Microeconomics: Theory & Applications
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Lecture – 26
Neoclassical Production Function

Hello welcome back to the lecture series on Microeconomics. Today we are going to start with theory of firm which is a very big component of this course. Now what do we mean by a firm? A firm is a production unit which transforms some inputs to some final goods and services we call them outputs.

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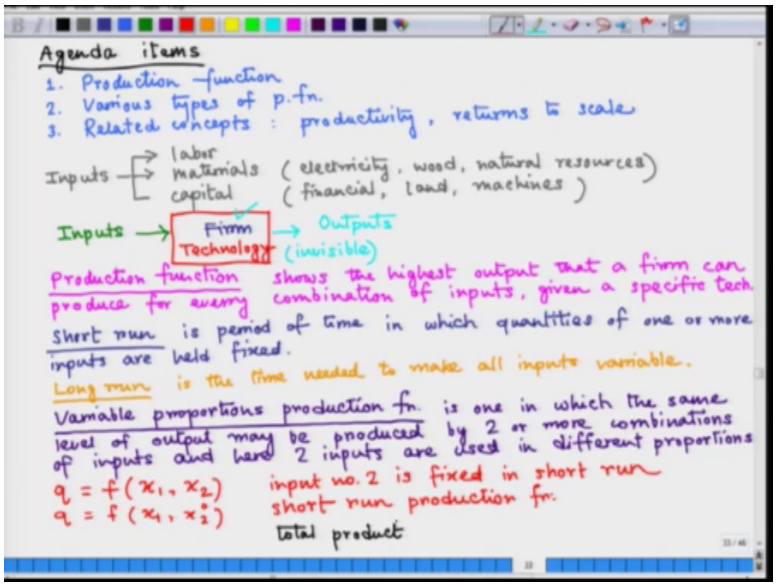

Before that let us have a look at the agenda items of today's talk. So, we are going to discuss a new concept of production function. We are going to look at various types of production function then we are going to discuss related concepts like productivity, returns to scale etcetera. First we are going to start with classification of inputs. Inputs can be classified in three major heads, number 1 would be labor.

And they can be skilled and unskilled human force. The second would be materials and these materials are like electricity, food then there can be some natural resources. Then a third type of input is called capital. And here capital could be again of various types. One can think about financial capital like money as a resource or there can be physical forms of capital as well like land, machines etcetera.

Now, what happens is the following so, we start our story from firm. So, firm is a production unit and firm has something known as technology and there is flow of these inputs into the firm or production unit and as a result, we see some outflow of goods and services which are known as outputs. Now what happens inside the firm that is kind of unobserved and technology is of course, invisible right.

And theory of production tries to have an idea about this processes or production processes what takes place inside a firm. So, we will now start with the concept of production function. What is production function? Production function is a technical relationship between inputs and outputs. So, let us have a look at formal definition of production function. Production function basically manifests technology. Some people say that if we have fair idea about production function, the technology is embedded in that.

So, one can write that production function shows the highest output that a firm can produce for every combination of inputs. There can be more than one inputs given a specific technology. When we are going to discuss theory of firm, there is a concept time which plays a very important role and now let us look at two concepts called short run and long run formally.

So first, we will start with short run. Short run is the period of time in which quantities of one or more inputs are held fixed. It implies that they cannot be changed. Now, what could be an example? An example of short run and fixed input could be from agriculture. So, when agricultural season say Kharif season or Rabi season, the farmer has already sown seeds to his plot and that plot is basically then becomes a fixed inputs.

Because the farmer now cannot bring in more land or reduce the quantity of land from agricultural production. So, in this case, land becomes a fixed input and the period of cultivation say 4 months or so, the season basically the agricultural season Kharif or Rabi becomes short run. Then we move to the concept of long run. So, long run is the time needed to make all inputs variable. It implies we can change quantities of any input at any point of time.

Now, there is a difference between technology and production function, although we say that production function is defined for a given level of technology or given knowledge of technology there is a difference. The technology basically provides information on all

possible combinations of inputs to produce outputs, but production function is basically only showing the highest level of output which can be produced from given bundle of input.

So, technology embeds all physical possibilities whereas, the production function actually presumes technical efficiency. So, we can talk about two types of production function one is known as fixed proportion production function and the other one is called variable proportions production function. As the variable proportion production function has more usage, we are going to first start with that concept.

