Microeconomics: Theory & Applications

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Lecture - 35
Long-run Cost Curves

Hi. Welcome back to the lecture series on Microeconomics. We have been discussing theory of cost. Last time we have finished our discussion on the short run cost curves. Now we are going to see how we can derive Long-run Cost Curves.

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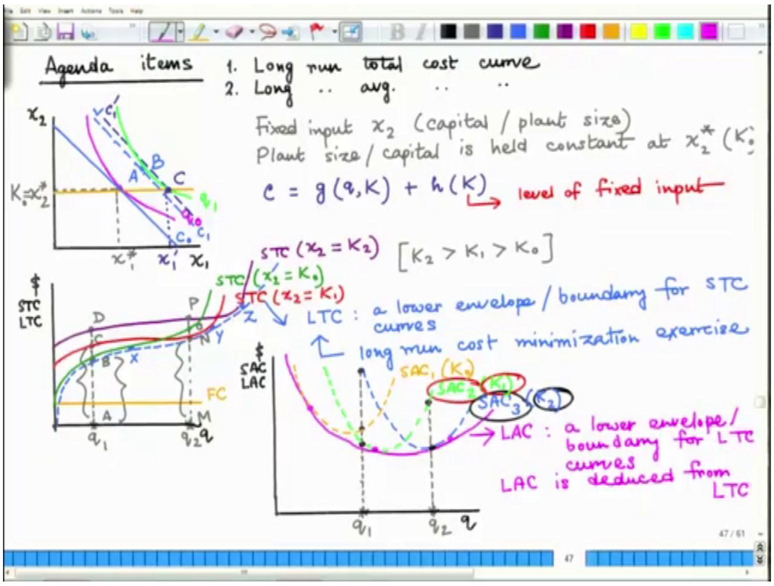

So, in this lecture we are going to learn about long run total cost curve, long run average cost curve on some related concepts.

So, in the case of short run cost function we have seen the total cost and average cost are the functions of the output level being produced by the firm. But in the short run there is one factor input which is held constant, rather than in long run all inputs are variable inputs. Now we can also say that short run is basically the period where the firm operates and the long run is basically planning horizon, because firm always plans for what is going to happen in near future.

So, now let us going to assume that; we start with a short run case where the x 2 input is the fixed input and in the long run we are going to be able to manipulate the level of the x 2 input. It implies that the firm is going to optimally choose the level of input 2 which is held constant in the short run. So, now let us see how that is going to impact our cost analysis.

So, as usual I am going to start with a diagram before I introduce new concepts. So, I am going to have a diagram where I plot the inputs along the axis. So, x 1 is the factor input which is a variable input in both short and long run whereas, x 2 is the factor input which is held constant in the short run.

So, it is a fixed input in short run, but it is a variable input in the long run. So now, let us look at the firms equilibrium in this case. So, let me draw the isoquant map here. So, suppose I start with an isoquant level producing output q naught and we know how to derive the firms equilibrium. We have to draw the iso cost line in this input plane such that, it makes a tangent with the iso cost or with the iso product curve or the iso quant curve at some point it makes the tangent. Say this is the point A. And at this level the optimal input combination is found and the firm gates to know what would be the optimal level of input 1 and what would be the optimal level of input 2; it should higher or purchase to produce the quantity level q naught.

Now, this is the short run equilibrium. Now note as this is already done, did these cost minimization exercise is already done by the firm, firm knows this numbers, and once the firm knows it locks the fixed input. So, in this case let me assume that the fixed input X 2 is basically capital and we can also assume that more specifically we can assume that this is plant size.

So, the firm has optimally chosen some plant size once it has targeted an output level q naught and then I say that this X 2 is held fixed ok. So, the plant size or capital is held constant at the X 2 star level. Let me introduce a new notation here and that is K naught, that is basically the plant size all right.

So, now, with this new notation of plant size, I can also write this plant size think here q naught that is equal to X 2 bar. Now let us see what happens if the firm wants to expand its production level.

So, of course, now you are not going to work with an oppose topping expansion, because this is short run. So, here let me draw this parallel to x 1 x is line. So now, the if the firm wants to produce higher level of output, say at level q 1, then it has to operate along this orange line because, I have already fixed the plant size or the level of my fixed input which is capital right.

