Microeconomics: Theory & Applications
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Lecture – 49
Oligopoly (Part-2)

Hello. Welcome back to the lecture series on Microeconomics. Let us continue our discussion on Cournot duopoly model. Last time we have seen how to derive reaction function of a Cournot duopolist. If we follow the same procedure we can also derive the same reaction function for firm 2 or duopolies 2. But, this reaction function for firm 2 will have a different slope and different intercept needless to say. So, now let us plot this reaction function for duopolist 2 and try to find out a solution of this Cournot model ok.

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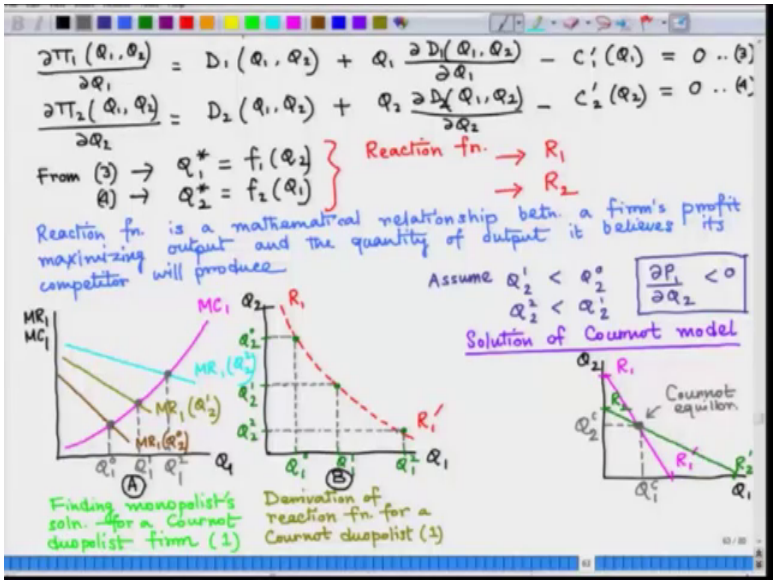

Now, we are going to draw two reaction functions in such a way that they intersect with each other, that intersection is required to find the solution of Cournot model and we will see why. Note that I can draw a straight line reaction function without loss of any generality it has to be downward sloping that is what we have observed previously, that there is a negative relationship between the quantity supply of firm 1 and quantity supply of firm 2.

So, let me call this reaction function of firm 1 R 1 R 1 prime right and let me now superimpose the reaction function of firm 2 on this and this reaction function for the

duopolies 2 will be a flatter line, it will have different slope and different intercept ok. So, let me mark this intercepts and this is basically my R 2 reaction function of R 2 right ok. So now, how to find the Cournot duopoly model solution? I claim that this is at the intersection point of these two reaction firms where these firms will find their profit to be maximized ok.

So, I call this Q c 1 as the profit maximizing equilibrium output level of duopolist 1 and Q c 2 profit maximizing output level of duopolist 2 and c denotes basically the Cournot model ok. So, basically this intersection point is my Cournot equilibrium. So, the question remains why this intersection point is referred as the solution to the Cournot model, why it is an equilibrium points? So, this is what we are going to discuss now.

So, if we assume that there is some output level of firm 1 say Q tilde that Q 1 tilde that assumes that some output level chosen by firm 2 at Q 2 tilde level, but what is the guarantee that actually the firm will firm 2 will choose the quantity or output level Q 2 tilde knowing that the firm 1 has already chosen Q 1 tilde output. So, basically the equilibrium in a Cournot model has to be mutually consistent, but there is no guarantee for that right. So, it is only at the intersection point of these two reaction functions that mutual consistency happens and we are going to now discuss that through another diagram.

