Microeconomics: Theory & Applications
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Lecture – 30
Firm's Optimization Problems (Part-1)

Hello. Welcome back to the lecture series on Microeconomics. We have been theorizing the production behavior of a firm. We have seen the concept of neoclassical production function, isoquants and various types of elasticities. These are basically alternative representations of the technology which is used by the firm. Now, with these concepts defined, let us look at the problem a firm faces. We are going to now discuss the optimization problems of a firm and they are of three types. We are going to discuss them one by one.

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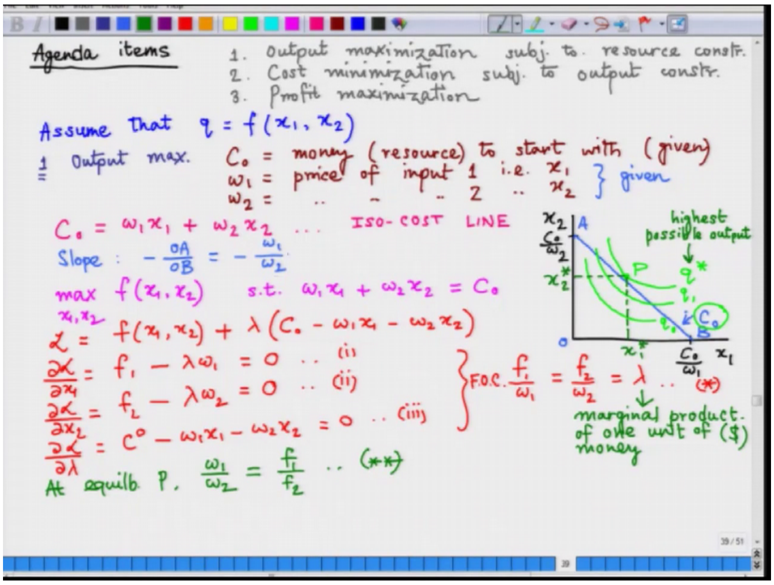

So, the first problem of a firm can be described as output maximization subject to resource constraints. The second problem can be described as cost minimization subject to output constraint and third and the final problem is basically the problem of profit maximization. We are going to assume the case of a two variable input production function as usual. So, you are going to, so assume that we have a two variable input production function written in general form. But, the analysis we are going to do now can be generalized in any time. So, let us now look at the first problem which talks about finding the maximum level of output that can be produced given a budget of a firm.

So, let us first look at the budget constraint of a firm. Here you will find that there is surprising similarity with what we have already done under the heading of theory of consumer behavior. In the case of consumer behavior there was a consumer, he or she was trying to maximize his or her utility subject to a budget constant. Here also we are going to follow the similar procedure to find a firms equilibrium. So, we are going to work with the problem number 1, which is basically output maximization.

Now, in this problem let us assume certain things. So, we can assume that the firm has C naught amount of money that is basically the resource to the firm to start with and that is a given parameter in this optimization problem. And, as there are two variable inputs we have to assume two input prices. So, this is price or unit price of input 1, it implies x 1 and w 2 is price of input 2 it implies x 2.

So, now, we are going to write something which you have already seen. So, in the case of consumer we wrote the budget constraint. Here also if the firm spends the entire amount of money C naught on two inputs, then we can write this particular equation, right and what does this give, this represents different combination of x 1 and x 2. It employees input levels which will perfectly utilize the resource it means that the money entire amount of money will be spent on this various combinations of inputs. And, as the cost level as the resource level is given and the resource is basically is cost here this is known as iso-cost line, right. This is exactly similar to the concept of budget constraint in the case of consumer behavior.

So, now, let us look at a illustration how we can. So, we are going to measure a variable inputs x 1 and x 2 to as usual and now, let us have a look at the iso-cost line equation. So, if the entire money or the resources are to be spent on one particular input say x 1 then how much units or how many units maximum one can purchase of input one that is given by C naught divided by w 1, right. So, similarly if the firm wants to spend the entire amount of money only on input 2, then it can purchase C naught over w 1 units of sorry w 2 units of input 2.

Now, of course, we can join these two extreme points and as we assume that these input prices are also given means they are constant. So, we get a straight line and this is basically the diagrammatic representation of the iso-cost line. So, we can denote a particular iso-cost level as C naught in this case. So, note that if I now increase the monetary resources to the firm to a level C 1 this curve will move up parallel, right. So, let us denote these by C 1.

So, note that we can generate series of iso-cost lines actually, but for this particular problem we will not require two different cost levels because we are talking about a particular cost level only. So, as of now let me erase the second iso-cost line for the higher amount of money. Now, what would be the slope of this iso-cost line? So, for slope we can let me note this points by A and B right and of course, O is my origin. So, the slope will be in this case minus because it is downward sloping OA aver OB, right and that will be nothing, but minus. Now, note that here I have used the geometric technique to find the slope, but of course, you can take total differentiation as we have done earlier to find this slop value.

