Microeconomics: Theory & Applications
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Lecture - 04
Basic Differential Calculus

Let us now look at the theme of Microeconomics which talks about 2 different types of economic analysis. They are known as positive economic analysis and normative economic analysis. Now, we are going to look at each of them and we will make a distinction between these two.

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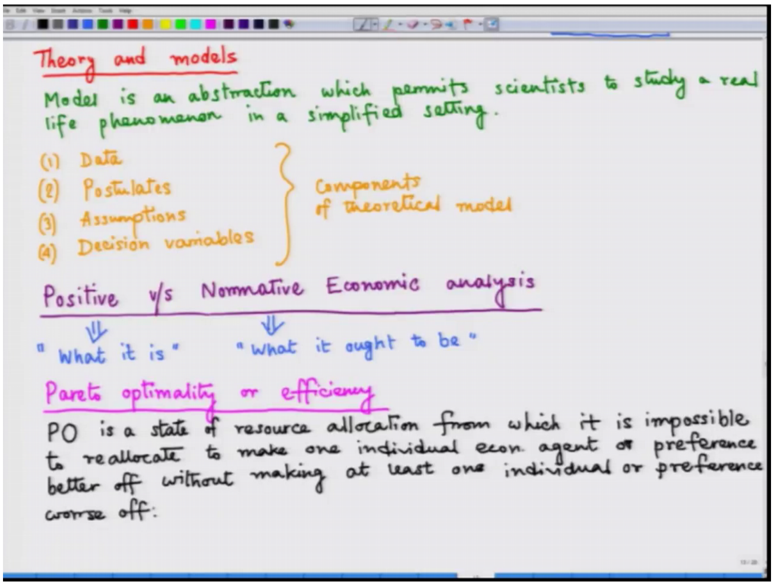

In positive economic analysis, economists are concerned with the description and explanation of economic activities. Here, we focus on facts and cause and effect based behavioral relationships. In contrast normative, economics is value based. There, economists make normative judgments on economic fairness. So, one can distinguish between these two saying that positive economic analysis actually explains, what it is and normative economic analysis explicit, what it ought to be.

Now note that positive economic statements can be empirically tested and either proved or disproved, but normative economic statements cannot be proved or disproved because these are opinion based statements. Now, we are done with the major themes of microeconomic analysis, but as a digression, let us look at another concept which is very well known and useful in economic analysis; this is the concept of Pareto optimality or Pareto efficiency.

So now, we are going to look at the concept of Pareto optimality or efficiency. If you remember we have listed a fourth question proposed by Professor Richard Lipsey on society's resource allocation problem. Lipsey has asked whether society's production and distribution activities are efficient or not, these concept of Pareto optimality are paired to efficiency is related to that very question.

So, let us first define Pareto optimality and Pareto efficiency and then we will explain the concept. I aggregate Pareto optimality by PO. So, Pareto optimality is a state of resource allocation from which it is impossible to reallocate to make one individual economic agent or preference better off without making at least one individual or preference worse off.

Let us now go back to the concept that we have already studied the concept of production possibility frontier or production possibility curve and let us see how this concept of Pareto optimality or efficiency is linked with that. So that way, we can show some example of Pareto optimality through a concept that we have already learnt. So, remember; what is a production possibility frontier that is basically giving us the maximum possibilities of outcome given resource constraints and technology constraint.

Now, all the points on the frontier are Pareto efficient or Pareto optimal why because you cannot produce extra units of one commodity without sacrificing some units of the other commodity. If you consider a point below production possibility frontier that indicates Pareto inefficiency in the production system, because the farm or the economy is unable to produce at per the production capacity the firm is unable to use resources fully and optimally.

Now, we are going to start doing little bit of differential calculus we have earlier focused on marginal analysis and we have told that marginal analysis is an integral part of economic analysis. So, we are going to now establish the linkages between marginal analysis and differential calculus. Here, I am assuming that you have fair idea about high school level calculus. So, you are aware of the concept of graph function limits derivatives and calculus of single variable.