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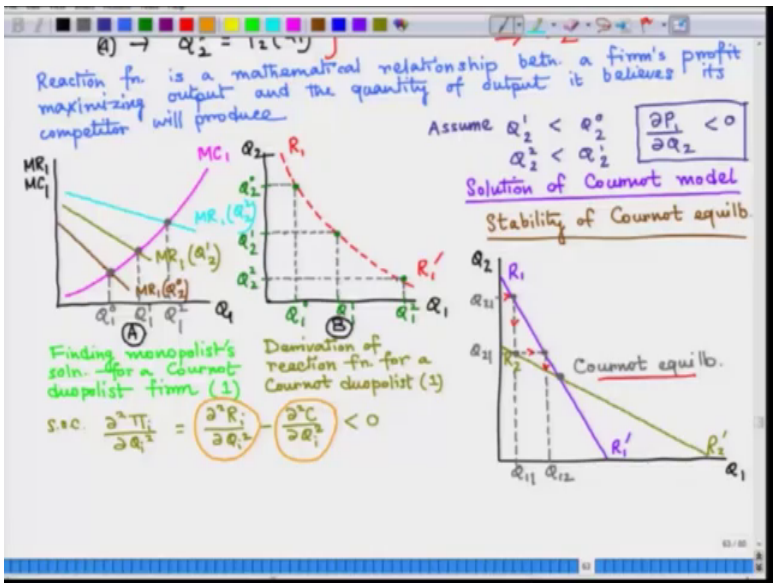

So, we have a larger piece of diagram here and let me first point that Cournot equilibrium again in this diagram. And now we are going to do a stability exercise, for that I need some space that is why I have redrawn this diagram ok.

So, now what does a stability analysis mean that if I deviate from this Cournot equilibrium whether I can come back to this original equilibrium or not. So, let me now start with some arbitrary level of output which is given by some number say Q 21 ok. And if the firm assumes if the firm 1 assumes that firm 2 is going to set its output level at Q 21. Then basically the firm 1 will refer to its reaction function and it then derives or finds out what is the quantity level it should produce.

So, you see that given the output level Q 21 which is basically the initial output level, the firm decides to firm 1 decides to produce this much of output level and that is basically Q 11 right. So, this is basically the optimal behavior of firm 1, if the firm 2 has chosen Q 21 output level.

Now, having seen firm 1 choosing this Q 11 output level the firm 2 will not sit idle it will go back to now its reaction function and find out what is the optimal level of output for each as the firm 1 has chosen Q 11 level of output and let me call that Q 21 ok. So, the same process follows as the firm 1, now sees that the firm 2 has chosen or we will choose some output level Q 21. It will now refer to the reaction function R 1 R 1 prime and from that reaction function it will come down to the Q 1 axis to find out what would be the profit maximizing output level in this case which is Q 12. And this process will continue until and unless these two firms which the initial Cournot equilibrium point which is given by the intersection of these two reaction functions.

So, basically what we see? There is a journey from here to here and then from here to there, then again from here to here, then from here and basically finally, it converges to the Cournot equilibrium point. So, the Cournot equilibrium, if it exists, it is a stable equilibrium ok. So now, we are in a position to state the second order conditions of the Cournot equilibrium. So, these are the second order conditions. So, for that we all know we need to take the second order derivative of the profit function of each firm right, and what do we get? We get this right.

So, this implies that the rate of change in the marginal revenue shall be less than the rate of change in marginal cost. So, basically that means, that the marginal cost must cut

marginal revenue curve from below, this is not a new result to us we have seen this before. So, this finishes our discussion on the Cournot market model. Now, we move on to another duopoly model which has been proposed by a German mathematical economist Heinrich von Stackelberg and this is basically a step ahead of the Cournot model. Because the Cournot model has a very naive type of market strategic interaction right, here we have assumed 0 value for the conjectural variation.

But the reality is not that simple why would a firm assume that if it produces some output level if it brings some change in its output decision then the competitor firm will not do anything. So, in that matter you know why we can assume that the output decisions of these firms are independent of each other. So, that is basically a very naive assumption right. So, Stackelberg actually relax that assumption and he came up with a more sophisticated duopoly model. So, now we are going to study that.

