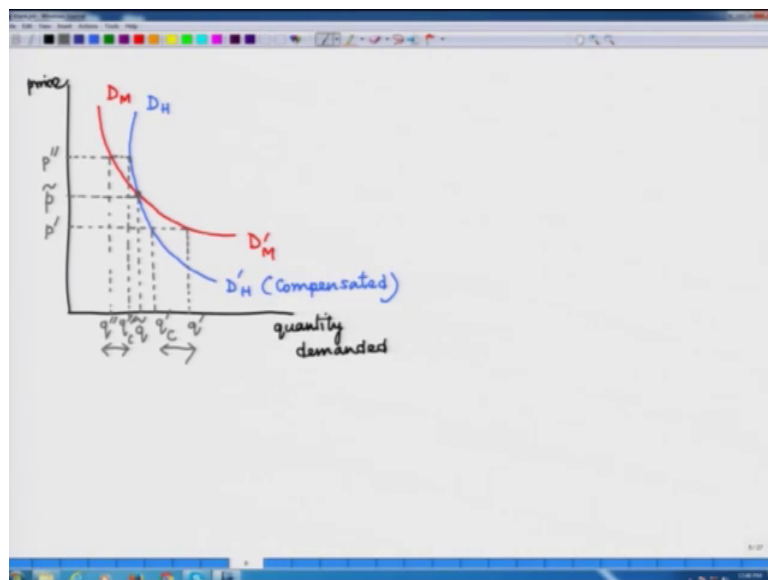


**Microeconomics: Theory & Applications**  
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**Lecture - 18**  
**Price Change & Consumer Welfare (Part – 1)**

Hello. Welcome back to the lecture series on Microeconomics. In the last lecture, we have studied Slutsky equation. In this lecture, we are going to study two different things. First we are going to study the merits and demerits of Hicks and Slutsky substitution effects and we are also going to study what will happen if there is a price change in terms of the compensated demand function and the Marshallian or ordinary demand function.

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Let us draw a diagram. So, here along the x axis, we are plotting the quantity demanded as this is a general result, I am not writing  $x_1$  and we measure price along the vertical axis. Now, we know that we can have the Marshallian demand function, we can have this time a curvilinear demand function does not matter, just to show the demand function can also be curvilinear.

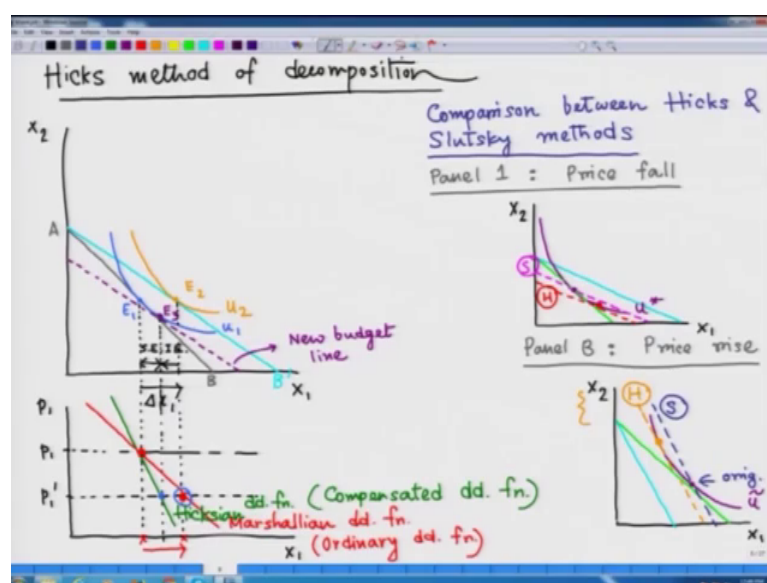
So, we can have something like you know this demand function  $D$   $D'$  and this is Marshallian to denote that let us have these subscript  $M$ . So, now, we can also superimpose the Hicksian compensated demand function on this diagram and we can have demand function like this. So, we can have  $D_H$ ,  $D_H'$ ,  $H$  denotes for Hicksian

or compensated. So, this is compensated ok. So, now, let us look at what will happen if we start from some initial price  $p$  tilde and if there is a change in price. First, we will talk about a fall in price. So, say price has gone down to price  $p$  prime. So, we can see that at price  $p$  tilde, the quantity consumed and demanded was  $q$  tilde.