So, the firm then has to find its equilibrium, short run equilibrium though at these intersection point of this isoquant and this fixed level of capital or plant size line the orange line. So, you see that to expand output now the firm cannot tweak the level of plant size but it can higher extra units of variable input. So, x tilde x dash 1 and that is the way the firm will now produce extra output and expand, but note one thing. Let us now look at the long run equilibrium of the firm, if the firm is allowed to change both the inputs optimally. So of course, the firm will then solve a Lagrange and we will try to find out a tangency point between these new iso cost line, which has the same slope.

So, the nu iso cost line has the same slope as of the old iso cost line, so at these nu equilibrium point B, the formal optimize its production and then accordingly it will choose the input bundle. But note that as it is not allowed, then the firm is force to operate at point C.

So, if the firm is force to operate at C, then we see that actually the cost of production is higher at point C compared to point B because, the iso cost line which passes through the point C lies above the iso cost line; say let me now denote decides the cost lines, so this is C naught, this is say c 1 and let me now called this c 1 prime, ok.

So, the iso cost line C1 prime lies above the iso cost line C1 that basically tells you that if the firm is in short run where it has to operate with the fixed plant size, then it can expand the output, but that expansion cannot be done in a cost minimal manner. So, the firm has to pay extra right.

So, we can write the firms cost function in this case as C equals some function g which is of defined over q and K, K is the capital or plant size q is the output level plus another function h which is defined over the plant size which is K alright. So, we are going to draw a diagram for the short run total cost curves.

So, here plotting output level along the horizontal axis and the cost indirect terms or repeat terms whatever, so this is basically my total cost and short run total cost of course. So now, we know that this is recap, but I think recap here sometimes. So, the short run total cost curve has 2 components; one is basically my fixed cost and the other one is variable cost, which start from the origin and it has the inverse a shape.

So, if we vertically sum these 2 curves then, we get the short run total cost curve. So, let these short run total cost curve is derived keeping some fixed value of some constant value of fixed factor input x 2 and let me denote that as the K naught plant size level alright. Now, there can be other plant size available of course. So, if that is the case, then let me draw a couple of alternative plant sizes, which could be made available to the firm in short run, but the firm has chosen to operate with K naught plant size and that can be like this. There can be, so let me assume that this is STC for a higher plant size and it takes value K1. Now we draw the third plant size which is even higher right something like this right and this is STC.

So, here the fixed input 2, plant size takes value K2. So, here let me write down the assumption that K2 plant size is greater than K1 plant size and that is greater than the K naught plant size right. So, that is basically the assumption that we make. So, basically what we observed: there are three different plant sizes which are possible to choose from and for each of these plant sizes, there is a corresponding short run total cost curve if the firm chooses a particular plant size right.

So, now let me see how that has an impact on the cost. Suppose my firm decides to produce q 1 level of output. Let us see which plant size will be the most beneficial for the firm ok. So now, you see that suppose there are intersection points with these 3 short run total cost curves. So, here we have points like B, C and d right.

So, you can see that, if the firm wants to produce q1 level of output with the plant size K naught then, the cost is A B right. Now if the firm wants to produce the same level of output, but for a higher plant size, then the short run total cost curve, which is relevant for the firm that is basically the red 1 and that will give a higher total cost given by a c. So, what we can see that for a lower level of output, if the firm wants to produce that level of output with the higher plant size, the total cost is higher because of the high fixed cost involved with high or the higher level of plant size.

Now, let us note that this story completely changes if the firm chooses to produce or decides to produce a higher level of output; say q 2 level of output. And then here you can see that to produce that level of output, we have to move up to find the points on the short run total cost curves for different plant size.

So, again you see that if I draw a perpendicular line that will create three intersections and let me name them MMP. So, this is say point M, then I call this point N, this is my o point and this is p point say. So now, you can see that for an output level q 2, it is beneficial for the firm if the firm, does not stay with the plant size k naught because, that will give a higher cost of production m o. So, the firm can optimally choose a plant size K1, which will give less amount of cost mn. So, optimally the firm can go for a higher level of plant size which results into a lower level of total cost as given by the distance MN.

Now, let us look at the short run total cost curve diagram. There, we got a family of short run total cost curves. Each curve corresponds to a particular plant size. So, what will happen to the long run total cost curve in this case? So, let us draw the long run total cost curve in the same diagram.

So, if I do not produce anything in the long run. That is an option. So, basically I will start with origin. So, the long run total cost curve I have to draw in such a way that, it becomes tangent to the short run total cost curves. Suppose we have a tangency point right here, say at point x then there can be another tangency point between the long run total cost curve and the short run total cost curve for the plant size K1 at point y. And there is another tangency point between the long run total cost curve and the short run total cost curve for the capital size or plant size K2 at these point z.

