

**Microeconomics: Theory & Applications**  
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**Lecture – 23**  
**Introduction to Risk**

Hello, welcome back to the lecture series on Microeconomics. Lastly we have seen how a consumer chooses between current and future consumption and that is what basically inter temporal choice theory is all about. And we have also seen how the concepts like future value and present value can be used in the financial markets. So, far we have assumed in microeconomics everything is certain. So, the price is, the money income and all other parameters that and economic model has are all given, but they can vary with time also and they can be highly random.

So, if the model parameters are random in nature, then how the consumer is going to choose optimally in this uncertain or random world that is going to be the subject matter of this and that next lecture. So, a consumer faces two types of problems in a risky world. First question would be how much risk that the consumer should take or bear and the second question is how the consumer is going to cop up with the risk. Now we are going to have a look at the agenda items for the lecture.

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Agenda items

1. Recap of basic probability Theory
2. Describing risk quantitatively
3. Linking NPV with risks → an illustration

Outcome is a possible result of a random phenomenon/experiment  
Probability is the likelihood that a given outcome will take place  
Pay off is the value associated with a possible outcome  
A random var. is a var. whose possible values are numerical outcomes of a random exp / pheno.

└ Discrete 1, 2, 3, ...  
Continuous

Expected value (Mean) is probability weighted average of the pay-offs associated with all possible outcomes.  
Variability is the degree to which possible outcomes of an uncertain event differ.

$X: x_1, x_2, x_3, \dots$   $K$  possible values  
Mean  $E(X) = \mu = \sum_{i=1}^K p_i x_i$  where  $p_i = \frac{Pr(X=x_i)}{Pr(X=x_1) + Pr(X=x_2) + \dots + Pr(X=x_K)}$   $\forall i=1, K$   
Variance  $V(X) = \sigma^2 = \sum_{i=1}^K p_i (x_i - \mu)^2$

So, first we are going to start with a recap of basic probability theory, I assume that you have fair idea about probability and distributions. So, its going to be a very quick recap. The second thing I am going to talk about describing risk quantitatively and third thing I am going to do is linking the concept of net present value that we have seen earlier with risks. And we are going to have an illustration through which we will understand how in pv shall we calculated in a risky world.

So now, let us start with some basic definition, which are useful to understand risk. So, we will start with outcome. Outcome is a possible result of a random experiment or phenomenon. So, an outcome may occur with the probability which will take value between 0 and 1 that we all know. So, when we role a die, we see that three is just one of the six possible outcomes. So, there is a likelihood attached to each of this possibilities and likelihood of an outcome in a random experiment or a random phenomenon is known as probability. So, we define probability is the likelihood that a given outcome will take place.

Next to introduce the concept of pay off. Pay off is the value associated a possible outcome. Now we define two important concepts random variable and its two types discrete random variable and continuous random variable; then we are also going to define to related concepts called expected value and variance. These are the instrumental concepts in risk analysis. So, random variable is a variable whose possible values are numerical outcomes of random experiment or phenomenon.

And random variable can be of two types discrete. So, if the values are discrete numbers like one two three etcetera then we are talking about discrete random variable and there is continuous random variable as well, where any value can be taken by the random variable, it can be a fraction as well. Now we are going to introduce a very important concept call expected value, which is also known as sometimes mean of the random variable also. So, expected value or mean is probability weighted average of the payoffs associated with all possible outcomes.

Next we introduce variability. Variability is the degree to which possible outcomes of an uncertain event differ. So, basically variability talks about the dispersion in the outcome. We have defined some key concepts from probability and statistics in order to learn risk better; now let us going to introduce a random variable  $x$  and let us going to measure

expected value and variance as a measure of dispersion in this case. So, let us assume a discrete random variable  $x$  which takes values like  $x_1, x_2, x_3$  so on and forth and we can assume that there are  $K$  possible values.

In that case the expected value or mean of this discrete random variable is given by the formula  $E$  of  $X$  sometimes it is also represented by symbol  $\mu$  and the formula is the following. So,  $p_i$  is basically the probability of the  $i$ -th value that the random variable takes. So, let me give a simple illustration here, probably it will help you to understand what is happening. So, the discrete random variable  $x$  takes various values say  $x_1, x_2, \dots, x_k$  and then  $p_i$  is basically probability that  $x$  takes value one for the first value  $x_1$  and then  $p_k$  would be the probability that the random variable takes value small  $x_k$ .

Now, we are going to introduce the concept of variance. And variance is denoted as  $V$  of  $X$  and also with this symbol  $\sigma^2$ . And, the formula would be again another weighted sum weights being probabilities, but this time we are going to take difference between the mean of the discrete random variable or any random variable, and the actual value of the variable which is actually observed and then square it that would be the variable for which we are taking the weighted average.

There are several representations of risk, but (Refer Time: 13:53) Professor Harry Marko is settled on the concept of variance as a measure of risk because it was already an accepted measure of dispersion by statisticians. Now we are going to introduce another new concept which is derived from variance and that is the notion of standard deviation. So, if you take the positive square root of variance, then we get standard deviation and it is denoted by  $\sigma$ .

So, last time we have seen how to compute net present value of an investment or of a project or a financial assets. Now we are going to assume that that net present value can take various values with different probabilities. And in this case how the consumer is going to decide whether to purchase that asset or not, whether to invest in that particular project or not, let us going to have a analysis regarding that question.

So, we are going to decide how a consumer is going to make a decision whether to invest in a project, where outcome is random.