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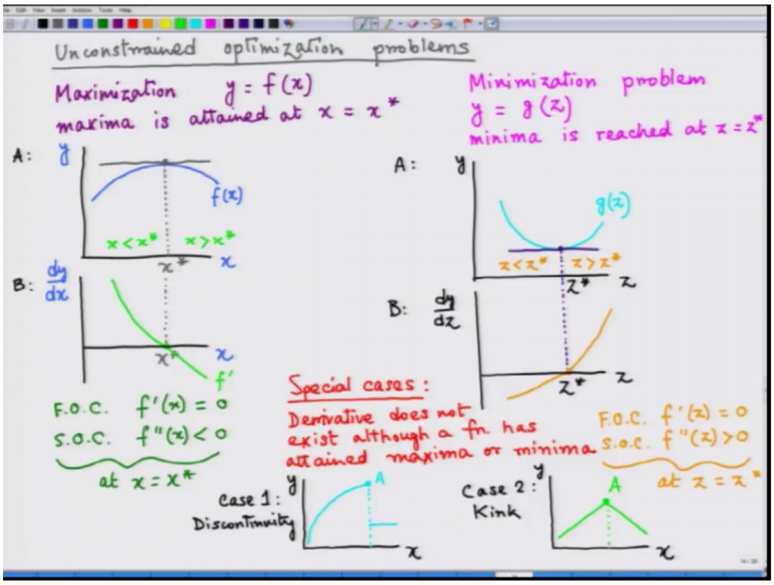

So, now let us have a very quick recap of a maximization or a minimization problem which is an unconstrained optimization problem. First, we are going to look at the maximization problem now we are going to assume a function y equal to f of x. So, y is a function of a single variable x and it is continuous let us assume that so, in that case, how to find the maximum of this function suppose the maximum is attained at x equal to x star. Now let us see; how we can represent the case through a graph and how the concept of derivative is going to be useful to find that maxima.

So, I am going to draw two different panels of graphs. So, in panel A, I am going to plot the actual original function and in panel B, I am going to plot the first order derivative or the slope of the function. So, let us now draw the function. So, suppose this is my function. So, I plot x, the original independent variable along the horizontal axis and along the vertical axis, I measure y and the dependent variable and here in panel B, I measure x along the horizontal axis and I measure the slope d y d x along the vertical axis.

So, we know from our basic calculus knowledge that to find the maxima we have to find a tangent to this function at some point where the slope of the tangent becomes 0. So, suppose this is this point. So, at x star if I draw a tangent to this function then slope of this tangent line is going to be 0, then we can say that the slope of this function will take a shape like this. So, this is f prime the slope.

So, the slope takes 0 value at that exact point x star where the original function fx attains a maximum and in the region, where x is less than x star the slope is positive because the function is increasing and any value of x which is higher than the maximum value x star the slope is negative because the function is sloping downward or it is backward bending.

So, for first order condition of a maximization problem we can write f prime x shall be equal to 0 and f double prime x which is the slope of the dy dx has to be negative now let us look at a minimization problem, alternatively, right. So, there we assume another continuous function y equal to g of say z and we assume that minima is reached at z equal to z star. So, let us draw a diagram I am not going to explain this diagram in detail because we have already seen the case of maximization. So, I will just quickly draw the diagram for the minimization exercise.

So, at z star the minimum value of the function is attained and of course, we can comment on the slope also in this case. So, here we can see that when z is less than z star then the slope is rising, but it is negative, and then when the z reaches z star it will of course, become 0 and when z takes a value higher than z star the slope becomes positive. So, when we plot the slope the dy dz will be 0 at z star and it will probably take a shape like this.

So, now, we have studied the case of minimization also. So, let us summarize what is the first order condition and the second order condition in minimization problem. So, here the first order condition would be similar if prime z equals 0 and f double prime z has to be positive, but note one thing, I forgot to mention in the case of maximization. So, when you are mentioning this first order condition and the second order condition, sorry, I should write a s o c and here s o c the sign of these derivatives are basically evaluated at the minima. So, is basically at this relationship of the signs of the derivatives will hold at z equal to z star and it will hold here in the case of maximization at x equal to x star.

Let us now look at two different cases and these I will call special cases and these type of cases can be found in economics. So, that is why it is important to have some idea about them. So, these special cases are the cases where the derivative does not exist, it is not defined although a function has attained maxima or minima.