So, this blue line which is this broken line that is basically a lower envelope of the short run total cost curves. And that is basically my long run total cost curve. So, long run total cost curve is basically a lower envelope or boundary for short run total cost curves. And this long run total cost curve comes from the long run cost minimization, exercise where all imports including plant size is variable and the firm chooses all of its inputs optimally all right.

So, there is a corresponding diagram in terms of the average cost function or average cost curve as well. So, let us look at the diagram. So, you have quantity produced and we have cost measuring terms of dollar or rupee ok. So, let me also write LTC here. So, here I am measuring shot run average cost and eventually long run average cost right.

So, now let me assume that the plant has 3 alternative plant sizes as we has assumed earlier and each of them we will give raise to a short run average cost curve. Of course, if you have a short run total cost curve, you will have a corresponding short run average cost curve right. So, this is basically my SAC1 that corresponds to my capital or plant size K naught.

Then I have a similar diagram short run average cost curve two and this time it is for the plant size K 1 ok. Note that the minimum short run average cost has gone down for the higher plant size. Another short run average cost curve associated with the third level of plant size which is denoted by S A 3 and K2 alright.

So, these are basically my short run average cost curves corresponding to the short run total cost curves. Now suppose the firm wants to produce, some output level say here say q 1, level of output the firm wants to produce. So, now, let me suppose the firm wants to produce q one level of output.

Now, note that I can draw a perpendicular line through this output level to judge which plant size. We will, give me the lowest average cost of production to produce this level of output in the short run. Now it is quite cleared here that to produce Q1 level of output the firm we will choose SAC 2.

So, it will basically choose K1, level of plant size because, at that plant size level the firm can produce q 1 level of output at the least average cost or at the minimum average cost, But of course, the same analysis as we have done earlier, if I now increase the output level, say q if the firm wants to produce a higher output level q 2. Say then if you draw the perpendicular line at that output level we can see that the intersections with that perpendicular and the short run average cost curves say that it can be produced at the least cost manner or with minimum average cost with this plant size K2.

So, we have to use the short run average cost 3 curve for that matter ok. So, here also we get a series of short run average cost curves corresponding to our choice of plant size. So, how to put a boundary on this Hamiltonian short run average cost curves. So now, we can derive the long run average cost curve from the long run total cost curve that we have already derived. And this will be and U shape curve and it will make tangencies with the short run average cost curves for different plant size levels at various points.

So, the long run average cost curve is basically a lower envelope or boundary for long run total cost curves all right and these long run average cost curve is deduced So, IAC is deduced from in the long run total cost curve alright.

So, now we are going to look at the relationship between scale elasticity which directly comes from the physical aspects of production technology and this cost curves.

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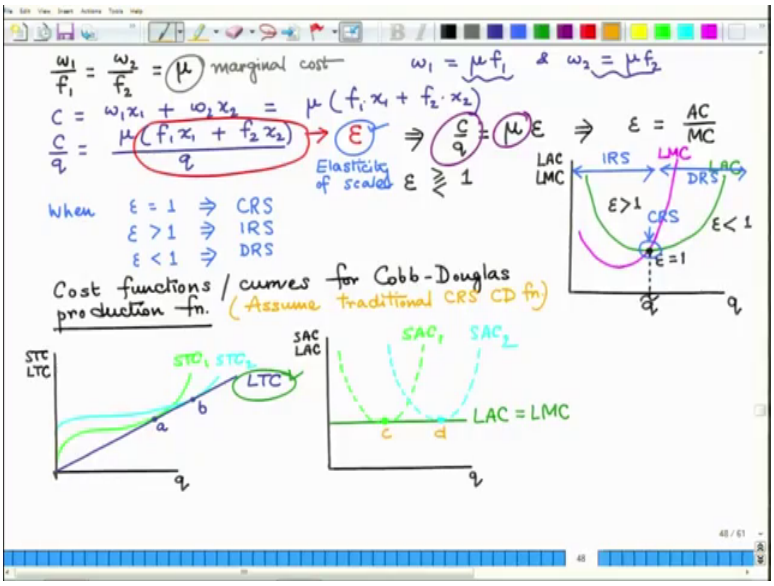

So, we know that if the firm is on expansion curve then, this condition will be made right. Where f 1 and f 2 are the marginal productivities of inputs x 1 and x 2 and mu is basically the marginal cost alright ok.

