Microeconomics: Theory & Applications

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Lecture – 27
Isoquants (Part-1)

Welcome back to the lecture series on Microeconomics.

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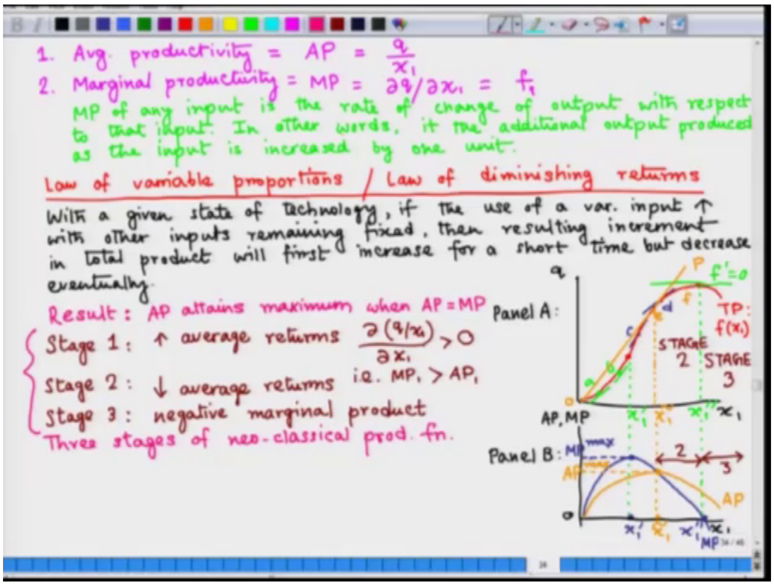

Now, we are going to look at the other formulation of production function and this time we are going to look at the production function in long run. So, in long run both the inputs are variable. So, we are going to now represent production function through a concept known as Isoquant.

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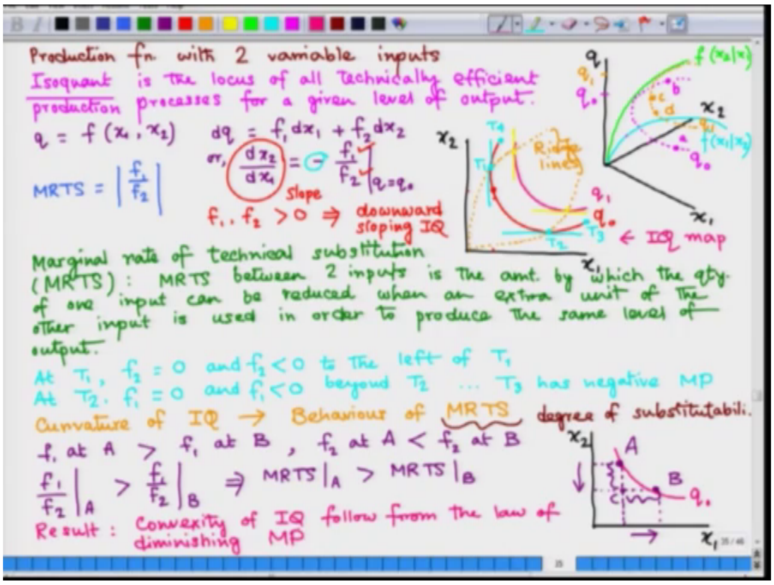

So, when we talk about isoquant, we have to talk about a production function of 2 variable inputs. So, we are if we have to represent total product curve then it is going to be a 3 D diagram. So, here I am trying to draw a 2 variable input production function in a 3 dimensional plane. So, along these red axis I am going to measured the total output q and we are going to measured variable input x 1 along this axis and x 2 along that axis.

So, I can now assume this law variable proportion holds. So, in that case with respect to each input axis we can have the curve moving like this. So, basically this will be f of x 2 when x 1 is given and we can also draw a similar diagram for production function from origin with respect to x 1 axis this time and we get f of x 1 given x 2.

