Microeconomics: Theory & Applications
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Lecture -33
Cost Function (Part-2)

Hello, welcome back to the lecture series on Microeconomics. Last time we have derived short run and long run cost functions through the Lagrangian method. Now, we know that a Cost Function of a firm can be deduced from the optimal input combination or optimal input bundle to produce a certain level of output for a firm. Now, let us see whether the same concept cost function can be derived from some other angle or some other tool that we have already seen. Now, I am going to take you to the concept that has been defined before and the concept is expansion path.

So, note that along and expansion path, we see lots of firm's optimal points and these optimal points are basically showing optimal input bundles. Each of this firm's equilibrium points lie both on isocost line and isoquant. So, the point when it belongs to the isoquant it gives a specific level of output and that very point when it lies on an isocost line gives a particular level of total cost of production. So, basically this way there is linking between the total cost of production and output being produced in the long run and as we all know from the expansion path diagram that as we move from origin towards right from left to right we see that output is expanding.

So, is the level of cost; so, we see that there is a positive relationship between the long run total cost and output level. So, as we have assumed that our input prices are given. So, the firm faces some constant input prices from perfectly competitive market, we can say that they are fixed and if that is the case then total cost becomes a function of the quantity level that the firm wants to produce. So, basically our conditional factor demand function or input demand function becomes function of the quantity that the firms has already chosen and hence the cost function. So, let us have a look at the mathematically expression of that and then we are going to get in to the deeper details of cost function and the cost curves ok.

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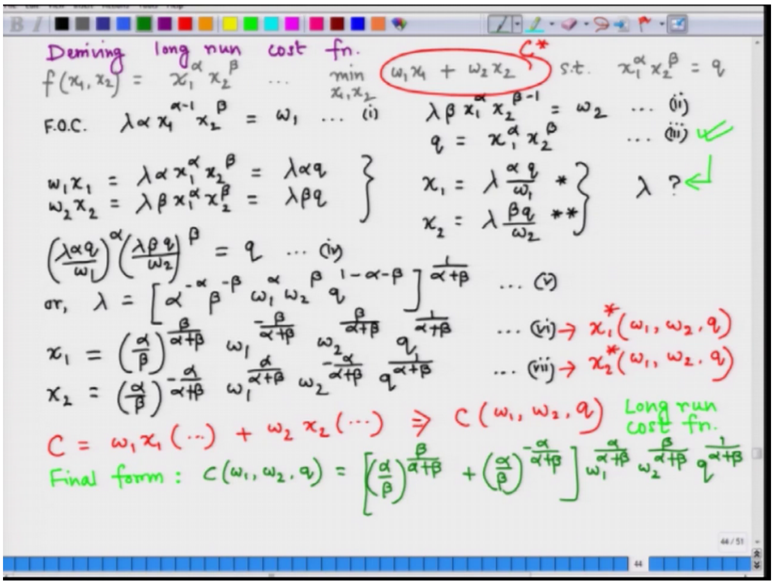

So, now let us start with general problem where we do not assume a constant return to scale or conventional form of Cobb Douglas production function. So, here we assume that the Cobb Douglas technology is given by in its general form, but for simplicity we can now ignore the capital A parameter the technical efficiency parameter we can assume its value as 1 and now we can say that my production function looks like this and I don't assume whether the alpha plus beta takes up value 1 or not.

So, given this situation; letus have a cost minimization problem in long run and this can be written as minimize w 1 time x 1 plus w 2 x 2 subject 2 x 1 alpha x 2 beta shall be equal to particular output level say q naught right. Now, for simplicity I can again erase this super script and let it be only q. So, here the same thing it will be minimized with respect to 2 variable inputs x 1 and x 2. So, now again you know this is quite straightforward.

So, you have to set the Lagrangian right. So, I skip the Lagrangian for you I straight away go to the first order conditions. I think that you can easily cross check my calculations here. So, here the first order conditions could be written as lambda where lambda is the Lagrange multiplier times alpha x 1 to the power alpha minus 1 x 2 to the power beta.

So, these will be equal to the input price for input 1. So, this is my equation number 1 which is derived for input number 1. Similarly, I can have lambda beta x 1 to the power alpha x 2 to the power beta minus 1 and that has to be equal to input price 2 w 2 and this is the fast order condition for input 2 right. And lastly, we have to have the constraint to be written right. So, this is our number 3 now multiply the first equation by x 1 to get.

So, this is basically lambda alpha q right. Now, you multiply the second equation by x 2 to get and that is basically lambda beta q right. So, from here we can get x 1 equal to lambda times alpha q divided by w 1 and similarly, we can get x 2 equals lambda beta q divided by w 2. I am skipping lots of steps with the hope that you can fill these missing steps.

So, here from this two what is missing we do not know as of we get the value of lambda right. So, we have to find lambda as well. So, we can use this third equation here to get the value for lambda. So, now if we substitute the solution for x 1 and x 2 in to the third fast order condition this one, then we get what then we get lambda alpha q divided by w 1 this will be whole to the power alpha and then lambda beta q divided by w 2 this will be whole to the power beta right. And we get this let me name this as 4.

