

Microeconomics: Theory & Applications
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Lecture - 48
Oligopoly (Part-1)

Hi, welcome back to the lecture series on Microeconomics. Now we are in the last part of our discussion on theory of imperfect competition. Today we are going to start our formal discussion on Oligopoly. What is oligopoly? Oligopoly means the presence of few sellers in the market. So, it is basically competition among few. So, we can start by giving some example of oligopolistic market competition, think about the case of telecommunication services in India.

So, here we see few players like Airtel, Vodafone, Reliance Jio, Idea etcetera right. So now, we will simply assume that we are dealing with only two competitor firms and see how market equilibrium is going to be achieved. But before we start talking about some classical models of oligopoly we will start with a general model ok.

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Duopoly models

Inverse dd. fn. $P_i = D_i(Q_1, Q_2) \quad i = 1, 2 \dots (1) \checkmark$

A. $\frac{\partial P_i}{\partial Q_j} < 0 \quad i, j = 1, 2 \text{ \& } i \neq j$

B. $\frac{\partial P_i}{\partial Q_i} < 0 \quad i = 1, 2$

$\frac{dP_i}{dQ_i} = \frac{\partial P_i}{\partial Q_i} + \frac{\partial P_i}{\partial Q_j} \frac{dQ_j}{dQ_i}$

Annotations:
 - $\frac{\partial P_i}{\partial Q_i}$ is labeled "direct"
 - $\frac{\partial P_i}{\partial Q_j} \frac{dQ_j}{dQ_i}$ is labeled "indirect"
 - "output variation" points to dQ_j
 - "conjecture / guess" points to $\frac{dQ_j}{dQ_i}$
 - "Conjectural variation" is written in a bracket next to the indirect term
 - "interpretations of Cournot conjo. variation ..." is written below the indirect term

$\frac{dQ_j}{dQ_i} = 0 \rightarrow$ Cournot model

• Each firm believes that the other firm's choice is independent from its own
 • Each takes the other firm's output as given

Cournot duopoly model

$\Pi_i = P_i Q_i - C_i(Q_i) = Q_i [D_i(Q_1, Q_2) - C_i(Q_i)] \quad i = 1, 2$

Annotation: "strategic interdependence" points to $D_i(Q_1, Q_2)$

F.O.C. $\frac{\partial \Pi_i}{\partial Q_i} = P_i + Q_i \left[\frac{\partial P_i}{\partial Q_i} + \frac{\partial P_i}{\partial Q_j} \frac{dQ_j}{dQ_i} \right] - C'_i(Q_i) = 0$

or, $P_i + Q_i \frac{\partial P_i}{\partial Q_i} = C'_i(Q_i)$ (2)
 Annotations:
 - "MR" is written above $P_i + Q_i \frac{\partial P_i}{\partial Q_i}$
 - "MR = MC" is written at the bottom right
 - "0" is written below $\frac{dQ_j}{dQ_i}$

So, we start with the market inverse demand function all right and that is given as P_i equal to $D_i(Q_1 \text{ and } Q_2)$ where i is basically 1 and 2. So, these i represents the number of firms in the market right. So, this is my demand function, let me give it equation number

1, and then from this inverse demand function what can be inferred? So, we can assume two results and let me name them result A and result B respectively.

So, in result A we can say that the partial derivative of the demand function of i th firm with respect to the j th firm's output is negative and that holds for both i and j equals 1 and 2 and note that here i is not equal to j . So, what does that mean? That means, that if the j th firm increases its output or market supply, then that has a negative impact on the i th firm's price.

So, now let us look at the result B, which is basically talking about the slope of the *ceteris paribus* demand function. So, I hope you remember that we have introduced this term *ceteris paribus* or *ceteris paribus*; that means, that everything else being the same. So, in this case I can write i equal to 1 to 2.

So; that means, that if every other thing remains the same, there is a negative relationship between firm i 's quantity supply and price that it achieves from the market ok, now we are going to look at the very interesting results from here. So, we know that there is this inverse relationship between price and demand if we are working with *ceteris paribus* demand function, but note that here in oligopoly we also talk about strategic interdependence.