So, now we can definitely write from here w1 equal to mu f1 and w2 equals to mu f 2 all right ok. So, now, let us go back to our cost expression which is w 1 x 1 plus w 2 x 2 and then basically if I now insert these findings in the cost equation, we get and expression like this ok.

So, now let us divide this cost expression by the output level to get the average cost. And then we have on the other hand all right ok, now note that these entity here is basically my scale elasticity of output that we have derived earlier while we were studying the production function and associated concepts right. So, from there I can now write: now this can also be rewritten then as epsilon equals average cost of production which is basically this and the average cost is divided by the marginal cost which is basically mu right.

So, this epsilon, now can take different values; it can be greater than or equal to or less than 1, depending upon the behavior of the average cost curve and marginal cost curve. So, now, let us look at various possibilities through a diagram ok. So, remember that we are in the long run.

So, you are basically majoring output along the horizontal axis and we are going to major long run average cost curve and long run marginal cost curve along the vertical axis.

Now we know from the analysis we have done so far that, our long run average cost curve will be and mu shaped one like a (Refer Time: 28:55) long run average cost curve and then we have the marginal cost curve, which is also u shaped right. If that is the case then we can draw it like this. And we know for sure that the long run marginal cost or marginal cost curve passes through the minimum point of the long run average cost curve ok.

So, now let us concentrate on this intersection point. Now let me denote this output level where the marginal cost passes the average cost curve. So, the output level which corresponds to the minimum point of the long run average cost curve by q tilde. So, the output level less than q tilde for those output levels, we observe the epsilon takes value greater than one alright. And at these particular output level, q tilde epsilon takes value 1 and finally, when we have output level higher than q tilde then we basically have epsilon less than equal to 1 alright.

So, now let us right, when we have epsilon equals to 1, we know that; that means, we have constant returns to scale right from our theory of production, then when epsilon is greater than one then we have the case of increasing returns to scale and when epsilons takes value less than one then basically, we have in the case of decreasing returns to scale.

So, just let me write down here. So, this epsilon here is basically elasticity of scale all right. So, when we can demarked these area, so basically the all the output level, the range of output which is less than q naught, sorry q tilde that basically gives the increasing return to scale production function. And for all output level greater than the output level q tilde we have the so called case of decreasing returns to scale only at this intersection point where epsilon takes value 1. We have the case of constant returns to scale ok.

So, now we are going to look at couple of special cases. So, suppose we have a conventional traditional form of Cobb-Douglas production function a times x 1 to the power alpha times x 2 to the power 1 minus alpha displaying constant returns to scale. So, that that means, that we are working with the linear homogeneous production function, then in that case how my cost functions are going to look like. That is what we are going to study next because Cobb-Douglas production function is a very common form of production function and cost function, which are used in a applied production analysis.

So, this case is for CRS technology. So, there we can first plot the total costs both in short run and in long run along the vertical axis and output level along the horizontal axis. So, of course, first we are going to plot the various inverse a shaped short run total cost functions say STC1 for a particular plant size. Then of course, we are going to have mode, so there will be a series of short run total cost functions for various plant size all right.

So, if these are the different cases. Then we can draw a straight line from origin which will be tangent to various short run total cost functions at various points. So, this ray gives me the long run total cost curve for a linear homogeneous production function, which could be Cobb-Douglas production function all right.

Now, let us look at the average cost diagram. q Now I am going to plot SAC and LAC all right. So, here let me first plot my long run average cost curve which is going to be a straight line that can be easily deduce from the long run total cost curve which we have just derived right. So, LAC shall be equal to LMC in this case of constant return to scale Cobb-Douglas production function.

So of course, in this case we can have series of short run average cost curves displaying various levels of plant size. But note that they will be tangent with the long run average cost curve and it is the minimum point of the short run average cost curve which becomes tangent with the long run average cost curve.

So, now we can draw another short run average cost curve corresponding to a different plant size, a higher plant size and here it makes another tangency. So, basically we can say that the tangencies likes point C and D will all lie along the straight line which gives the long run average cost curve, in the case of a constant return to scale Cobb-Douglas production function.

So finally, we are done with our discussion on long run cost curves. So, for we have finished our discussion on various types of short run and long run cost curves. Now we are going to look at their implications while a firm chooses and equilibrium in the short run and in the long run.

So, we are going to continue with this discussion in the next lecture.