Remember we have assumed a smooth continuous function, but suppose if we do not have that; then what sort of possibilities may emerge again, let us focus on the maximization case, you can very well draw similar diagrams for minimization case as well. So, again I am going to draw two panels. So, this is you can say case number one and this is basically case number one is the case of discontinuity in the function and there is a case number two and that is the case of existence of king points.

So, here plotting y and x y and x; so, in the case of discontinuous function, we can have the following shape, suppose, there is a discontinuity or break at a particular point say point a and then there is functional value after the value x equal to A. So, we can clearly see that this function y equal to f of x has attained maxima at point A, but as there is a break in the function there is discontinuity and the derivative is not defined.

Similarly, let us talk about the king points now in that case we can have a functional relationship like this where again at point x equal to A, you see the function has reached its maximum, but at this point a infinite number of tangents can be drawn. So, basically the slope is undefined as the derivative is undefined we cannot use the first order condition f prime x equal to 0 to test whether there is maxima or minima. So, earlier we were assuming f of x. Now we are going to assume f of x 1 to x n suppose, there are n number of independent variables.

So, here in this case, we are going to study how our concept of derivative and differentiation will change to accommodate marginal analysis.

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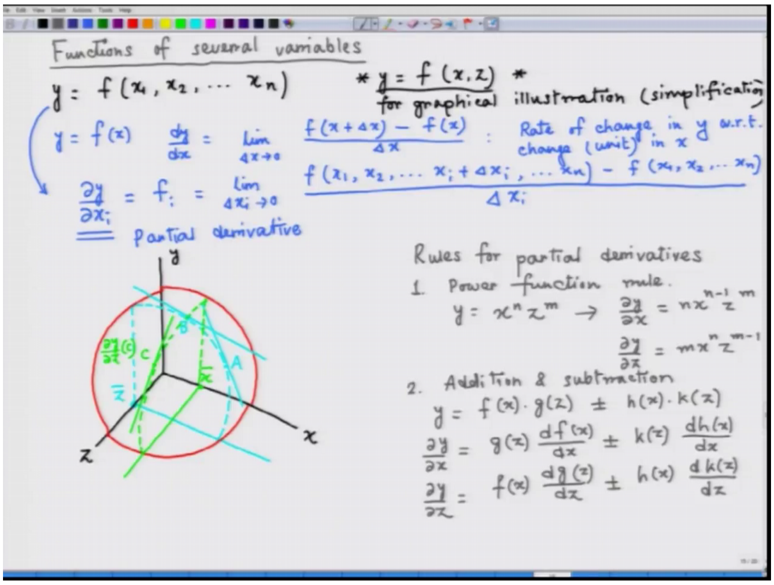

So, the general form we assume. But for simple graphical purpose we can also assume that well there are two independent variables; so, f of x and z. So, this form we are going to utilize when we are going to draw some diagrams. So, let us start with the basic definition of the derivative in the single variable case then it will be easy for us to follow what modification we need to do. So, we again start with this simple case y equal to function of x. So, in this case, the derivative dy dx is defined as limit of delta x tends to 0. So, we are talking about infinitesimal small change in the variable x and how that has impacted the function on value that is the change in function value and we need to divide by delta x.

So, basically here in this case derivative says it is a rate of change in y with respect to change in general unit change in x. Now let us move to this function of several variables. So now, let us consider this y and in this case, we require this concept of partial derivative. So, remember that assumption of ceterris paribus everything remaining same everything holding constant. So, here also we are going to assume the same thing suppose we pick 1 x and we change its value holding, other x values constant at their previous level. So, if that is the case then we get what is called partial derivative and that is basically this statement.

Now, you observe that infinitesimal small change in x i we are talking about a particular variable x 1, x 2, there are no changes and then for i-th variable there is this infinite decimal small change in x and this expression is divided by delta x i. Now let us have a graphical illustration of this concept of this thing called partial derivative this is a very useful concept in microeconomic analysis. So, let us look at a graph to see what exactly happens in this case we are going to draw three axes because there are three variables. So, here for this graphical illustration purpose I am working with this simplified functional form. So, I measure y here I measure x here and I measure z here.