So, you know if we look at the last result which is result B which talks about the partial derivative of demand function with respect to i th firm's output and that is negative the result is not actually that simple as it looks like. Why? Because if the firm decides to produce some units extra or some units less than then its competitor firm may also decide to respond to this output variation and it may wish to change its output level or market supply level.

So, then there will be an indirect impact on the price of the i th firm as well. So, now, we are going to break down the change in price due to a change in the quantity supply decisions of i th firm into a direct effect and an indirect effect. So, let us take help of the total differentiation and write this equation for firm 1.

But you know you should note that this can also be written for firm 2 right. So, basically we are working with this inverse demand function given by expression 1 for firm 1 right and we are taking differentiation of that function, as if the firm 1 has changed its output

level. So, we are going to study what will happen if it happens, if the firm decides to change its output level.

So, now let us look at these two components right. So, this is basically my direct impact which is coming from the ceteris paribus demand function and this is basically my indirect impact ok. So, this indirect impact is basically coming from the strategic interdependence between these two firms ok.

So, now let us concentrate on this particular term which is basically the crux of the oligopoly theory. So, what does this say? These basically says if the firm 1 decides to change its output level by 1 unit then by how many units its competitor firm is going to change its output level or market supply?

Now, nobody knows the answer to this particular question. So, this is a question for which answer is required right. So, in a sense it is talking about output variations right and we need to know the answer, but unfortunately there is no clear answer to this question. So, the firm 1 has to guess right or conjecture that if it changes its output level how the competitor firm is going to respond?

So, basically there is a conjecture to be made ok. So, this is known as the concept of conjectural variation in oligopoly theory right and we will see that for different values or different schemes that the firm assume for this term leads to different types of models. So, now, we are going to study the case when a firm assumes the value of the conjectural variations to be 0.

So, the derivative dQ_j/dQ_i takes value 0. So, now, let us see what is the implication of that so; that means, that if the i th firm or j th firm changes its output level there is no response from the competitor. So, these assumption leads to the Cournot model, proposed by a French mathematical economist Augustin Cournot in mid 19 century. Now when Cournot assumes this, then what does actually he mean? So, there are two interpretations of this and both are basically same, but you know representing them in representing the same concept in different words.

So, we will write it down. So, first of all it can be said that each firm believes that the other firm choice is independent from its own or it can also be said in other words that each takes the other firms output which is basically the market supply as given right ok.

So, now let us formally start discussion on Cournot duopoly model ok. So, now, we are going to start with the profit function of the i th firm where i takes two values 1 and 2 because it is a duopoly case. I assume that these two firms has different cost functions I can assume 0 cost and same cost also it does not matter, but to be to have a genuine model let me assume that this cost functions are different.

So, note that the interdependence are in the Cournot model comes from this demand function right. So, the output decisions of i th firm impacts the price that is received by the not only the i th firm, but of the j th firm as well ok. So, now that is basically the revenue side ok. So, now, let me take out the cost as the complete profit equation and i equal to 1 to 2 ok.

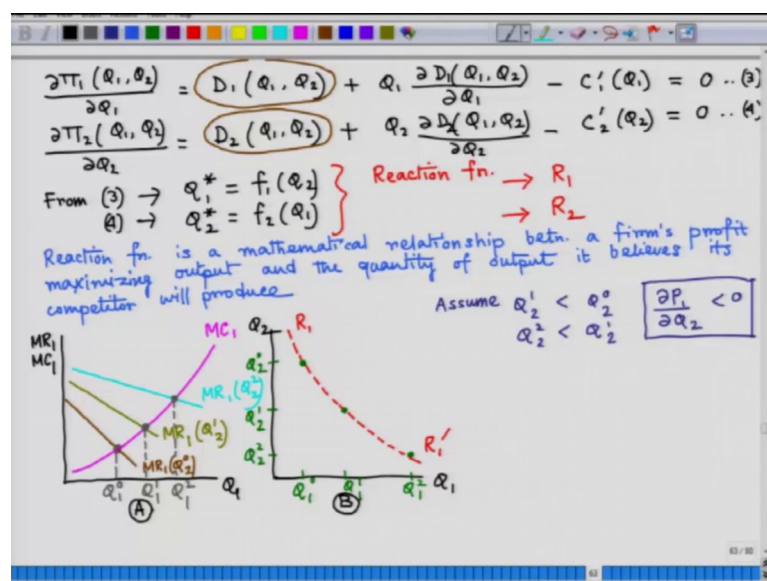
So, now, we have to derive the first order condition of this i th firm here. So, we know what to do, we have to take partial derivative of the profit expression with respect to the output of the i th firm and if we do so we will get P_i plus Q_i . Now this is going to be little complicated here because they are the strategic interdependence between these two firms, one response others output decisions and of course, we have this.

