Microeconomics: Theory & Applications
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Lecture – 28
Isoquants (Part-2)

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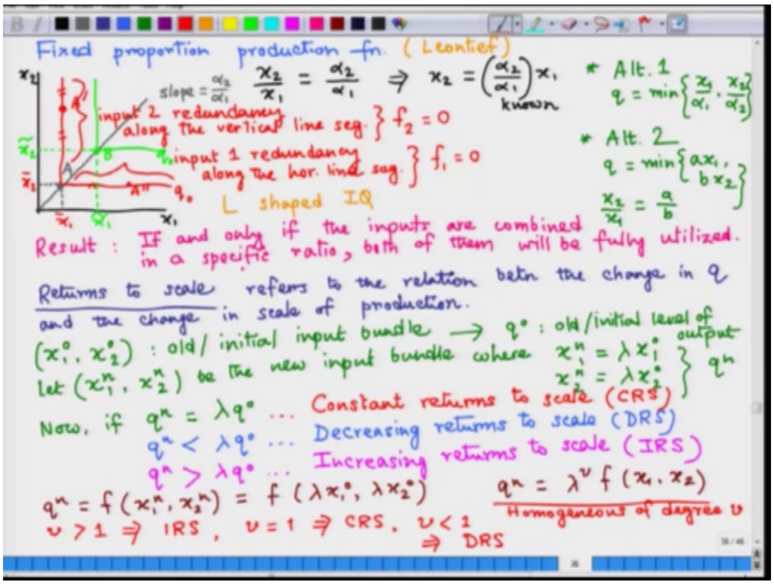

Welcome back to the lecture series on Microeconomics. Now, let us move to the other type of production function, which we have termed as fixed proportion production, function. This is exactly opposite to what we have seen so far. So, far we have seen that the inputs can be combined in variable proportions. So, the input ratio x 2 over x 1 can change along the isoquant and all these points along the isoquant will give us the same value of production, but here comes an interesting case, where we assume that there is only one specific ratio at which, these inputs can be combined to produce a particular level of output and if that is the case then, we have fixed proportion of inputs and we get a fixed proportion production function. This has been made popular by Nobel laureate economist professor Wassily Leontief and these type of production function has found several uses in planning exercises in various countries. So, let us have a look at the simple model of a fixed proportion production function here also, we are going to assume that, we are in the long run. So, both inputs are variable inputs. So, we measure 2 variable inputs along the axis.

So, let us assume that our inputs are mixed or used in a proportion like this. So, if that is the case, I can represent x 2 in terms of x 1 only. So, in this expression as alpha 2 and alpha 1 is known to me; then I can basically draw a straight line through origin whose slope is going to give me the ratio and that is basically alpha 2 over alpha 1. So, take any point on this line, this point will give me a particular input bundle, which will give me a particular level of output. So, let me call this point A ok.

Now, let me talk about another input bundle, where I keep the level of x 1 fixed, but I give more units of x 2. So, suppose this is some point A prime right. So, what will happen? So, this many extra in units of input 2 is totally useless because, then the ratio of input 1 and input 2 is changed and with these new ratios, I cannot produce the output in a technical efficient manner. So, basically all these levels of inputs here become redundant right. Similarly, I can keep the level of input 2 fixed at this level say x 2 bar.

So, let me also call it x 1 bar and if I now take another input bundle say a double prime where, I increase the input units of input 1, then also I see that these points along this joint line will give me input bundles which are redundant. So, we see some input redundancy along the horizontal line segment similarly, we see input 2 redundancy along the vertical line segment.

So, what does this mean actually? So, this means that if 2 the first order partial derivative or the marginal product of input 2 is 0 along this vertical line segment and here, we get similarly f 1 to be 0 and as we do not work with the points, where marginal product is 0, we will not choose points like A prime or A double prime.

