Microeconomics: Theory & Applications
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Lecture – 32
Cost Function (Part-1)

Hello, welcome back to the lecture series on Microeconomics. We are now going to conclude our discussion on the physical aspect of production technology and move to the monetary aspect of the production technology.

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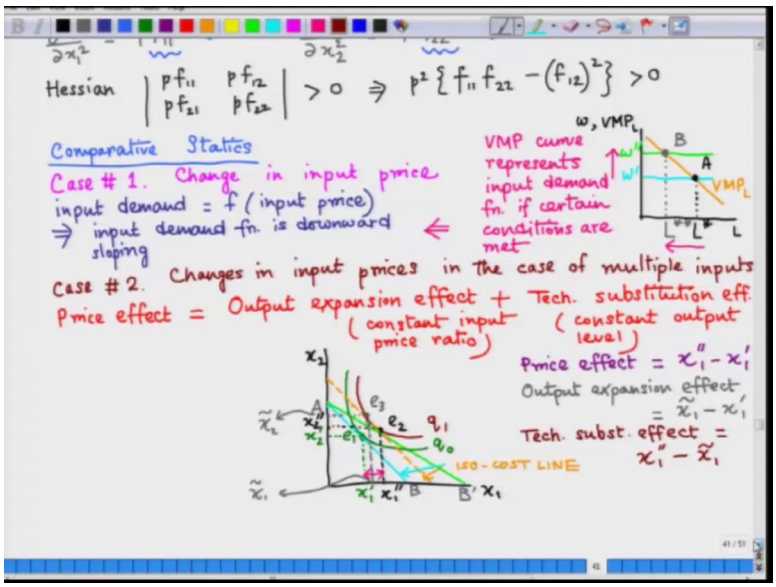

So, now we would like to end this discussion with a definition on expansion path. From this price effect decomposition analysis we can find out an interesting concept which is known as output expansion path. So, let us first draw a diagram and then we will have a formal definition.

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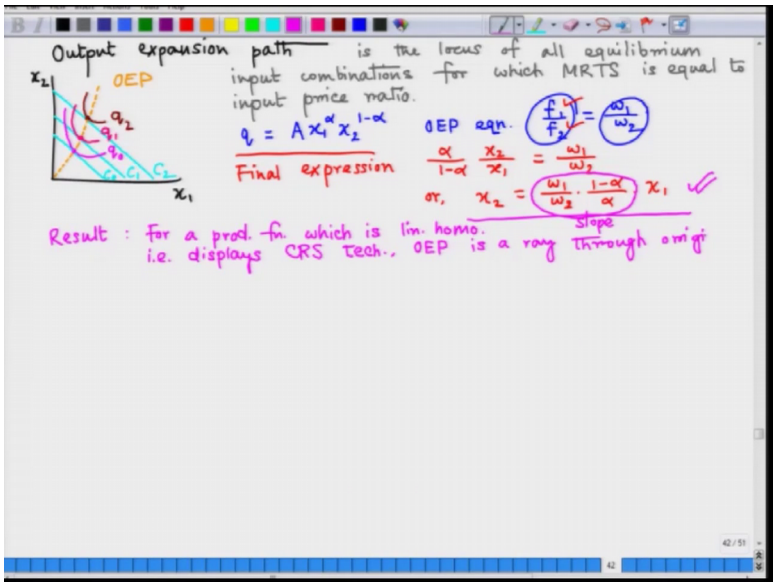

So, here we are going to have a series of isocost lines with the same slope a higher level of isocost line basically represents higher volume of resources available to the farm. So, this is denoted by isocost level C naught, this is denoted by C 1 and this one is C 2 right. And we can find out a series of equilibriums for the single farm where tangencies are obtained.

So, say this is for output level q naught, then we can get another tangency here for isoquant level q 1 and similarly we can have another tangency with isoquant level q 2. And if I join these equilibrium points forms equilibrium points with a curve, then the locus of these equilibrium points is known as the output expansion path or abbreviated as OEP. So, now we are going to have a formal definition. Output expansion path is the locus of all equilibrium input combinations for which the marginal rate of technical substitution which is the absolute value of the slope of the isoquant is equal to input price ratio.