So, variable proportion: so a variable proportions production function is one in which the same level of output may be produced by two or more combinations of inputs and here 2 or more I am assuming here two for simplicity are used in different proportions. So now, we are going to study variable proportions production function in greater details mathematically.

Now, we are going to introduce three related concepts known as total product curve average, product curve, and marginal product curve. A variable proportions production function can be shown in two different or alternative ways. One for short run and one for long run; in the short run, we assume that there are 2 inputs one of them is fixed the other one is variable. And in the long run we are going to assume that both the inputs are variable in nature.

So, we are going to first start with the short run case. Here we are going to talk about some concepts like total product curve, average product curve and marginal product curve. Let us have a look at them one by one. So, we write a production function mathematically as q, q denotes output as a function of two inputs. There can be more, but let us assume for simplicity there are only 2 inputs. And, if we have the set up on the top of that if we assume that our input number 2 is fixed in short run then we have a short run production function given as where this x superscript naught subscript 2 x naught 2 or x 2 naught is basically fixed level of input 2.

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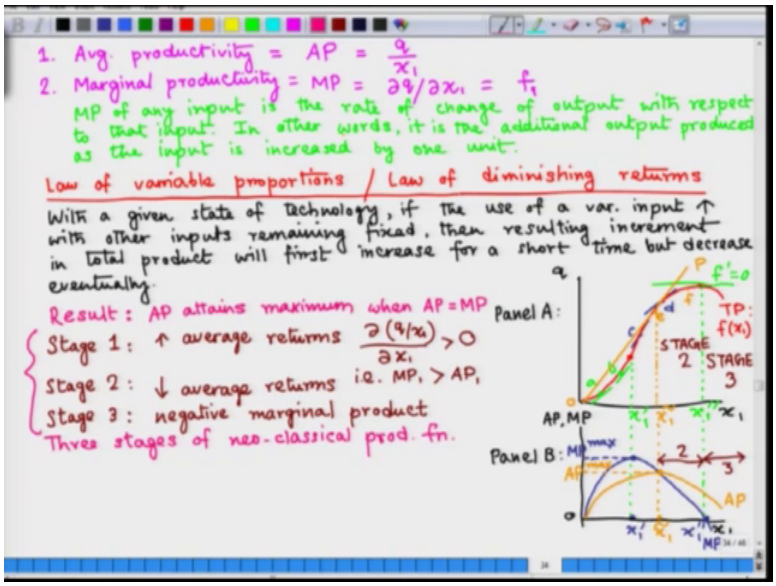

Two very important concepts are average products or average productivity and what is this. This is given, this is also abbreviated this can be abbreviated by AP and this is basically q divided by x 1 the volume of output divided by the volume of variable input. Now, here comes the second concept marginal productivity which is very important in microeconomics and that is given as MP the short hand notation for that.

And that is basically the first order derivative of the production function with respect to the variable input. Sometimes it is also shown as f 1, the first partial of the production function with respect to the variable input. Now, let us look at the concept of marginal product formally through a definition. So, marginal product of any input of course, it has to be a variable input is the rate of change of output with respect to that input.

So, this definition directly follows from the partial derivative representation of marginal product. In other words, it is the additional output produced, as the input is increased by one unit. So, this is most straight forward definition. Now, what about the shapes of this total product curve, average product curve and marginal product curve if we want to draw them?

Now, the shapes of these particular functions or curves are dictated by hypothesis known as the law of diminishing marginal returns. This is alternatively known as the law of variable proportions. Now, what is the origin of this law or principle? Now as a concept production function predates the idea of utility function in microeconomics. We have to

go back to the middle of the 19th century where a bunch of classical economists were independently working on the concept of production function. And, this law of variable proportions or diminishing returns has evolved over time with these people.

Now, we are going to look at a formal statement of this law of variable proportions. So, now, we formally define the law or state the law. With a given state of technology, if the use of a variable input increases with other inputs remaining fixed, then resulting increment in total product will first increase for a short time, but decrease eventually.

So, this law can be best understood through diagrams. So, let us have a look at the diagrammatic representation of this law. So, we are going to have two panels of diagram. So, along the horizontal axis, I measure my variable input x 1 and I measure output along the vertical axis and this is my panel A diagram where I am going to plot the total product curve. Now at the bottom, I am going to draw a panel B diagram.