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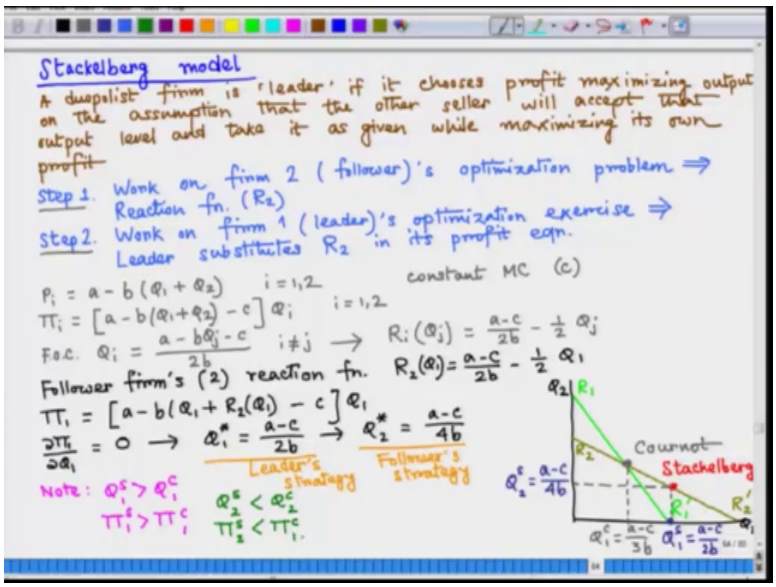

So, in a Stackelberg model the duopolist firms role differ from the Cournot one. So, in the Stackelberg model one duopoly firm becomes the leader firm and the other duopoly firm becomes the follower firm. So, who is a leader and who is a follower. So, let us have some formal definition for these two concepts and then we will come to the formal mathematical model. A duopolist firm is leader if it chooses profit maximizing output on the assumption that the other seller will accept that output level and take it as given while maximizing its own profit right ok.

So, on the contrary a duopolist firm is the follower who accepts the others output choice ok. So, now, we are going to have a mathematical model which will basically describe how the leader exploits the follower. So, in this case we will assume that the leader has access to a very critical piece of information, which is basically the information on reaction guard. So, if the leader firm knows the exact mathematical firm of the reaction function of the firm 2 or the here we assume that the leader firm is firm 1, without loss of any generality. So, if the firm 1 or the leader firm knows the exact mathematical firm of the reaction functions of firm 2. Then basically he makes uses of that information to set an output level at the first hand and then enjoys a first movers advantage.

So, basically we have to solve this problem in two steps. So, it is quite interesting to note that the Stackelberg model is basically more of a sequential type game rather than the Cournot model where the firms, the duopolist forms have chosen their output simultaneously ok. So, here while solving the Stackelberg model the first step would be to work on work on firm 2 which is a follower optimization problem. And these will basically lead to the reaction function which is a private piece of information, but somehow if that is stolen then the firm who has this information becomes the leader.

So, in this case basically firm 1 is our leader and actually what happens there we will see that the leader substitutes these reaction function R 2 say in its profit equation before it maximizes its profit right. So, this is a two step procedure ok, to solve a Stackelberg model will be very complicated if we do not assume a functional form. So, for simplicity let us assume some functional forms and that assumption or this simplification will also help us to compare the Cournot model outcome with the Stackelberg model outcome.

So, we will go for some simple functional forms, we will work with linear demand functions and cost functions so that we can get nice looking functional forms and get some nice looking solutions. So, our Stackelberg model will be solved with respect to these simplified functional forms. So, here I assume inverse demand function p i equals a minus b Q 1 plus Q 2 right. And that is basically for both of these firms right ok.

Now, I assume that my cost is such that I have a constant marginal cost. So, now, I assume my cost structure is such that that it gives rise to constant marginal cost and I assume that to be denoted by c and if I assume these two. Then my profit function profit equation for both the firms will become a minus b Q 1 plus Q 2 minus c times Q 1 right

and this is again for i equal to 1 to 2 ok. So now, we know how to proceed this problem, we have to basically find out the reaction functions right. And that will come from the first order conditions.