So now, let us look at one simple example with this conventional form of Cobb-Douglas production function. How can we find expansion path for a Cobb-Douglas production function? So, we start with the original form of a Cobb-Douglas production function displaying a CRS technology and the equation which represents OEP is given by f 1 divided by f 2 equal to w 1 divided by w 2. The slope of the isoquant and the slope of the iso cost line.

So, now, we can plug values for this we can derive the partial derivative from this given form of production function. So, if we do so, we get an expression. So, I am giving you the final result. So, we can rewrite this expression as. So, now, have a look at this final expression. This is an equation of a straight line where this component is basically the slope and this does not have any intercept. So, this provides me an equation of a straight line from origin.

So, we get this result that for a production function which is linearly homogeneous, it implies displaying CRS technology. The output expansion path is array through origin. So, this result has interesting implication, if you know the type of your production technology and if it is CRS, then you know that you can expand the scale of production via a straight line through origin.

Now we are going to distinguish between short run and long run cost minimization problem of a firm. Remember that we have already spelled out the role of time in production and we have differentiated between long run and short run. I will remind you again. So, in short run some factor input cannot be manipulated easily. So, its quantity level is fixed for the entire time period; that is short run.

So, now let us see, how that kind of distinction has an impact on farms economic behavior. So, let us see no talk about an example. Suppose there is a farm it hires labor purchases raw material from the market to produce a particular good, but this farm has an office space which is owned by the proprietor or the entrepreneur

So, the person does not pay anything for the office space. So, if you ask an accountant the accountant will say that the cost component for space office space is 0. But if you ask an economist he or she will have a different view to it and the person will say, look this is a resource and this resource can be put under alternative views from where the owner of the resource can earn some money. So, the interpreter or the farm manager can lend this office space to another person or to another in organization and earned some rent.

So, this is the money that he or she is losing out when he or she is utilizing this office space for his own business. This is the notion of opportunity cost of a resource and economists will argue that this should be incorporated while computing cost of a firm. So, economists make a distinction between accounting cost and the economic cost by adding opportunity cost to the accounting cost of a production unit.

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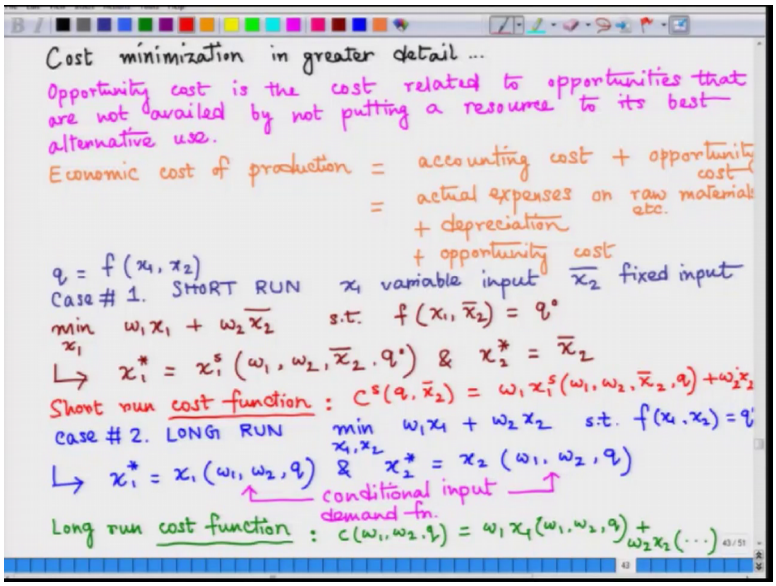

So, let us have a formal definition of opportunity cost. Another example of opportunity cost would be the time given by the entrepreneur to his or her own business. Now in lieu of running this production business, the person could have got a job from market and would have earned some decent salary. So, the salary for gone is basically the opportunity cost of the entrepreneur or the person who is who is running the production business production running the firm or running the business.

So, we can write that economic cost of production is equal to the accounting cost which is computed by the accountants plus opportunity cost which is contributed by the economists. Now the accounting cost will be the sum of the actual expenses on raw materials etcetera means all sort of inputs, but the accountant will also add the depreciation. So, that is basically the wear and tear of the capital the fixed input used by the firm to produce its output plus there will be this opportunity cost. So now, let us make distinction between the fixed cost and the variable cost.