So, we can see that it is going to give us a shape like a dome or like a hill right. So, the height of the hill from the level measures the degree of the volume of output. Now, one can think about a conical cross section of this production hill. So, suppose I fix some value of output at q naught level, so, then basically what I am saying that you know I have to take a conical cross section and most likely I am going to get a shape of that cross section.

So, if I now superimpose this curve that is just generated in the 2 dimensional input plane then what do I get? Suppose, I take a point here say point a and I take a point there say point b. So, I can take these 2 points a and b on this contour line q naught or iso product line q naught and then I can superimpose these 2 points in the input plane. So, here I can take another conical cross section and the height along the output axis this time will be q 1 at which I am going to take the conical cross section. So, I am going to get another curve like this and this will give me again like previous one all the input combinations like say c d which will produce the same level of output say q 1.

So, these generated curves which are superimposed on the input plane are called isoquants. So, now, let us have a formal definition of isoquant, isoquant or iso product curve is the locus of all technically efficient production processes, what do we mean by production processes, these are basically combination of factory inputs for a given level of output. In a simple word we can also say that isoquant curve shows all input bundles that produce exactly same level of output ok.

So, now let me have a simple diagram and this time we are going to draw isoquant in 2 dimensional plane x 1, x 2 are the variable inputs and now I am going to draw the curve, something like this will happen some sort of curve like this we are going to expect in the input plane right q naught is basically the given level of output at which the conical cross section of the production hill has been taken.

Now, why it is called technically efficient, because note take any particular point say here let me call this point a now note that this shows a specific input bundle to produce the level of output q naught and let me denote this as x 1 prime and x 2 prime, note that there are other possible input bundles available which could produce the same level of output. For the example we can consider this point b where more units of input one is given. So, b can also produce the same level of output q naught.

But b is not part of the isoquant. So, that is why it is only giving us the technical efficient input bundles. So, now, we do not need b anymore. So, let me remove this let me also remove this a and other things as I do not require let me clean this diagram as do not require that point a anymore.

So, whenever we have a curve in economics we are also interested in the slope of that curve and here also the slope of the isoquant has special interpretation. So, let us have a formal definition and derivation of the slope of a isoquant. So, here we can start with the production function q x 1 comma x 2, 2 input production function we can go for the total differentiation dq right. Then we can have first partial with respect to input one times dx 1 we have f 2 dx 2 similarly right.

Now, this item will become 0 because there is no change in the output value as we are on the isoquant and the definition of isoquant says that it is for a particular level of output so, there is no change in output. So, we get this is the slope of the isoquant and generally when we write slope as this is for a specific level of output we write that as well say q equal to q naught say that is more proper.

So, now let us make a comment on the slope. So, f 1 and f 2 these are the marginal products and we assume that f 1 and f 2 are both positive. So, we are not in the stage 3 of the new classical production function. So, these implies that we have a downward sloping isoquant and we can abbreviate isoquant as IQ. So, now, let us look at a very important concept known as marginal rate of technical substitution which is abbreviated by shortened MRTS.

So, MRTS between 2 inputs is the amount by which the quantity of one input can be reduced when an extra unit of the other input is used in order to produce the same level of output and now let us going to look at a formal definition of MRTS. As in a 3 stage neoclassical production function, we have a stage where the marginal product becomes negative here also we can assume that marginal product can become negative.

When we wrote the initial slope of the isoquant we have assumed that marginal products are positive to have a downward sloping isoquant, but we do not have to assume that. If we assume that we are in the third stage of neoclassical production function which is anyway pretty unrealistic, but suppose theoretically if we are there then what is going to be the shape of the isoquant. Now let us have a look at that possibility. So, now, we are going to go back to the isoquant q naught and of course, one can have series of isoquants displaying various levels of outputs.

So, one can have another isoquant say q 1 level of output we need at least 2 isoquants to tell the story and when we have series of isoquants the higher level of isoquants display higher level of output and that is known as an isoquant map. So, here we have what is known an isoquant map, we would like to end our discussion on the isoquant saying that 2 isoquants cannot cross each other in the isoquant map and I leave it to you to find out why.