So, what next? We have already seen in the case of a constant optimization problem we have to adopt the technique called Lagrangian method. So, here also we have an objective function and a constraint and we can frame the Lagrange expression, let us have a look at the problem. So, here what is the firm's problem? Firm wants to maximize output which is represented by my two input production function and the firm faces a constraint, a budget constraint you can say which basically is given by w 1 x 1 times w 2 x 2 the total expenditure on inputs shall be equal to the given level of resources for full. So, basically we are assuming full utilization of resources, right.

So, here this is the optimization problem and the decision variables of course, will be x 1 and x 2 because the others are parameters in this model. Now, we need to set up the Lagrange. So, for Lagrange we can write, right so, where lambda is basically my Lagrange multiplier. So, now, what do I need to do; we all know that a partial derivatives with respect to the decision variables, hence we get so, say this is my equation number 1, then of course, the second one will follow. So, this f 1 and f 2 are basically the first partial derivatives with respect to the input levels. And, lastly we have to take derivative with respect to the Lagrange multiplier as well to enforce this equality of budget constraint, right.

Now, what do we get from all this? So, you can probably anticipate the result as we have derived this kind of Lagrange before, right. So, now, let us see how can we interpret this particular expression as denoted by star. So, this is basically the crux of my first order condition. So, these are my first order conditions. So, now, let us try to find out and expansion for the expression given by asterisk. So, what do we have if we take this expression 1 over w 1 and 1 over w 2? It simply gives the number of units of a particular input that can be purchased through 1 unit of money; say 1 dollar or 1 rupee.

So, when we have the number of number of units of a particular input and if we multiply the marginal productivity of that input, then basically we get the increase in output, right. So, that way the first element f 1 over w 1 help represent the increase in output if we spend 1 unit of money entirely on input 1 or x 1. So, similar explanation is valid for the second component as well, which is f 2 or w 2.

So, the lambda basically gives the marginal productivity of money. So, why it is marginal productivity of money? It shows if we spend 1 unit of money, say 1 dollar or 1 rupee then how many units of additional output we can produce? So, lambda has this interpretation of marginal productivity of 1 unit of money and that can be given by dollar or rupee whatever. So, mathematically we have derived the optimization the solution to optimization problem for firms first type of problem.

Now, let us have a graphical way to find the solution to the same problem. So, different levels of different levels of output is given by different isoquant or isoproduct level curves and a higher level of isoquant gives the higher level of output or shows a higher level of output, right. So, in this case basically if we super impose the isoquant map then the firm will try to be on the highest possible isoproduct curve or isoquant. So, that will happen if firm finds a point like this, represented by say point P, where the isoquant represented by q star level of output is tangent to the iso-cost line which is represented by C naught.

So, now, we can see that the firm finds optimal levels of input x 1 star and x 2 star such that highest level of output. So, this is basically highest possible output, right. So, what happens at that tangency point which is the equilibrium? So, at equilibrium which is determined by the Lagrangian as well what happens the slope absolute value of the slope of the iso-cost line which is given by w 1 over w 2 shall be equal to the absolute value of the slope of isoquant which is given by MRTS which is basically the ratio of the marginal products, right. So, we basically get back the same expression from graph, we get the same condition from the graphical analysis as we have seen in the case of Lagrange which is given by the single aesthetics expression.

Now, we have obtained the first order conditions for the optimization problem of the firm. Shall we check for the second order condition? Yes, you can follow the Hessian technique that we have seen earlier to check for the to check whether the second order condition is satisfied or not. But, as we have presumed nicely behaved isoquant means convex to origin and downward sloping we are ruling out abnormal cases of isoquants. So, let us not get into the second order conditions. We assume that that they are satisfied. Now, let us move to the second problem of the firm which is basically finding the least cost possible way to produce a particular output level.

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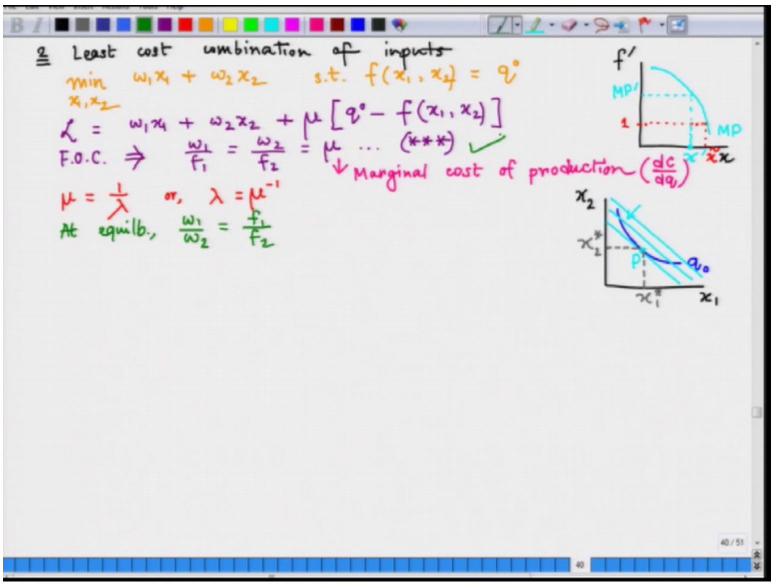

