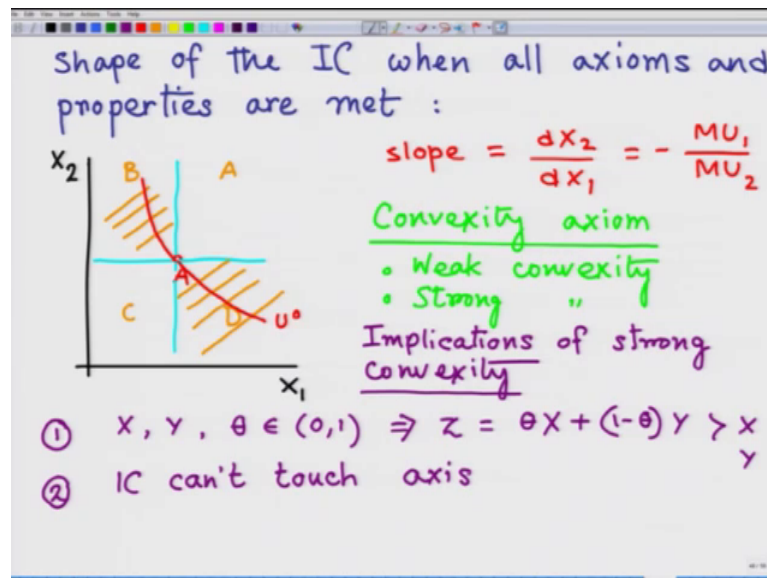


Microeconomics: Theory & Applications
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Lecture – 12
Indifference Curves (Part -2)

Welcome back to the lecture series on Microeconomics. Let us continue with the discussion on the properties of Indifference Curve and various shapes of Indifference Curve. We have listed down 4 properties of Indifference Curves. Now if the Indifference Curve follows these properties, what kind of shape we will that be?

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So, let us examine that focus more on the Convexity axiom. Convexity can be of 2 types; Strong and Weak. When we have drawn these convexs to origin indifference curve we are assuming Strong convexity.

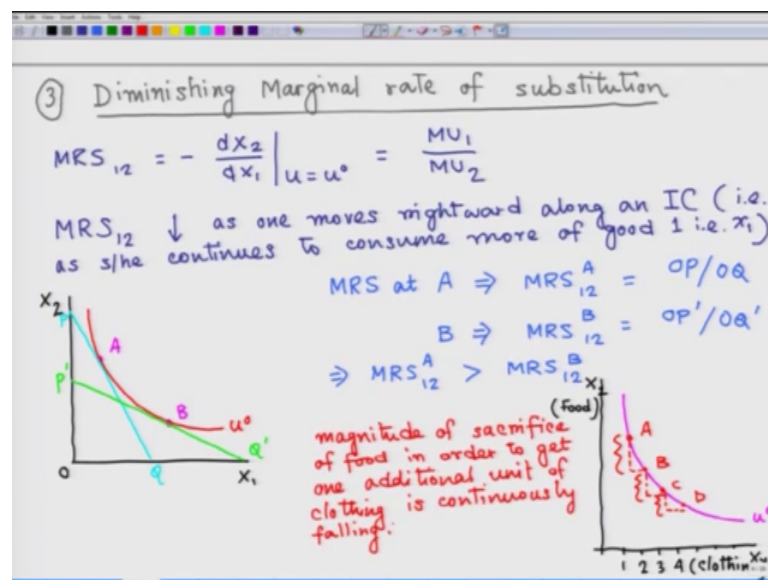
So, by Strong convexity, we mean that if you take 2 points on this indifference curve and if you join these 2 points or commodity bundles by 1 straight line, then that straight line we lie right to the curve entirely. And what do we mean by weak convexity? So, again you take 2 commodity bundles or points on this indifference curve, if you join them by a straight line. Then, either the straight line, we will lie right to the curve or it will coincide with a part of that curve. So, if weak convexity holds, there is a possibility that we may

not get a convex to origin indifference curve. So, in that case the indifference curve can have linear segments.