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Investment decision under conditions of risk

NPV =  $-C_0 + \sum_{t=1}^T \frac{C_t}{(1+r)^t}$

A numerical example

$T = 2$ ,  $r = 10\%$ ,  $C_0 = 25000$

Outcomes ... Business/market conditions (Good, Average, Bad)

$T = 1$  (1st yr. analysis)

$E(C_1) = 4400 + 10800 + 2800 = 18000 (\text{₹})$

$V(C_1) = 6,400,000$   $\sigma = \sqrt{V(C_1)} = 2529.8$

$T = 2$  (2nd yr. analysis)

$E(C_2) = 22000$

$V(C_2) = 10,800,000$

$\sigma = \sqrt{V(C_2)} = 3286.3 \approx 3286$

$E(NPV) = E\left[-C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2}\right] = -C_0 + \frac{E(C_1)}{1+r} + \frac{E(C_2)}{(1+r)^2}$

$= 9534 (\text{₹, approx.})$

Outcome	Pay-off (₹)	Probability	$C_1 - E(C_1)$
Good	22,000	0.20	4000
Average	18,000	0.60	0
Bad	14,000	0.20	-4000

random

Outcome	Pay-off (₹)	Probability
Good	28,000	0.15
Avg.	22,000	0.70
Bad	16,000	0.15

So, first let us look at the net present value formula that we have introduced before. Let me write down the formula, then I am going to talk about the notation. So,  $C_0$  is basically the value of the initial investment and that is the current value ok. Now  $C_t$  is going to be the net cash flow to the consumer or to the investor and we are also going to have a positive rate of interest, and then finally  $T$  is basically life of the project or asset. So, you can also assume that there are cash flows from an asset.

Now as we are going to basically talk about a numerical example, let us assume that we are going to work with two time periods to have a simplified analysis and the value of rate of interest would be 10 percent; means assumed for simplicity. Now we are going to look at the outcomes that one can get in an uncertain world. So, this outcome could be; so this is a project or this is an asset. So, outcome can depend upon business condition and there can be three possible states that can arise: one is good state the other one is reasonable or average and the other one is the third one is a bad state. So, we can name them state 1 then state 2 and state 3 respectively.

Now, pay offs will depend on this business conditions and they will also vary from one year to the other. So, let us now assume the case of year 1 or  $T$  equal to 1. So now, let us look at a simple table which has monetary figures. So, we can have outcome and that is basically the market condition, then there can be pay off that is basically cash flow and

we can have the probability associated with that outcome or the occurrence of the state. So, we can have good average and bad states and the payoff will depend.

So, now let us assume that the payoff is represented in Indian rupees, so 22,000 if good state occurs. If it's an average market condition then the net cash flow would be 18,000, and it's a bad state of world not favorable market conditions then we will receive 14,000 net cash flow. And there are probabilities for this outcome, so my pay off is now random or cash flow is random. Now we are going to give some arbitrary probability values associated with this number, so 0.2 0.6 for average market condition and again 0.2 for the bad state or poor market condition.

So, in this case now let us compute the descriptive statistics measures. So, first we talk about the mean of cash flow in period 1. So, what we need to do? We need to multiply these two numbers, then these two numbers and these two numbers and then sum. So, if we do so, then we get what? 4400 plus 10800 plus 2800 and we get 18000 Indian rupees that is basically the mean of the cash flow in period 1. Now we can compute the variance also the variance would be: so for variance what we need to do? We need to create another column to show the calculations so that you understand it fully. So, we have to now take the difference from the actual value of the random variable and the mean of the random variable. So, we get 4000 in good state, 0 in average state and minus 4000 in the bad state right.

Now let us go back to the formula. So, we need to square these difference terms and that we will give us some numbers. Then we need to multiply those numbers with this probability values and if we sum them up then we get a number. So, that is basically my variance. Now we are going to take the case of second period numbers will change for the second period. So, let us have another table to represent the numbers for this case.

So, again outcome three possible states of the world; good, average and bad, then we have pay off in Indian rupees and the pay off this time will change its different here. So, let us assume different values 28,000, 22,000 and 16,000. Let us also assume probability values and they are different in this case 0.15, forever a state 0.7 and again for the poor market condition bad state of the world we assume 0.15.

So, in this case also we are going to look at our mean and variance, I am not going to show you the full calculation I am just going to write the number, hope you can drive this numbers yourself and the variance is right.

Now from here we can get the value of sigma, which is basically is positive square root of variance of  $C_2$  and that is basically around 3286.3; approximately we can assume that to be 3286. So, let me also get the sigma value in period 1. So, sigma in this case will be positive square root of variance  $C_1$  and that will give us 2529.8, we can assume this to be 2530. So now, we are in a position to compute the mean of the random variable net present value. Let me also assume that  $C_0$  as a value of 25000 the initial investment.

So, we know that we have to take expectation of this random variable  $C_1$ ,  $1 + r$  plus  $C_2$ ,  $1 + r$  square and that would be minus  $C_0$  because its fix its not a random variable. So, we get expectation of  $C_1$  divided by  $1 + r$  plus the expected value of  $C_2$ . So, that is basically the expected value of the cash flow in period 2, we have this. Note that we have already obtained this expected values. So, we can directly plug this numbers, we also know the value of this and we also know the value of  $r$ . So, if we plug these numbers in this expected net present value expression, we get around; we get a monetary value approximately close to 9534 right.

So with this, let us stop our discussion for the movement, in the next lecture we are going to continue that discussion.