So, we measure the variable input x 1 along the horizontal axis this is 0. So, this is the positive side of this and this is negative side. So, in the panel B diagram I am going to measure the average product and marginal product along the vertical axis. First, I am going to draw the curves and then I am going to explain. A neo classical production function has three stages and it follows this kind of a shape.

So, TP is basically my production function f of x where x 2 is the given input right. Now, note that as I move along from one point to the other point, the slope of the production function changes. So, of course, marginal product will change. Now note that up to some point the slope is increasing means that marginal product or the first derivative of the production function is increasing. Let us assume that we are talking about some input level x 1 prime till which the marginal product is increasing.

And that can be seen as the slope of the production function at any given point is increasing as well. So, for that matter considered these two points a and b. The marginal product given by the slope is increasing. So, the marginal product function starts from 0 and reaches a maximum at this input level x 1 prime. So, at x 1 prime marginal product reaches its maximum right.

Now, let us look at another point on the production function say here. Here what we can see? We see that the production function has basically attained a maximum and at this

maximum point the slope is 0. So, what do we mean? We mean that the first order derivative equals 0 at this point means that the marginal product of the variable input has become 0.

So, let us now look at that input level and we plot. So, here we are talking about some x 1 double prime level of variable input 1 and we see that marginal product has attained a maximum, but after that it starts falling. So, you can see any two points here say c and d, and you can note that the slope at these two point is basically decreasing.

So, the slope at point d is less than slope at point c. So, basically marginal product is falling. So, we can have a downward sloping segments or falling segments of the marginal product curve and at the level of x 1 double prime the input use when we get the input use up to x 1 double prime. Then the marginal product becomes 0 at that input level and if the firm uses more units of variable input then marginal product even can be negative and that will result into a backward bending production function. So, we have obtained the shape of marginal product curve. Now let us have a look at the shape of the average product curve.

So, how do we get average product from diagram? So, we have to take any particular point on the production function and we have to join that particular point with the origin through a straight line and the slope of that joining straight line will give us the average product right. So, we can continue having lots of points on the average product and you will see that first average product rises and then it starts to fall after some point. So, now, average product curve will also have an upward sloping segment and the downward sloping segment. Let us look at the same diagram to display this behavior of average product curve.

So, how do we get such a straight line which basically gives the highest slope if we join a point on the production function and the origin. So, we can start from the origin, draw a straight line in such a manner that it becomes tangent to the production function. Say it become tangent to the production function at a point e. So, that is the highest slope that one can make in order to get the average productivity and slope of this function gives the suppose you know I want to call this straight line OP.

So, the slope of the straight line OP gives the average product of the input level x 1 tilde. So, now, let us superimpose that level on panel B diagram x 1 tilde. So, at this x 1 tilde

level average product attains its maximum. So, if you take any other points say b or c on this production function, you see the value of the average product is less right. So, before this input level x 1 tilde average product curve is basically rising. So, it will attain a highest point at the input use level x 1 tilde and then, as we move on to point like d and then maybe another point say f, here then you see that average product is falling as the slope of the straight line joining this point to origin has gone down.

So, the average product beyond this point or input level x 1 tilde actually is about to fall like this. So, this orange line gives me my average product curve. So, we see a result that the marginal product curve passes through the highest point of the average product curve. Based on the diagram that we have just drawn, we can talk about three distinct stages of neoclassical production function. Let us discuss these stages one by one.

Stage 1 is about increasing average returns and that is basically mathematically represented as the following. So, here we see from the graph; that means, the marginal product of the variable input will be always higher than the average product in this stage. Then comes stage 2 and that is known as decreasing average returns and then comes stage 3, the last stage and there we see the phenomena known as negative marginal product.

So, at the beginning quantity of the fixed input is abundant relative to the quantity of the variable input. When more and more units of the variable input are used the fixed input is utilized more effectively, but once a point is reached where the amount of the variable input is used in such that most efficient utilization of the fixed input is accomplished. Then further increase in the variable input makes the fixed input limiting compared to the variable input. And hence, we see this law of diminishing marginal returns happening.

So, this is basically our stage 3 where the marginal product is negative. Then we have this decreasing average returns so; that means, this part is basically my stage 2. So, this is stage 3, I just you know write 3 here. Then stage 2 would be this area, I can write 2 here and in the panel A diagram let me denote that stage as well and that is basically stage 2. And of course, the other area is stage 1. So, in total what do we get? We get three stages of neoclassical production function.

So, we are going to continue with the discussion in the next lecture.