So, in short run we have already said that some of the factor inputs are held constant. So, basically their quantity of use does not depend on the volume of output being produced and that is basically the fixed factor input. Now the cost which is associated with this type of factor input is called a fixed cost. Now some of these fixed costs are recoverable and some are not. So, let us go back to the example of office space again.

So, suppose a person starts a business and he or she has this factory shed, but also this office space is required and the person has taken up us office space in lease. So, he or she got some room which he or she now needs to decorate. So, the person has to paint the room and also has to purchase some furniture to decorate the room.

So, the expenses that are incurred on painting the room is basically sunk cost, because if tomorrow or next year the person has to go out of the business he or she cannot recover the money that he or she has paint on the painting; but, what about the furniture? Although the furniture is also a fixed factor and contributes to fixed cost of production, but some part of that expenses made on purchasing furniture can be recovered by selling them off in the market.

So, furnitures are not the expenses associated with furnitures is not a sunk cost; it is just a fixed cost. So, with these distinctions made, let us now look at the cost minimization problem in short run and long run. So, we assume two input production function given by q equal to function of x 1and x 2 as we have assume previously now we are going to study case 1 which is in the case of shorts run .

So, in short run, let us assume that x 1 is my variable input which can be varied with respect to the level of output production, but x 2 is my fixed input which cannot be varied with level of output example of that would be like land and some machineries heavy machineries capital items basically. So, I can now put a bar on this x 2 to denote that it is a fixed input in short run. So now, we have to write down the firms cost minimization problem.

So, the cost minimization problem now would look like. So, this is the objective function as x 1 is the only variable input. We have to optimize this with respect to only one decision variable, but wait there is a constraint also right and that is basically the output level that the firm wants to produce right. So, that is basically the optimization problem in a nutshell.

So, now I am not going to said Lagrange etcetera. I am I have already shown you how to do that. So, now, I am going to revisit the solution straight. So, now, if I solve this problem, I get the optimized value of my input 1. I denote this by s, s stands for shortrun. So, of course, this will be the functions of w 1, w 2 my input prices. Now as the value of second input is fixed. So, that is also a parameter in my model. So, my shortrun input

demand for input 1 will be function of the quantity of the fixed input as well. And lastly it will be a function of the output level to be produced right and of course, x star two will be nothing, but x two because it cannot be changed in shortrun right.

So, if I now plug this x extra value. So, the optimized values of input in the cost equation, then what do we get? I get what is known as shortrun cost function. So, a cost function gives the minimized value of cost to produce a particular level of output and it will be w 1 x s 1 and two times x 2 bar right. So, now, let us look at the long run cost minimization problem where the input 2 becomes a variable input as well. So, now, we talk about case 2 where we have long run where both the inputs are variable.

So, here the optimization problem will become w 1 times x 1 plus w 2 times x 2. This objective function has to be minimized with respect to 2 inputs and there will be of course, a constraint which is basically the output constraint right. So, now, again I skip the Lagrange etcetera. Let me quickly take you to the results. So, in this case, probably you remember that I get my input demand functions as function of the input prices and the output level.

So, now note that these two functions this one and this one x 1 and x 2 are known as conditional input demand function. Now why do we say that these are conditional factor demand functions? Because note that they are different from another type of input demand functions that we have derived in previous lectures.

So, if you remember our exercise on profit maximization, they are also we have derived the cost minimizing input bundles and hence the input demand functions, but note there we did not specify a particular level of output as the constant. So, there the input demand function was function of only the input prices and not the output level to be produced. But here as this is a cost minimization exercise we see that the target output level q naught becomes a parameter. Hence, the input demand will certainly depend on the choice of that parameter.

So, choice of that parameter value: so, here it is known as a conditional factor demand or input demand function. So, we have obtained our conditional factor demands x 1 star and x 2 star. Of course, if we have more than two, this can be generalized very easily. So, now, if I plug these optimized values of input levels in the cost expression or equation what do we get we get what is known as a long run cost function.

So, this cost function is very important concept in applied production economics literature and we are going to study various types of cost curves next as of; now let us have an expression for the long run cost function. We write the same three components w 1 same three variables or same three factors on which the input demand depends which are input prices two input prices and the output level.

We will continue with this discussion in the next lecture.