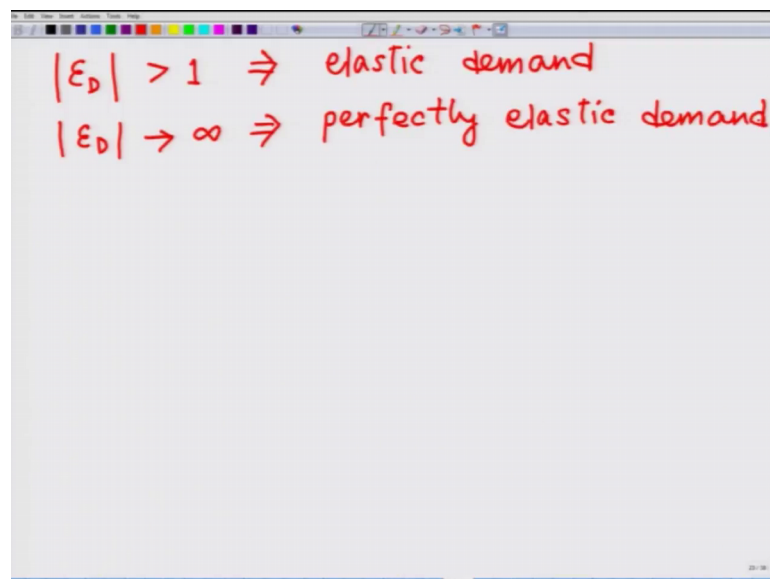
So, this can be given as NB over ON, right. Now these can be stated as ratio of lower segment of demand curve to the upper segment of demand curve. Now if we have a straight line demand function like this as you can see although the slope is constant along the straight line demand function, but the PQ bundles are changing. So, of course, the elasticity value will also change, right.

So, in a very general case when we are having a general demand function like this in PQ plane and when we have a demand function like this then the midpoint of this straight

line the exact midpoint of the straight line demand function at that point the elasticity value will be exactly equal to one then at this intercept with the quantity axis, there the elasticity value would be 0, and then the intercept with the price axis at that point elasticity will tend towards infinity. So, interim points the point between the point between say point C and point B, the elasticity will be less than 1 and greater than 0 and the points along the demand curve segment AC, the elasticity values will be definitely greater than 1.

As elasticity takes value 0, we call it perfectly inelastic demand so; that means, if there is a small change in price absolutely no change in quantity demand is seen if epsilon D takes a value between 1 and 0, then we call this is as inelastic demand so; that means, that if there is x percentage change in price quantity demanded changes less than x percent when epsilon D takes value equal to 1, we call that as unit elastic demand. And, that means if there is x percentage change in price of a commodity quantity demanded will also change by x percentage.

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If epsilon D takes a value greater than equal to 1, then we call that as elastic demand. So that means, that if there is x percentage change in the price of the commodity more than x percentage change is observed in quantity demand. And finally, if epsilon D tends towards infinity, then we call that this is perfectly elastic demand.

So, this is the basic setup of a simple model. We will continue with this discussion in the next lecture.