So, this is my first order condition; now note that if we are in a Cournot model setup, then basically we assume the conjectural variation which is this element is no rule because it takes the 0 value right. So, this entire expression basically disappears right. So, we can then rewrite our first order condition as all right ok.

So, now do you recognize this first order condition that we have obtained here? Let me call this equation number 2. So, let us you know study these first order condition element by element; so, first the simplest case. So, this is basically nothing, but my marginal cost. So, we get this then we write equal to now let us look at the other side of the equality and you know if you remember our discussions on monopoly, this is basically marginal revenue right.

So, we get back the MR equal to MC condition that basically we have seen in the case of monopoly or to be general. So, the first order condition what we have derived now basically is the same first order condition that we had in the case of a pure monopoly right. So, basically the Cournot duopoly model boils down to a pure monopoly model, now we are going to analyze the solution of the duopoly model in terms of a simple graphical and algebraic analysis ok.

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So, now let me write down the first order condition in detail assuming that we have the 0 conjectural variation as per Cournot's assumption. So, this is the first order condition for my firm 1 duopolist 1 ok. So, now, let me name this 3 and 4 equations fine.

Now, from 3 what do I get? We get optimal value of firm ones profit maximization problem. So, that is basically through the optimized value of market supply or output Q_1 star, but note that this is a function of the belief of firm 1 firm 2's output or market supply decision right.

So, now from 4, we get what? As per the normal interpretation of the first order condition I get the profit maximizing output level of firm 2, but note that this now depends on Q_1 . So, basically my each of this firms profit maximizing output level becomes the function of the firms belief about its competitors choice of output level right. So, some kind of reaction or interdependence is seen through these two first order conditions right and this is known in economics as reaction function ok.

So, from 3 equation or first order condition we get the reaction functions of firm 1 and from the first order condition number 4 or expression number 4, we will get the reaction function of firm 2 right. So, as reaction function plays a very important role in finding oligopoly equilibrium, let us have a formal definition of that to get our understanding clear.

So, reaction function is mathematical relationship between a firm's profit maximizing output and the quantity of output, it believes its competitor firm will produce. So, now, we are going to have a diagram to analyze how to derive reaction function graphically and with the help of intuition. Now we are going to concentrate on graphical analysis, we are going to have two panels of diagrams we are going to do the analysis only for firm 1.

But needless to say that one can very easily follow the same steps to get it done for firm 2 as well. So, panel A diagram and panel B diagram, we are going to first work with panel A diagram. So, in panel A diagram, we are going to measure the marginal revenue and the marginal cost of duopolist 1 and we are going to measure root of the duopolist 1. Now note that we have already said that the duopolist problem becomes a monopolist problem and so, we are going to find firm one's equilibrium by equating marginal revenue and marginal cost right.

So, let us first plot the marginal cost curve. Now we want to make a general diagram so here I assume a non-linear marginal cost curve. So, firm 1 now we are going to draw a series of marginal revenue curves; now why series of that we are going to discuss now. So, let us first draw the initial marginal revenue curve and let me name this MR 1, now note that when this marginal revenue curve is derived the firm 1 assumes a specific value of output level or market supply chosen by its competitor firm 2.