So, now as we follow the Marshallian demand function, the quantity demand increases up to say  $q$  prime, but if we follow the Hicksian demand function the compensated one, then the increase in demand is low and the quantity demanded this time will be  $q$  prime say  $c$  compensated. Now, if we think about a price rise this time say price has gone up from  $p$  tilde to  $p$  double prime. So, we can see that if I follow the Marshallian demand function, then there is a fall noticed up to  $q$  double prime, but now if I refer to my Hicksian or compensated demand function at the same price, we observe the new demand to be  $q$  double prime  $c$ .

So, in this case we can see that the fall is less of this magnitude due to difference in the demand function we are referring to and here also we can see the rise is also less, if we the rise in the quantity demand is less if we go by the Hicksian or compensated demand function. Now, we are going to look at the difference between the Hicks method and the Slutsky method, what are the implications on consumer theory through the aid of two graphs. So far we have studied the case of a price fall, in this new discussion we are going to look at both price fall and price rise.

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Before we draw graphs and look at them carefully, let me comment that the Slutsky method is more pragmatic, it is more practical because of course, you know we really do not know at what level of utility the consumer you know was before the price change happened. So, it is really difficult to execute a Hicks type decomposition for a total price effect decomposition

Whereas, Slutsky's method is more practical because Slutsky is suggesting that we need to push the consumer back to the original consumption bundle. Now, as we can observe the original consumption bundle it is much more easier to implement Slutsky's suggestion. Now, let us look at two different panels of graphs. So, panel 1 will be the Hicks and Slutsky method shown for the case of price fall. And, there will be panel B which shows both these methods for comparison purpose for the case of price rise and the price changes are of course with respect to commodity 1; price of commodity 2 remains the same.

Let us start with the same old diagram, initial budget line, then there is a fall in price of commodity 1. So, budget line becomes flatter. Now, let us draw the initial indifference curve so that we can create tangencies and all. So, assume this is the indifference curve which depicts the initial utility level of the consumer denoted by  $u^*$ . This is the consumers equilibrium point that we start with. So, now, if I follow Hicks, then I have to take out money in from consumers wallet in such a way that the consumer is forced to go down to the initial utility level and then he or she finds himself or herself at this new equilibrium point.

So, this is you know the monetary adjustment following Hicks. So, let me denote by H now if I follow Slutsky, then Slutsky suggest that well we need to compensate the consumer, but the consumer should be pushed back to the original consumption bundle. So, money should be deducted in such a way that the consumer can at least consume the previous bundle. So, this new broken line is the compensated budget line following Slutsky principle. Let us denote by capital S, now note that this Slutsky method suggest a new budget line which lies above the budget line compensated budget line or, income adjusted budget line proposed by Hicks.

So, of course, these Slutsky budget line is going to make tangent with a higher level of utility depicting indifference curve. So, in the Slutsky's method actually, utility does not

remain the same due to this monetary income adjustment, utility actually rises. Now, let us see what happens in the case of a price rise same diagram same. So, we start with the initial budget line. Now, we go for a price rise. So, if price of quantity on rises, the new budget line will become a steeper one like this and let us start with the initial indifference curve. So, we can have an indifference curve which is tangential to the initial budget line at this point. So, this is the consumers initial equilibrium, we depict this level of utility by  $u$  tilde.

Now, note that in this case if I follow the Hicks method of decomposition, then I have to compensate the consumer in such a way that the consumer is given extra amount of money such that the consumer can gain in the initial consumer can go back to the initial level of utility, right. So, that is the idea of Hicks substitution. So, this broken line will be the new budget line. So, some extra amount of money is given to the consumer and the shifted budget line is denoted by capital H. So, here we see that the consumer makes a new tangency point between the dashed line, the new income adjusted budget line and the initial indifference curve and he finds his or her equilibrium.