So, 3D diagrams are difficult to draw, but let me try my level best now let us talk about the partial derivative. So, when you take partial derivatives suppose you want to take partial derivative with respect to x then what do you have to do you have to fix a value of the other variable z at a particular value say z bar. So, what you do you fix that value of z z bar and then you draw a line a vertical line as I keep value of z fixed and now I change my value of x; how the values of y variable are going in a change suppose you take a point A.

So, then the partial derivative will be basically the tangent to the surface the slope of the tangent to this surface now if you move to a point say b here similarly you have to draw another tangent at that point and the slope of the tangent line will give the partial derivative value. Now we can reverse the process we can fix the value of x at say x bar and we can change the values of z to see its impact on y. So, we have to follow the same procedure.

So, the partial derivative is basically slope of a cross section through the surface rules for partial derivative first is the power function rule in this case we assume a function say y equal to x to the power n and z to the power m, I am showing the case of 2 variable, but this can also be generalized that is not a big problem. So, in this case if del y del x takes the value of n x n minus 1 z m and del y del z takes the value of m x to the power n z to the power m minus 1.

Now the second rule is about addition and subtraction. Now, here we assume functional form like this f of x times g times z plus minus h times x times k times z. So, in this case, rule says del y del x can be derived as and del y del z equals fx times d g z dz plus minus h x d kz dz.

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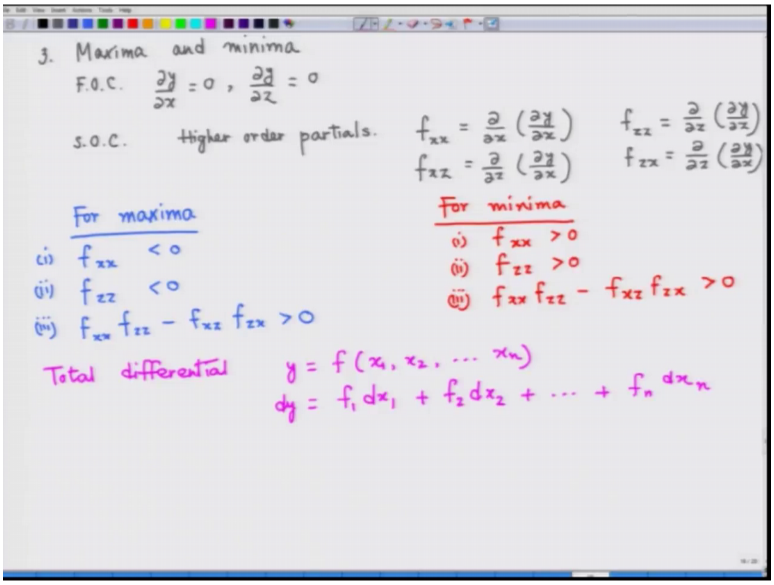

Now the third rule is about finding maxima and minima here to maximize or minimize a function f of x z, we have to take partial derivative for each of the unknown independent variable and set them equal to 0. So, basically for both maximum and minimum for both maximization and minimization problem, we know that we have to deal with this first order condition del y del x equals to 0 and del y del z equals to 0.

Now to judge whether we have reached a maxima or minima we need higher order partials. So, now, we are talking about second order condition to judge whether we have reached a maxima or minima. So, for that we need higher order partials. So, in this simple case, there will be 4 higher order partials. So, this is the second order partial and another second order partial will be f zz equal to del del z del y del z and then there will be some cross partials ok.

Now, let us write the rule for maximization problem first for maxima these conditions are required; I am not going for mathematical proof just remember these results now for minima, we require the following set of second order conditions. Now we are going to introduce another important concept which is heavily used in economic analysis and this is the concept of total differential this is based on partial derivatives only. So, here we assume the general function f of x 1 x 2 x n. So in that case, we write total differential dy equals f one which is the partial derivative with respect to the variable x 1 and multiple this is multiplied by change in the variable x 1 plus f 2 which is the partial derivative with respect to the variable x 2 times the change in the variable x 2 and so on so forth.

In the next lecture, we are going to continue this discussion on some mathematical concepts which are very useful for microeconomic analysis.