So, now we can solve this for lambda right. And we can get a very complicated expression, and this will be whole to the power 1 divided by alpha plus b right let me call this number 5. So, now we can plug this lambda value in to this input demand expressions say these 1 denoted by asterisk and input demand for 2 denoted by double asterisks.

So, if we do so then what do we get? Again we get some complicated nasty expressions right. We can call this 6 similarly we can get x 2 as alpha over beta minus alpha beta w 1 to the power alpha divided by alpha plus beta w 2 to the power minus alpha; alpha plus beta and finally, q to the power 1 over alpha plus beta; we call this expression 7 right.

So, now note that this expression 6 which is basically the input demand function for input 1 can be now written as a function of input prices and the output level only as alpha and beta are given parameters and similarly we can write the same thing for my input 2 as well right. So, these are the conditional factor demand functions or input demands. Now, as these are the optimized values from a cost minimizing excise, we can plug these values in the cost expression here. And if we plug then we get what we get the minimized cost which can be denoted by c star and if we do so then, we get the long run cost function which is written as w 1 times x 1 that complicated function w 2 time x 2 which is a complicated function now note that as a whole this will give a function of w 1 w 2 and q as well right.

So, this is basically my long run cost function now the final form of cost function in the Cobb Douglas case can be given as. So, I am again given you the final expression. I wish that you would be able to feel amazing steps very complicated form, but you do not have to remember this you can always derive this. So, now note that we have derived the long run cost function which is an expression involving the input prices and the output level, but also note that the firm has some control over the quantity that he the quantity of output that it wants produce, but it has no control on the input prices.

So, we can safely assume that cost function long run cost function is the function of the output level only. So, from here we would like to (Refer Time: 16:06) our journey to derive various types of cost curves which has very nice economic interpretations and use in practical decision making problems of a firm.

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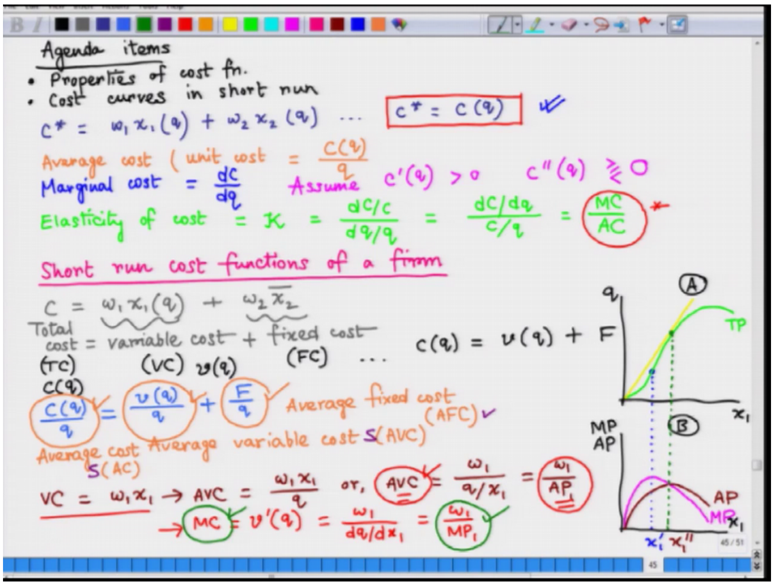

So, basically in the long run I can write my minimized cost which is represented by the cost function can be written as when I have 2 variable inputs x 1 and x 2.

So, if that is the case I can write cost function as a function of output only and not the input prices. So, now we take a more simplified version of the cost function where the total cost is a function of the quantity of output to be produced only and now we are going to define some concepts which are very critical in the theory of firm and they are the concepts of average cost and marginal cost. So, first I will start with average cost. Sometimes it is also known as unit cost because this talks about the cost of production per unit of output.

So, we can define it simply total cost divided by the number units of output being produced. So, this is my average cost now when we talk about a function in micro economics we have always shown some interest to the slope of that function. So, here also we are interested to know if we take fast order derivative of these cost function given here what do we get? We get what is known as marginal cost. So, the marginal cost is defined as dc dq.

So, what does that mean? So, marginal cost can be interpreted as the additional cost to be incurred by the firm if the firm wants to produce 1 additional unit of output. Now, let us see how these marginal costs behave as the firm decides to produce more number of outputs. So, basically we are now talking about curvature of the marginal cost curve.

So, one can assume that the marginal cost which can also be denoted as c prime q is positive. Now, the slope of marginal cost curve which is c double prime q can take any value. So, safely we can say that it can that secondary derivative can take any particular any value it can be greater than less than or equal to zero depending upon different production conditions we are going to visit this issue soon. So, next concept which is of some interest too and production economist is the concept of elasticity.

So, let us define total cost elasticity for a cost function. So, that can be given by represented by the Greek word Kappa and we defined that to be equal to proportional change in total cost divided by proportional change in the output level right. Note that alternatively these can be represented as dc dq divided by c over q and we have seen the interpretation of these expressions. They are basically the marginal cost and average cost.