So, let me work with the first order condition first for both the firms and it will look like a minus b Q j minus c divided by 2 b and here I assume that i is not equal to j right ok. So, these leads to the reaction function of the i-th firm given j-th firm's quantity or output choice and this will have a shape like this ok. Now let us write down specific functional form of the reaction function for firm 2, because that is what our step 1 suggests right. We need to know the reaction function of the follower firm. So, the follower firm is firm 2 here so let us note down its reaction function right ok.

So, now we have to move to the step 1 assuming that the leader firm or firm 1 knows the reaction function that we have just derived, it will now write down its profit equation as. Note that it will not write Q 2 here anymore because it now knows that if the, if it chooses some output level Q 1 tilde how its opponent firm 2 is going to choose its output level and that is through the reaction function. So, here he will simply substitute the expression that we have derived here and complete its profit equation.

So, now it is simple for firm 1, firm 1 has to find the first order condition for profit maximization right and that he will set it to 0 and then that will basically give output level Q 1 star profit maximizing output level for the Stackelberg leader firm as this. You can solve this yourself. And then if you plug back this value of Q 1 star in the firm 2 or the follower's reaction function which we have just derived then you can also get this expression. This is firm 2 or the followers profit maximizing output level when the leader has already set the output level Q 1 star in the market ok.

So, we will draw a graph depicting these linear reaction functions which will also show the Cournot equilibrium and Stackelberg equilibrium solutions ok. So now, we have to first plot the reaction functions as we know the specific functional firm we can draw and this I say to be the firm ones reaction function R 1 R 1 prime. Now I am going to superimpose firm 2's reaction function on this R 2 R 2 prime is the firm 2's reaction function ok.

So, we all know that at the intersection of these two reaction functions we find our Cournot solution. Now if we solve our Cournot model which I leave up to you; you will

see that we get the output of the Cournot duopolist firm from the axis and let me now note down that output level. So, this is Q c 1 say depicting or denoting the Cournot equilibrium output level for firm 1 and that is going to be a minus c divided by 3 b in this case fine. So now, what is the Stackelberg outcome? Now the Stackelberg outcome we will come here where these reaction function R 1 R 1 prime meats the quantity one axis. And this is basically the highest quantity that the firm 1 or the leader firm can produce given the situation and the reaction functions. So, basically this is my Stackelberg output for firm 1 which is also a leader firm and that you know can be found as a minus c divided by 2 b right ok.

So, now we know what to do: we have to find out the followers output level or supply in the market and we know what to do. We have to basically move up from this Stackelberg solution of firm 1 to the reaction function of firm 2 and then from the reaction function firm 2 we can read the output level what it must produce in order to maximize profit. So, basically the firm 2 which is a follower firm accepts these output level Q base 1. And then produces this output level Q is 2 and that is equal to if you solve you can get that a minus c divided by 4 b right ok.

So, we can compare our Stackelberg and Cournot solutions here. So, note that this is basically the Stackelberg solution. So, this is Stackelberg equilibrium point ok. So, basically leader firm chooses a particular point on the reaction function of the follower firm so as to maximize its own profit. And that has an implication on the profit levels of these competitor firms.

So, now, we are going to compare our model outcomes. So, here note that Q s 1 is greater than Q c 1 and same can be said for profit levels right and then we can. So, this is basically for the leader right, this is for the leader and then for the follower we can write Q s 2 is less than Q c 2 and pi s 2 is less than pi c 1.So, basically in the Stackelberg case we see that one of the Cournot duopolist firm suffers succumbs to the leader and it receives lower level of profit.

So with this, we finish our discussion on Stackelberg duopoly model next time we are going to study another type of oligopoly model which is known as collusive oligopoly model. So, we will be continuing our discussion on the theory of oligopoly markets even in the next lecture.