So, if weak convexities ruled out, if we assume only strong convexity. Then, that also has this implication that the indifference curve cannot touch either of these axes. Now what are the other implications of strong convexity? Convexity is really important in economic analysis and that is why we are focusing more on this convexity property of the indifference curve or iso utility curve. So, now, we are going to talk about the first implication of convexity. So, if the consumer has 2 commodity bundles X and Y and if we can assume a theta such that it takes value between 0 and 1.

Then, we can construct a consumption bundle, an average consumption bundle which is a convex combination of the bundles X and Y and that average bundle will be preferred over either the X bundle or the Y bundle. So, the implication is very simple here. The implication is that a consumer in general likes average over the extremes. Now, let us look at the second implication of convexity. We have already said that if we assume strong convexity IC can't touch axis.

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Now, let us move on to the third implication and this is very important in consumer analysis. This is known as the Law of Diminishing Marginal rate of substitution.

Now, what is Diminishing Marginal rate of substitution? For that we need to first define marginal rate of substitution. Marginal rate of substitution gives the maximum amount of a good that the consumer is willing to give up in order to get 1 extra unit of another good such that the consumer still gets the same level of utility. So, this is the definition in words. Let us see what does this mean in terms of mathematical language. We have already seen the definition of marginal of substitution. Now, let us see how we can define this in mathematical terms. So, Marginal Rate of substitution between 2 goods; 1 and 2 is defined as the negative of the slope of the indifference curve.

Remember on Indifference curve, the utility is fixed. So, we need to write this; so, these tantamounts to the ratio of marginal utilities. Now, we say that MRS_{12} declines as one consumer moves rightward along an Indifference Curve. It implies as he or she continues to consume more of good 1. It implies x_1 in our case. Now, if we want to show this graphically, then let us draw this graph. So, x_1 measured along the horizontal axis x_2 quantities of quantity 2 measure along the vertical axis and we have an indifference curve convexed origin a conventional Indifference curve denoting utility level u naught.

Lets draw a line which is tangent to this point A. So, that we can measure the slope of these are indifference curve at point A and let me name it P Q. Let me have another straight line drawn this time tangent to point B and let me name that in P dash Q dash; in that case MRS at point A would be OP / OQ . Similarly, MRS at point B would be $MRS_{12B} = OP' / OQ'$. So, from these expressions, it is obvious that MRS at point A is higher than or greater than MRS at point B.

So, what is happening here the absolute value of the slope of the indifference curve is falling as we move downward along an indifference curve. Now what does it mean economically for that. Let me construct another graph. Now note as the consumer consumes 1 unit of clothing and some unit of say food, we are on this point. We are at this point on the indifference curve. Now as the consumer he has increased the units of clothing to keep him or her on the same indifference curve or the same satisfaction level, the quantity of food has to go down and suppose we say this is the point and then, the consumer continues to purchase more and more of clothing to keep him or her on the same indifference curve. We have to reduce it his or her consumption of food and the consumer moves to a point say C.