So, my marginal revenue is actually a function of the output level chosen by my competitor, why is this so? Because, if you note the demand function, there is strategic interdependence in both the demand functions for this individual duopolists. So, of course, when you derive the marginal revenue from the demand function then these interdependence will be there embedded in the marginal revenue function itself. Ok.

So, if the duopolist behaving like a monopolist assumes that its competitor is going to produce and supply Q_2 level of output in the market and then it finds its equilibrium at the intersection between this conditional marginal revenue curve and the marginal cost curve right. And finds the profit maximizing level of output all right denoted by Q_1 , this is my initial level of equilibrium.

So, now let us study how the firm is going to find another equilibrium if it assumes another value of Q_2 . So, basically if I change the value of Q_2 , if I change my belief in

other words then I am going to get another marginal revenue curve. Now let us study how we can generate this series of marginal revenue curves.

So, here I assume a case. So, let me write that this is assumption of course, assume ok. So, what does this mean? This means that now I am going to assume that my competitor firm; firm 2 supply less to the market. So, if that happens then how it is going to impact my price?

So, now we have to look at the result that we have talked about right. So, remember we started with some results A and B. So, now, if you go back to the result A then; that means, actually this right. So, if the firm 2 my competitor firm increases its output level then that has a negative impact on my price. And the reverse side of the story which actually which is assumed by us says that if the competitor firm reduces its output level then basically there is an increase in the market price that is observed by me.

So, if I believe that my competitor is going to produce and supply a lower level of output then I am expecting higher price in the market. So, basically I am talking about an upward shift of the demand function which I started with. So, if that is the case similarly marginal revenue will also shift upward in response because my demand function is now changed it has gone up it has moved up. So, the marginal revenue will also move up. So, now, let us draw that in our MR equal to MC diagram. So, now, let us have another marginal revenue curve corresponding to our output choice Q_1 which is less than Q_2 right.

So, needless to say that again this duopolist one will adopt that monopoly type of maximization exercise in this case and finds another equilibrium and it will now produce a higher level of output Q_1 ok. So, now, this exercise can be repeated for another output level, let me assume this time a level Q_2 which is even less so it is less than Q_1 alright.

So, if that happens then basically we can have another round of shift in demand and the correspondingly my marginal revenue curve shifts also and this shift all towards right side. So, there will be a MR 3 sorry I made a mistake here it should be MR 1 ok.

So, now let us go back to our diagram. So, we are basically now drawing the third marginal revenue curve in the series right and of course, we get another equilibrium and

correspondingly we get basically another output level and the output level produced by my firm 1 has further up in response right ok. So, now, we have completed our diagram in panel A; let us move to the diagram which is panel B. And there we are going to plot the relationship that we have seen between the output levels chosen by firm 1 depending upon its assumptions of firm 2's output level.

So, now I plot Q_1 along the horizontal axis and Q_2 along the vertical axis fine. So, now, note that I can now plot these points. So, suppose this is a point which talks about whose coordinates are basically Q_1 and Q_2 right ok. So, this is the initial point at which we started, but this is basically conditional upon the assumption of firm 2's output level.

So, if the firm 1 assumes that my competitor firm 2 is going to produce and supply at this level, I am going to choose my firm 1 is going to choose this much of output level and then we have seen the repeated reduction in the output level that I, that my firm assumes for its competitors Q_1 . And then we have seen that in response the firm as produced higher output level itself. So, Q_1 right and lastly we have chosen another low level of output Q_2 and we have seen a higher level of output produced by firm 1 in response ok.

So, this is what we have seen from panel a diagram right. So, now, if I join these points then what do I get? To look up of all these points will give me my reaction function for firm 1 right; it can be non-linear as well. So, I call this R_1 ; R_1 prime so this is basically firm 1's reaction function.

So, these two diagrams in two panels A and B are helpful to find firm ones reaction function, these graphical exercise can be repeated to find similar reaction function for firm 2 as well. So, we will continue our discussion on oligopoly models in the next lecture.