So, we will always operate on point A to produce say q not level of output. Now let us move to a different output level a higher output level to produce a higher output level. Now, I need to combine these 2 inputs in the same fixed proportion alpha 2 or alpha 1, but now I need more of both these inputs as I need to produce more units of output and say, I have x 1 tilde and x 2 tilde and this is my point B. So, similarly I can now formulate this, parallel lines 2 x 1 and x 2 axis and these line segments along with this point B will present the isoquant for the output level say q 0.

So here, what we see is the following. So, for a fixed proportion production function, which is also known as Liontief production function named after the inventor that, we get l shaped isoquants. Now, how to represent mathematically, this type of production function? Well, there are 2 alternative representations alternative one; we can write q as minimize minimum value of x 1 over alpha and x 2 over alpha 2. There is another alternative, which also is seen at times in textbooks and other scholarly work that is to be written as a x 1 comma b x 2.

So, in this case basically x 2 and x 1 are mixed in a over b ratio right. So now, we are going to write a result. So, if and only if the inputs are combined in a specific ratio both of them will be fully utilized. So, along the vertical segment of the l shaped isoquant, the marginal product of input 2 is 0.

So; that means, that we have the formula of a MRTS remember that is to be f 1 over f 2. So, if f 2becomes 0 then basically, we have MRTS as equal to infinity and as if we take a point along the horizontal segment line segment of the Leonteif l shaped isoquant, then we have the marginal product of input 1 to be 0. So, in that case the MRTS, we will take value 0 along this line segment at the kink point of the corner point the MRTS will be undefined.

Now let us move to a different concept, which is also very interesting and important in production economics and that is the concept of return to scale. So, we are in the long run, where all the inputs are variable and they can be changed at any point of time in any manner. So, let us see how the production is going to respond to these changes?

So now, we are going to study return to scale, a change in scale means proportionate change in all inputs. Now we are going to study, how output response to this kind of change. So, return to scale refers to the relation between the change in output level q and the change in scale of production right. In simple words, it is the rate at which output changes, when all inputs are changed either increased or decreased proportionally, now to show the concept mathematically suppose, we start with an input bundle x o 1 x o 2.

So, that is basically, my old or initial input bundle and I assume both my inputs are variable inputs right and this gives me q old level of output ok. Let x n 1, x n 2 be the new input bundle, where x n 1 equals lambda times x 0 1 lambda is a scalar and x n 2 is lambda times x0 2, then the question is, what about the new level of output denoted by q n right? So now, if q n is equal to lambda times q 0, then the output has actually increased by the same proportion right and this is known as constant returns to scale and this is abbreviated as CRS, but there can be other cases as well right.

So, if q n is less than lambda q 0 then what happens? Then basically, output increases less than proportionally and that is known as decreasing returns to scale abbreviated as DRS and there can be another case, where q n is greater than lambda times q 0 and that is known as the increasing returns to scale. So here, output increases more than proportionately than the proportionate change in input. So, this is the concept of returns to scale in a very simple manner alternatively, we can use mathematics to represent the concept of return to scale and that is to be done through the concept of homogeneous functions.

So, let us also look at the alternative representation and return to scale with respect to homogeneous functions. So note that q nu can be actually written like this, which can be rewritten as right. So, one can now factor out lambda and we can rewrite the production function after scale change as lambda to the power v function of x 1 and x 2 and if we can write in this manner then, we say that the production function is homogeneous of degree v. So, this is a very important result that, we are going to use again and again in this course. So, this is known as homogeneous of degree v and then we can look at various values. Now we can take various values right, it can be greater than 1, it can be equal to 1, it can be less than 1 as well.

So, depending upon the value of v actually, we can call whether a function production function shows CRS, VRS or DRS, if the value of v is higher than 1, then we see that the output is increased more than proportionately and we call that the technology is displaying and increasing return to scale v takes value 1 then we know that the output has increased at the same rate with the input and there is proportionate change and that is known as CRS technology and if v takes value less than 1 then we call that we have a decreasing returns to scale technology. So, I can summarize v greater than 1, IRS then v equals 1 means CRS and v less than 1 implies DRS. So, we are going to continue with the discussion in the next lecture.