Microeconomics: Theory & Applications
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Lecture - 44
Monopoly (Part-1)

Hello, welcome back to the lecture series on Microeconomics. Today, we are going to start our discussion on imperfect competition. The first market model that we are going to study is the Monopoly model. What is the monopoly? Monopoly is a market where there is only one or single seller selling its output. Now, it may be difficult to get real life examples for pure monopoly case. So, Indian railways could be an example of pure monopoly in Indian context.

Now, we are going to study, how in this case the market is going to find an equilibrium. Before we jump into the formal model of monopoly, let us revisit the simple model of profit maximization once more and there from there we will see how we deviate from perfect competition to find the monopoly model or to develop our monopoly model.

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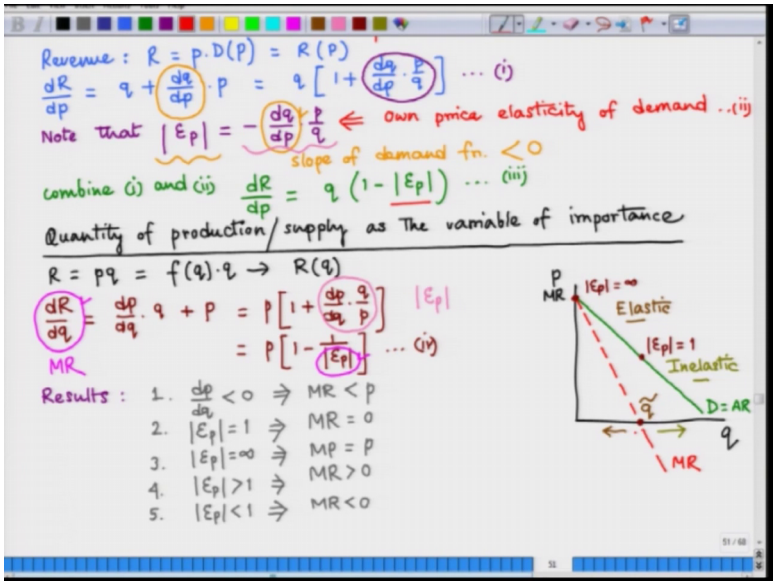

So, let us revisit our profit maximization exercise by a firm which we studied earlier. So, their profit is defined as total revenue minus total cost and total revenue is defined as market price of the output times quantity of the output minus c of q is basically the cost function. Now, note that in the case of perfect competition, we assumed that this market

price is held constant at p bar and this is beyond control of the individual firm selling its output in a competitive industry right.

So, the implication of this assumption is that we are working with a constant marginal revenue. We have defined marginal revenue earlier. I hope you remember the definition of marginal revenue right. But it is quite unlikely to assume that p will remain fixed as the supply is going to change in the market. In any market we have studied that there is a inverse relationship between the market price and the quantity demanded.

So, if market price changes of course, that will have an impact on the market demand and as the demand changes responding to a price change there will be an impact on the total revenue. But in the case of perfect competition, we have assumed that marginal revenue is equal to the market price.

So, the firm sells each and every unit of its production at the same price. But, if we do not assume that how our story is going to change that is what exactly we are going to study next. Let us have a simple mathematical model, where we focus on total revenue side. So, our revenue of the firms revenue is equal to R times market price P times the demand function D of P right ok. So, that means, that my revenue has become a function of price as well. So, if there is a change in market price, how to study its impact on revenue? We can consider the slope.

So, we differentiate total revenue function with respect to the market price to get q plus dq dp. So, that is basically the slope of the demand function right times P ok. We can rewrite that as q 1 plus dq dp times p over q right ok. Now, let me name this expression 1 ok.

Now, note that this expression here in equation number 1 is somewhat familiar with what we have studied long back. So, let me remind you we have studied a concept called elasticity of demand own price elasticity of demand and that used to be defined as minus of dq dp times p of q right. So, this entity dq dp is basically the slope of the demand function and as the demand function slopes downward we can say that this takes negative value.

So, the slope of demand function is negative, but generally we work with we find it more convenient to work with a positive number. Hence, we take the absolute value of the

elasticity and this is the way we define own price elasticity of demand right ok. So, now, let me call this my expression number 2. Now, if I combine 1 and 2, what do we get? We get dR dp equals q times 1 minus the absolute value of E 1 elasticity right. This could be my expression number 3, right.

So, we found finally, a mathematical expression which talks about the relationship between total revenue and the market price. Of course, we can see that this sign of the derivative dR dp depends on the value of own price elasticity right. So, far we have studied the relationship between the market price and the total revenue of a firm. Now, let us study the relationship between quantity and the total revenue of a firm because the relationship is quite useful in the theory of market. We are going to see various uses of this concept that we are going to derive shortly ok.