So, the second problem can be summarized as cost combination. So, here straight away we start with the firm's objective function in a mathematical manner. So, basically here the problem is of a minimization type. Here the firm wants to minimize cost of producing a particular level of output. So, that constraint is represented assuming level of output that is prefixed, say q naught, right. So, here what are my division variables? Of course, my input variables, right. So, here also we formulate Lagrangian as well and we write this time we assume a new Lagrange parameter mu. So, now, you will see that that is that is different from lambda in the previous case and hence we are differentiating with symbols here, right.

So, I am not showing you all these partial derivatives and all I assume that you are confident by this time to take derivatives yourself and finding the first order conditions. I am giving you the crux of the first order condition. So, you can find this first order conditions to get these result which is given by this equality or this expression, right. Now, the task at hand is to find an interpretation of this expression, right which is given by 3 asterisks.

To find out the interpretation for 1 over f 1 and 1 over f 2 terms let us have a look at the marginal product diagram, that will help us. So, I take any particular input say x here I am not writing subscripts 1 and 2 to make it general and then let us write f prime. So, that is basically the marginal product which is basically the case f 1 and f 2 if we have specific inputs with respect to which we have to take derivative, ok. So, now, we know that in conventional cases, in standard cases marginal product is a downward sloping function. It can be concave to origin, convex to origin, straight line to origin we do not care as of now. We will just say that what does it give actually.

So, we fix a particular level of x and then we go up to the marginal product curve and that basically represents the level of increment in output which is basically marginal productivity at this particular level of input use say x prime. So, that is MP prime, right, ok. So, now we can also look at the reverse site reverse side of it. So, suppose I now say that I want to see increment of 1 unit of output. So, I can fix the value one here along this axis and now, I can always go back to my marginal product curve and figure out the number the unit of x which will give me that value of marginal product which is basically 1, right. So, we can say that that is basically an input level x tilde.

So, this inverse of the marginal product function which is given by 1 over f 1 and 1 over f 2 can be represented or can be explained as the number of units of a particular input required to produce 1 unit of output only from that vary input and keeping the other input fixed. So, what is the interpretation of w 1 divided by f 1? So, that is basically the cost if the firm decides to produce one extra unit of output through employing only the variable input x 1. Similar explanation holds for the input 2 as well, right.

So, now, let us look at the interpretation of the language multiplier in this case. So, certainly what you observed here is basically an increase in the cost on firm's side to produce one extra unit of output this is named as marginal cost of production. So, mathematically we write note that there is this relation mu; mu is basically 1 over lambda or alternatively I can also write lambda is basically nu inverse. So, there is a relationship between two Lagrange multipliers.

Now, let us look at a graphical solution to firm's cost minimization problem. So, here we have to start with a particular level of isoquant displaying output level q naught and we have to achieve the minimum possible cost which for which the iso-cost line will become tangential to the isoquant, because that is required by the Lagrangian method solution.

So, here what we are going to do? We are going to start with one isoquant displaying the output level cannot naught and now, you are looking for a series of iso-cost lines in this input plane one given by this, then there can be one more. These are all parallel translations of the same iso-cost line having different resource values or cost levels, but with the same input price ratio. So, our target is to go down, right.

Now, how far you can go down? You can find and iso-cost line such that it it becomes tangential to the isoproduct curve or the isoquant at a particular point say denoted by point P and that is the least cost at which you can produce because if you go down further say to this level, then you cannot produce that output level q naught. Hence I erase this invisible iso-cost line. So, what do we get? We basically get the least cost input bundle which is represented by the equilibrium point P. So, we get x 1 star units and x 2 start units of inputs that will be able to produce q level q naught level of output in the least cost manner.

So, now, let us look at the first order let us look at the firms equilibrium from the graphical analysis and try to match the condition with the first order condition we have derived from the Lagrangian exercise. Here what do we see; at equilibrium, so, at point P basically we observe the input price ratio which is basically the absolute value of the slope of the iso-cost line which is given by w 1 over w 2 shall be equal which is given by f 1 over f 2, right. That is exactly equal if you see the first order condition right here, that we have to derived earlier.

We will continue this discussion on optimization behavior of firm in the next lecture.