So, this is an interesting result that elasticity of cost depends on the marginal cost and average cost of the cost function. Now, let us look at the distinction that we have made earlier between the fixed factor and the variable factor. So, just you remind you again when we are in short run if we cannot change the level of a particular input then you know that’s called the fixed factor and the associated cost is known as the fixed cost. And all other factors which can be changed even in short run they are basically the variable factors and the costs associated with this type of factor inputs are known as the variable cost.

So, our cost of production in short run can be broken down into 2 components component number 1 is a fixed cost and component number 2 is a variable cost. So, now we are going to study short run cost functions of a firm. So, this is the departure from whatever we have done so far. So, so far we have done everything in the long run.

So, here we start with a total cost expression which is given by w 1 times x 1 of q. So, that's basically my variable input and the expense on the variable input right. So, this is basically my variable cost. And then, the expense made on the fixed level of fixed input that is given by x 2 bar and that is basically my fixed cost right and this is basically my total cost right let us first introduce some abbreviations.

So, total cost is abbreviated popularly as tc then we have variable cost which is abbreviated by VC and the fixed cost is abbreviated by fc right. So, here note that my cost function as I write c as a function of q, I can also write my variable cost function as a function of output level 2. So, I can write v as a function of q.

So, I can finally, write an expression like where f is basically denoting the fixed cost now we can define certain concepts which are basically the concepts of average variable costs and average fixed cost. So, basically our target is to represent average cost into 2 components.

So, we can divide this c of q by the quantity of output total cost divided by the units of output produced and we get this. So, now note that this component the first component is known as average variable cost which is again abbreviated as AVC and then we have the second component and that is known as average fixed cost and that is abbreviated as AFC and the some of these 2 gives us average cost of production which is abbreviated as AC.

Now, if you want to add S in front of this that is also because we are dealing with short run, but there is no need to add and S in front of AFC because this things exists only in the short run having defined the concepts of average cost average variable cost and average fixed cost let us now look at the possible shapes of these curves in short run.

So, first I am going to draw diagram to show the possible shapes for the marginal cost and the average variable cost in the short run now also try to show you that whatever we have learned in theory of production is also linked with theory of cost. So, I will have two panels of diagram in panel a I will draw neoclassical production function and let me mark this as TP total product curve right and then I will have.

So, so I am basically measuring my variable input along the horizontal axis and my output as along the vertical axis. So, now I am going to have a panel b diagram where I am going to represent marginal product and the average product along the vertical axis and the same variable input x 1 along the horizontal axis. So, this is kind of recap, but lets have a recap. So, now the first target is to have a graph for the marginal product right.

So, we know that marginal product takes the highest value at the point of inflection. Say, somewhere here where the slope of the total product function stops raising and it hits a maximum value and then it starts falling. So, we see a change in the curvature of the total product or the production function. So, earlier it was increasing then after the inflection point it starts increasing at a decreasing rate right. So, let us have this denoted by x 1 prime level of input right.

So, now if we want to draw the marginal product curve we know that it will start from origin and then it will heat a maximum value and then it will start to fall. We are not interested in the negative segment of the marginal product. So, we stop our marginal product right here marginal product curve. Now, let us move to the average product curve right. So, here how to get the average product we know the trick.

So, basically we have to draw a straight line through origin and you know it should be tangent to the total product curve at a particular point and at that point where it becomes tangent to the total product curve at that point the average product becomes highest right. So, now let us let us have that input level mart here also. So, now note that for the average product it will for start raise, then at this level of input say x 1 double prime it reaches the maximum and then it starts to fall right. So, this is what we have from the neoclassical theory of production function.

Showing the variable return to factor or vary or the diminishing marginal productivity right now let us link these concepts from the neoclassical theory of production to the theory of cost we have already a derive the expression for average cost and marginal cost let's have a look at them again. So, I start with the total variable cost which is abbreviated at VC. So, what is VC? VC is basically w 1 times x 1 ok. So, now, if I write the average variable cost AVC from here what do, I need to do I need to just simply divide this by my level of output right.

So, now, note that this can be written or rewritten as w 1 divided by quantity divided by the level of variable input right. So, what do we get we get w 1 divided by average product of the input 1. So, this is an interesting result. So, here what we see? So, the average variable cost if the ratio of the price of the variable input and the average productivity of the variable input ok.

So, as w 1 is constant here we see that there is a negative relation between this average product of the input and the average variable cost of the production unit. So, when average product curve is increasing; that means, that average cost curve should be falling and when the average product curve is falling; then the average variable cost should be rising. So, there is an inverse relationship between these two ok.

So, now let us have a look at the marginal cost aspect. So, again we start with this VC right. So, from here if I want to derive marginal cost I know what I need to do I need to basically take the derivative right like this and what that would be? So, that would be. So, what we observe is this. So, again we observe another interesting result. So, the marginal cost of production from the cost side of the production technology has a relationship with the marginal product of the variable input right. And again we see the same inverse relationship right.

So, when marginal product is rising then the marginal cost should be falling and when the marginal product curve is falling then marginal cost should be rising due to this inverse relationship right. So, we will continue our discussion in the next lecture.