So, now let us look at the case where the firm treats quantity of production or output as the variable of importance. So, in that case let us write down the revenue function again and in that case that will be p times q. Now, note that in the case of the monopoly. Monopoly being the single seller in the market it caters to the entire market. So, basically the monopolist faces the entire market demand function. So, if the monopolist has proper idea about the market demand function, it can write its revenue function as this.

So, what does that mean? So, that means, that monopolist revenue function becomes a function of quantity that it produces and supplies to the market. So, now, with this let us start with our usual analysis. So, if now the monopolist is concerned to see how its decision to produce 1 unit more or 1 unit less on the revenue of the firm, then you know the monopolist has to take derivative of the revenue function with respect to the q right.

So, we can write this as dp dq times q plus price right. Now, note that these can be rewritten as if we take P common, then it turns out to be dp dq times q over p ok.

So, now note that this expression that we have just obtained as striking similarity with one concept that we have seen earlier and that is basically the concept of price elasticity of demand which we introduced long back and said that we it is basically a negative number, but we take the mod value of that for analytical purposes.

So, now, with this concept in mind we can rewrite this expression as 1 over 1 minus the mod value of own price elasticity of demand right. So, this is a very important result in

monopoly and in general in the case of imperfect infer in perfect competition. So, now, let us look at this expression here.

So, what is this? So, this is basically the marginal revenue right, we know the definition of that ok. So, we can now see that marginal revenue of a monopolist has some relationship with the elasticity of demand which is basically given by this epsilon expression right mod value of the price elasticity of demand. So, of course, as the value of epsilon p changes, it will have an impact on the marginal revenue of the monopolist. So, if we now want to draw the curve of marginal revenue, then we have to see various cases for the value of elasticity of demand.

So, let us now reach down some results which will help us to draw such a diagram right. So, first we can say that as the demand function is downward sloping and we have this dp dq representing the slope of the demand function, this implies that marginal revenue will always lie below the demand function which is given by this market price here right. Now, we can talk about another result which is dependent on the elasticity value.

So, if we talk about the middle point of a linear demand function that basically represents the unit elastic demand point right. So, if that is the case, then we see from that expression 4, marginal revenue will be 0. Now, the third case is basically another extreme case where we think about the elasticity value being infinity. So, in that case again from the expression 4, we see that MR will turn out to be exactly equal to p right.

Next, we consider the case of elasticity value being greater than equal to 1 that is basically the elastic case, elastic demand case and for that we have marginal revenue positive. Now, lastly epsilon p can also take value less than 1 so that represents the inelastic demand case. So, in that case marginal revenue will become negative right.

So, now, we have these cases listed. Now, it is easy for us to draw the marginal revenue diagram all right. So, so, we have the monopolists quantity of production given by q, p and marginal revenue MR are measured along the vertical axis. So, as usual we start with our demand function which is a straight line downward sloping straight line it is. So, I say that this is my demand function. Now, note that this demand function is also my average revenue function I left it to you to figure out why. Now, let us plot my marginal revenue function right.

So, we know that at this point, the starting point of the demand function, the elasticity value is basically infinity right. So, in that case the marginal revenue will be exactly equal to the market price. So, we get a point of the marginal revenue curve. Now, let us come down along the demand function. Let me assume that this is the midpoint of the demand function. So, here the price elasticity of demand will take value 1 and we know that the corresponding marginal revenue would be 0. So, it should be somewhere here right ok.

So, now we get two points and now we are going to utilize the result number 1 which says their marginal revenue will always lie below the demand function. It means, the marginal revenue is always less than market price that is charged by the monopolist. So, by employing that result, we can draw marginal revenue curve. So, it is a line, it is a straight line downward sloping. So, you see that suppose, this quantity level we call it q tilde at which marginal revenue becomes 0.

So, for all quantity levels less than q tilde, the monopolist firm actually is on the elastic part of the demand market demand function and hence marginal revenue is positive. But, for all quantity level, output level, which are higher than q tilde the monopolist firm is actually operating on the in elastic part of the market demand function. So, this is the marginal revenue curve, but there are two important there is one interesting result, that I want to draw you attention to.

So, if we deal with a general linear demand function, the marginal revenue curve actually is twice as steep the market demand function that can be shown, but we are not going to that detail. So, now, we move to the equilibrium analysis of the monopolist firm.