Now, if I follow the Slutsky approach, then we can see that Slutsky suggest that you have to give money to the consumer in such a way that the consumer can purchase and consume the initial consumption bundle, the original choice. So, if I follow Slutsky suggestion, then I have to give consumers an amount of money and we have to push the consumers budget line in such a way that this new budget line, this broken budget line denoted by S passes through the original equilibrium point. Now, note that as the Slutsky budget line the broken budget line is lying above the Hicks proposed budget line, then the Slutsky budget line will create tangency with a higher level of indifference curve representing higher level of utility. So, again we see in the case of price rise also, the Slutsky method if we follow then utility does not remain constant.

Now, note that one of the problem with Marshallian demand function is that that as we move along the demand function, Marshallian demand function as we go down point by point along the demand function or go up point by point along the demand function, the utility is changing right. So, of course, that is not desirable because you do not want to change anything else you know apart from the price to have relationship between price of a commodity and quantity demanded of a commodity. Now, Slutsky approach if we follow the same problem remains. So, theoretically Hicksian method is much more

robust and that is why we find application of Hicksian demand function and Hicksian approach of total price decomposition in advanced economic analysis; especially in the fields of public economics environmental economics.

If time permits, we are going to discuss one application of this Hicksian demand function in later part of the course. So, we are done with our discussion on total price effect and its decomposition through Slutsky equation. Now, a partition question which emerges is the following as prices change, how a consumers welfare changes with that. To answer this particular question, we have to now introduce a new concept called consumer surplus. Before we define consumer surplus in proper way, let us look at another concept which is known as reservation price which is fundamental to the definition of consumer surplus.

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Consumer's surplus

Assumptions :

1. Discrete good ( $x$ )
2. Utility function is quasi-linear
3. Fixed money income ( $M$ )
4. Constant market price ( $p$ )

Quasi-linear  $u(x, M) = v(x) + M$

Reservation price ( $R$ )

The price at which a buyer/consumer is just indifferent between purchasing and not purchasing an unit of the good.

$R_1 \Rightarrow u(0, M) = u(1, M - R_1)$  for the 1<sup>st</sup> unit of  $x$  ... (i)

$R_2 \Rightarrow u(1, M - R_2) = u(2, M - 2R_2)$  for the 2<sup>nd</sup> unit of  $x$  ... (ii)

Assume  $v(0) = 0$

$v(0) + M = v(1) + M - R_1 \rightarrow R_1 = v(1)$  (iii)

For the 2<sup>nd</sup> unit,  $v(1) + M - R_2 = v(2) + M - 2R_2 \rightarrow R_2 = v(2) - v(1)$  (iv) marginal utility

$R_3 = v(3) - v(2)$  (v)

$R_1 + R_2 + R_3 = v(3)$

Generalized result :

$v(n) = \sum_{i=1}^n R_i$

Gross benefit from consumption of  $n$  units

Let us make the following assumptions to analyze reservation price and consumer surplus we are going to assume that consumer derives utility from a discrete good; that means, that the good is available in integer numbers like 1, 2, 3 etcetera. Then, we also assume that the consumers utility function is of quasi linear type. We assume that the discrete good is denoted by  $x$  and let us also make an assumption that the consumer has fixed money income,  $M$  and the consumer faces a constant market price  $p$ . This is per unit price for discrete good  $x$ .

Now, let us have a look at the quasi linear utility function which is key to our discussion. A quasi linear utility function can be defined as  $u$  as a function of  $x$  and  $M$ ;  $M$  can be any other good  $y$  also, but let us for simplicity assume that the consumer derives utility from the discrete good  $x$  and all the money he or she spends on all other goods which is denoted by  $M$  and we can write this quasi linear function. It is two components  $v$  of  $x$ , the utility that is derived from the consumption of discrete good  $x$  and the money spent on other commodities which is  $M$ . Now, we are going to introduce this concept of reservation price and reservation price is denoted by the symbol  $R$ .