So, now let us talk about the possibilities of 0 and negative marginal products right. So, let us work with these q naught isoquant and let us have couple of points. So, let me have one point where I can draw a straight line which becomes tangent at that point and it also becomes parallel to the x 2 axis, let me name this particular point as T 1.

Similarly, I am looking for another point along the same isoquant where I can draw a tangent which will become parallel to the x 1 axis and suppose this is the point and let me name that T 2 and we can have some points like T 3 and T 4. Now any point such as T 3 and T 4 are basically inefficient points, now consider the point T 1. So, at T 1 what happens at T 1 the f 2 or it implies the marginal product of the input 2 becomes 0.

And it becomes negative to the left of T 1 right. So, if that is the case we see that we have a negative sign in front of the slope, but if one of the marginal product becomes negative then the slope will become positive and then we can have upward sloping isoquant and this is what exactly we are getting. So, the point T 4 here actually lies on the positively sloped isoquant where we see the marginal product of input 2 is negative.

Similar explanation could be given for the point T 2 as well. So, at T 2 the slope of the isoquant is 0, right. So, if the slope of the isoquant 0 then; that means, f 1 becomes 0 of course, and it becomes negative beyond T 2. So, we get to see that a point like T 3 has negative marginal product. These are theoretical cases, but in reality they are kind of ignored, similarly we can find points along the q 1 isoquant as well where the marginal product of one input becomes 0 at one point of time.

So, suppose these 2 are the points. So, now, if we join such points with origin we get a curve. So, these curves joins all the points along which marginal product of input 2 is 0 and let us have another curve joining the points along which the marginal product of input one is 0 and these 2 curves are known as ridge lines and economists are basically interested in the area which is bounded by these 2 ridge lines.

So, in traditional economics we are going to study downward sloping isoquant only we will not have upward sloping segments of the isoquant, to rule out the negative marginal productivity of any input. So, we have looked at the slope of the isoquant and now we assume that we have a downward sloping isoquant, but what about the curvature is it convex to origin or is it concave to origin. So, for that also we have to assume certain things and we have a law which will give us a convex to origin isoquant and this is what we are going to study next. We have to talk about the behavior of MRTS; that means, whether it increases or decreases with respect to one input say we are increasing one input x 1 then, what happens.

Now, what does this MRTS actually gives. So, MRTS basically gives a degree of substitutability. So, now, let us look at this substitutability issue in greater detail which will lead us towards the curvature of the isoquant. So, again we are going to measure 2 variable inputs along the axis. Now we will first not draw the curve we will first start with 2 input bundles suppose we have this initial input bundle A with some coordinates.

Now, suppose I want to produce the same level of output, but not using the input bundle A. So, then I know that the isoquant is downward sloping and I can have an alternative bundle say here denoted by B which also gives me some input combination that will allow me to produce the same level of output. So, what happens really. So, let me name this intersection point C. So, as we move from the input bundle A to B along an isoquant I have not drawn it.

But assume that there is an isoquant then we imply B C units mode of the input one and there is a reduction corresponding reduction in input use 2 and reduction or the magnitude of reduction in input use in input twos use is given by the line segment A C. So, if that is the case then productivity of the input one decreases because we are utilizing more units of it. If we follow a conventional production function a concave to origin production function showing the decreasing returns to the variable input.

So, similarly as the use of input 2 has gone down we can expect that the marginal product of input 2 has gone up by following the same logic. So, we can write the marginal product at point A is greater than the marginal product at point B and we can also write if to the marginal product of input 2 at A is less than the marginal product at input bundle B.

So, now let us take ratio why we take ratio because we need to get down to the form of MRTS. So, f 1 divided by f 2 at input bundle A shall be greater than f 1 over f 2 at input bundle B. So, this means the MRTS at point A is greater than MRTS at B. So, the absolute value of the slope has increased as we moved along the isoquant from A to B.

So, if absolute value of the slope has actually increased then we can draw a isoquant which gives us this convex shape. So, here is the result convexity of isoquants follow from the law of diminishing marginal product. So, we are going to continue with the discussion in the next lecture.