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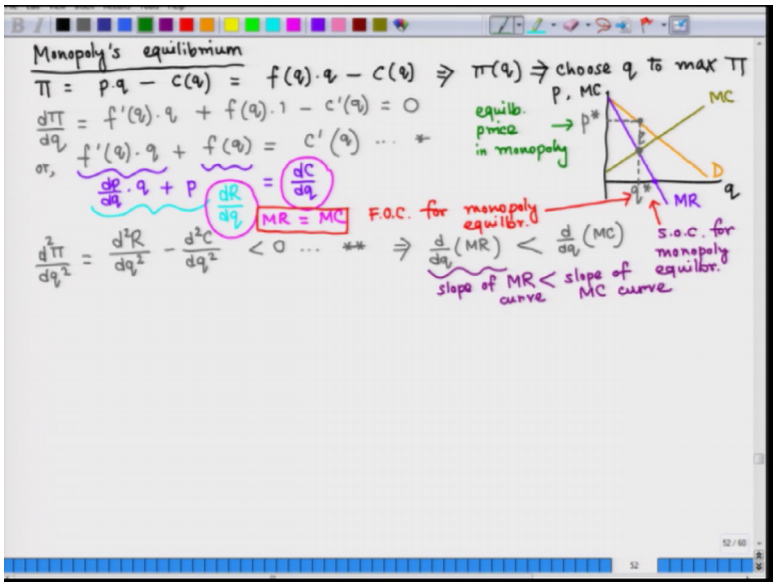

So, we are going to start with that same profit expression; pi equals total revenue which is p times q minus the total cost which is c of q. Now, note that as there is only one single seller in the market, the monopolist actually can read the demand function or the market demand function. Hence, he the monopolist actually knows the exact functional firm of the demand function. So, we can replace p with f of q, the demand function right.

So, this is the way we can rewrite the total revenue and of course, then cost will be deducted from the profit expression. So, this is basically now giving rise to a profit function which is function of q. So, basically the monopolist has to choose, so, the implication is the monopolist has to choose a proper quantity level to maximize its profit ok. So, how to do that? Of course, we have to now get the first order and second counter conditions of the profit maximization problem. Note that this is an unconstrained profit maximization problem.

So, now let us take derivatives that has to be equal to 0 right ok. So, we can rewrite that first order condition as right, now let me denote this by star ok. So, now, let us have a look at this expression here. So, what does that give? That is basically the slope of the demand function dp dq times q right and this is basically my market price. So, this is an alternative expression of the same first order condition, which is basically the marginal cost of production right.

So, now, I am explicitly writing the derivatives. So, now, what is this? Again, this expression is not unknown to you. That is basically the change in revenue with respect to small change in quantity or output being produced and sold.

So, this is basically the marginal revenue and that expression we already know. So, that is basically marginal cost. So, we get this very important condition again for profit maximization that is the case of MR equal to MC. Note that this is not the first time we have got these first order condition. Even in the case of perfect competition also we got the same thing, but there the price is constant and that is equal to the marginal revenue. So, we had MR equal to p equal to MC or in short p equal to MC, first order condition.

So, this is basically my first order condition for monopoly equilibrium right, excellent. Now, we move to the second order condition. We have to assure ourselves that indeed we got the maximum value of profit. So, for that we need to take derivative again the second order derivative, this is not new to you. So, we get what? We get d square R dq square minus d square C dq square that shall be less than 0. So, that is basically my expression number double star ok.

So, if I rewrite that would lead to another known second order condition that we have seen earlier ok. So, the rate of change in the marginal revenue shall be less than the rate of change in marginal cost at the equilibrium point right. So, what does that mean? So, that means, that slope of the marginal revenue should be less than the slope of marginal cost at equilibrium right. So, that is basically my second order condition for monopoly equilibrium ok.

So, let me write this, slope of marginal revenue curve has to be less than slope of marginal cost curve at the equilibrium output level q star ok. Now, let us going to look at a graphical illustration to see how a monopolist finds it equilibrium through a graph by utilizing the first order and the second order conditions, that we have just derived; q we are going to plot marginal cost of production and the market price p.

So, let us first plot the demand function here. So, this is my demand function D. Now, we know that there is a marginal revenue function like this which slopes downward and cuts the quantity axis at some point. The corresponding point on the demand function basically gives the unit elastic point on the linear demand function.

So, now, we are going to superimpose the third and last graph which is basically the graph for marginal cost. Now, we are going to assume that we are working with a straight line marginal cost curve upward sloping. This is for simplicity.

So, now if that is the case we know that the monopolist finds equilibrium at the intersection point of the MR equal to MR and MC curves because that is dictated by the first order condition. So, basically and if we drop a perpendicular line from that intersection point say e on the quantity axis M, we get the monopolists equilibrium output level q star right.

So, this is basically the monopolist equilibrium and that is obtained by following the first order condition. And note that here as the marginal revenue is downward sloping curve and marginal cost is an upward sloping curve, it also satisfies this q star point, also satisfies the second order condition of the monopolist profit maximization problem as well ok. So now how the market price is going to be determined? So, we know the demand functions.

So, monopolist now will go up from this equilibrium point, it will hit the market demand function, it knows the functional form of the market demand function. So, the monopolist now will charge a price which is the price, the consumer is going to ready to pay for this q star level of output. So, this is the way the monopolist will set the market price. So, this is basically the equilibrium price in monopoly ok. So, we will continue with our discussion on monopolies equilibrium in the next lecture.