For the time being, let us forget that we have a market price  $p$  for the discrete good  $x$ . Let us assume that the, let us assume that we are interested in knowing the maximum willingness to pay by the consumer for 1 unit of this discrete good and that is what reservation price is all about. So, now, let us look at the definition of reservation price. Now, how to represent reservation price or how to mathematically manifest reservation price? For that we have to go back to the concept of utility function. So, let us see how utility function will be used to mathematically manifest reservation prices. Let us assume  $R_1$  is the price at which the consumer is indifferent between consuming 0 unit or 1 unit of the discrete good.

So, if that is the case then we have  $u(0, M) = u(1, M - R_1)$  giving the utility from 0 unit of discrete good and all the money spent on other goods that should be equal to the utility level of the consumer. If the consumer purchases and consumes 1 unit of the discrete good, if he or she does so, then this is the highest price  $R_1$  that the consumer is willing to pay for that unit and that amount will be deducted from the money income that he or she has. So,  $M - R_1$  amount of money will be spent on all other goods. So, if that is the case then we say that  $R_1$  is the reservation price for 1 unit or the first unit of the discrete good  $x$ . Similarly, we can assume there is reservation price  $R_2$  and we can write  $u(1, M - R_1) = u(2, M - R_1 - R_2)$ .

So,  $R_2$  is the reservation price or highest price at which the consumer may have purchased 1 unit of commodity and the utility that he or she derives from this consumption bundle shall be equal to the utility from the following consumption bundle 2 units of discrete good and  $M - R_1 - R_2$  units of other good. So, this is for the second unit of consumption. Let me number this equation. So, this is number 1, this is number 2. Now, let us assume for simplicity. So, this is another assumption coming in that  $v(0)$  is 0.

So, if we assume that then what do we get from the identity or equation 1, we get  $v_0$  plus  $M$  equal to  $v_1$  plus  $M$  minus  $R_1$  that will give  $M$  equals  $v_1$  plus  $M$  minus  $R_1$  because we are assuming  $v_0$  equal to 0.

So, finally, we get  $R_1$  equal to  $v_1$ . So, the reservation price of the consumer for the first unit of the discrete good  $x$  is basically the utility that the consumer is deriving from that very unit of the good. Now, let us continue over discussion for the second unit of the discrete good. So, following the same logic, we can write for the second unit of discrete good consumption, the following shall hold;  $v_1$  plus  $M$  minus  $R_2$  shall be equal to  $v_2$  plus  $M$  minus  $2R_2$  or this will lead to, what this will lead to  $R_2$  equal to  $v_2$  minus  $v_1$ .

So, this is also very important result. So, the mark result for the second unit of the discrete good consumption is also very interesting. So, what we see that reservation price for the second unit of discrete good consumption is equal to the difference in utility what the consumer derive from the second unit of the goods consumption and the first unit of the goods consumption. So, if you remember this difference  $v_2$  minus  $v_1$  is nothing but marginal utility. So, the reservation price can be expressed in terms of marginal utility as well. Now, let us continue this discussion for the third unit. So, for third unit, there will be a reservation price  $R_3$  and for that the following shall hold.

So,  $R_3$  the reservation price for the third unit of the discrete good shall be represented as the difference between the utility derived from the third unit of the consumption and the second unit of the consumption or equivalently or alternatively, the marginal unit marginal utility of the third unit of discrete good. So, now, let us again number this equations or expressions. So, this will be 3, then this will be 4 and finally, this will be 5. Now, from 3, 4 and 5, one can write the following. You just sum these reservation utilities to see what happens. So,  $R_1$  plus  $R_2$  plus  $R_3$  will be equal to  $v_3$ .

So, this is a very interesting result that shows that the utility that a consumer is deriving from the  $n$ th unit consumption of the discrete good is basically the sum of reservation price of that particular unit and all the previous units. We can generalize this result and write, this expression that we have just written has a name. So, this is also known as the Gross benefit of consumption. In the next lecture, we are going to continue this discussion on consumer surplus and we are going to look at some other interesting topics in the theory of consumer behavior.