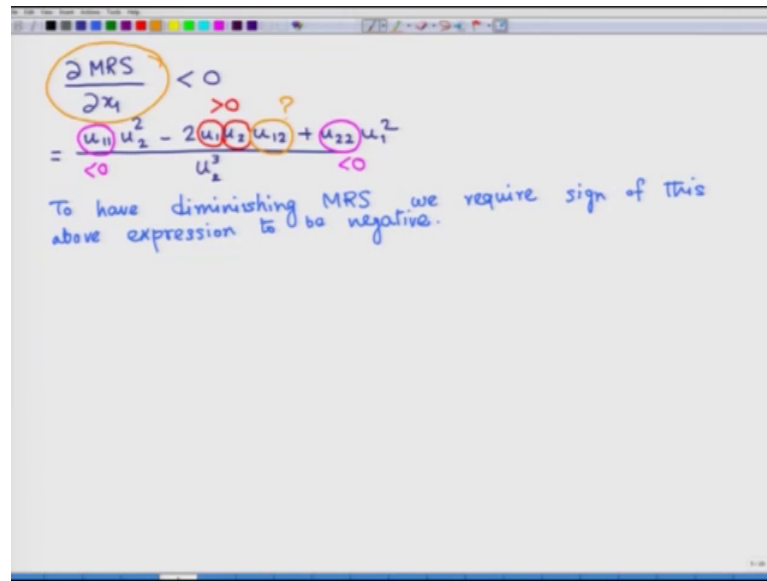
So, this way the consumer makes his or her journey along the indifference curve and he or she is moving right word right. Now, what does that mean in terms of MRS. So, note that the consumer initially was willing to give up this much of food for 1 unit of extra clothing. Once, the consumer reaches point B to get another unit of extra clothing. Now, the consumer is willing to sacrifice some unit of food, but that magnitude of sacrifice has gone down.

Now the consumer suppose moves from point C to point D and you can very well see from the graph that although the consumer is still willing to give up some units of food, it is fewer units of food compared to the previous cases. So, one may ask what is the reason for this. One can sight 2 different reasons for this; one is this as the consumer increases consumption of 1 particular commodity at the expense of the other, the intensity of desire to have more and more of that commodity falls because of the law of diminishing marginal utility. Because as the consumer is sacrificing 1 good, that goods the marginal utility is increasing because the consumer is having less units of that good.

The other reason could be that in world when we consume goods not all of them are substitutes of each other. So, in that case the consumer would like to consume an average bonding where the consumer has you know something of you know almost all commodities that he likes or requires. So, here in this case what we have seen? We have food and clothing now can the consumer leave with only food or can the consumer only leave with clothing? No, the consumer requires both. So, of course, after some point of time the utility of 1 extra unit of clothe or 1 extra unit of food, starts falling for the consumer and you know that leads to this convex indifference curve or the diminishing marginal rate of substitution. Because finally, the marginal rate of substitution basically gives a tradeoff between 2 commodities such that the consumer remains with same level of satisfaction.

Now, a question that automatically comes to our mind is that is there any linkage between law of diminishing marginal utility that we have seen in the case of martially on consumer theory and this diminishing margin rate of substitution, what we are finding in the ordinal approach to consumer theory. We are going to study that right now. So, for that we need to look at the mathematical expression of the slope of MRS.

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$$\frac{\partial^2 MRS}{\partial x_1^2} < 0$$

$$= \frac{(u_{11})u_2^2 - 2u_1u_2u_{12} + u_{22}u_1^2}{u_2^3}$$

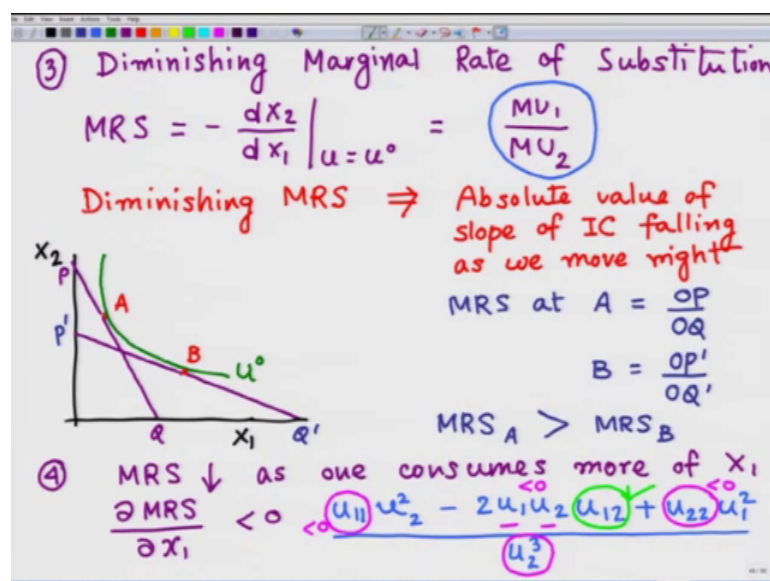
To have diminishing MRS we require sign of this above expression to be negative.

Now, let us look at the final expression. I am not going to derive step by step I am just going to state the final result and then, we will explain. Now note that these u_1 and u_2 are the marginal utility.

So, these are the first partial derivative of the utility function with respect to the respective commodities and as we know that there is axiom of non satiation, we can assume safely that they are positive. Now, let us look at these double partial or the second order partials. So, u_{11} and u_{22} as we have already assumed that there is Law of Diminishing Marginal utility, we can again assume that these will be negative. Now to have diminishing MRS, we require sign of this above expression to be negative. Now, it is impossible to comment on the sign of this expression because although we can certainly assign sign to u_{11} u_{22} u_1 and u_2 , we cannot do so for this entity u_{12} .

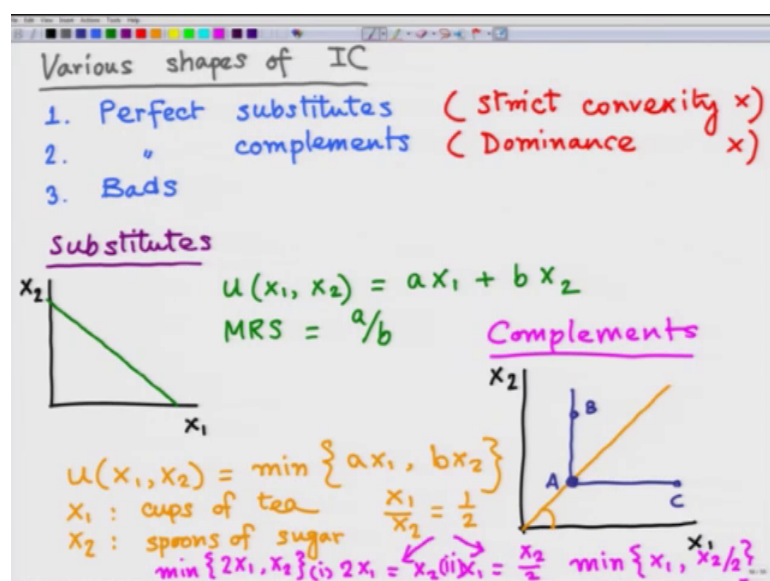
And hence, we cannot comment on the sign of this $\frac{\partial^2 MRS}{\partial x_1^2}$ in a straightforward manner.

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Now, having stated these properties of IC, we have seen that in general indifference curve is going to be convex to origin. Now, will a question automatically comes to our mind, if we violate some of the axioms, if we violate some of the properties of indifference curve; then, what sort of new shape of indifference curve are going to emerge and this is going to be the subject matter in last few minutes of today's discussion.

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First, we are going to talk about 3 different shapes of indifference curve and we have names for them.

So, we are going to consider or we are going to study 3 types of indifference curves which has non convex shapes and we are going to discuss them one by one. So, first let us start with the case of perfect substitutes. Now, in the case of perfect substitutes which assumption or axiom is violated? Let us study substitutes with the help of a graph. So, the substitutes are considered to be very similar kind of goods, you can treat them as almost 1 good or the same good. So, you know it is easy to understand the substitutes through examples. Let us take the case of pen. So, if you know I have to write down something on a piece of paper and somebody gives me a black pen or somebody gives me you know blue pen, it does not matter; it is a pen to me and you know with that pen I am going to write.

So, a black pen or a blue pen is basically perfect substitutes. It gives almost same kind of satisfaction to the consumer. You can also think about Coke or Pepsi. Suppose, you are you know willing to have a cold drink, you are thirsty in a hot summer day. So, it you know until and unless you really have some kind of restriction set outside or you know if you are some kind of very strong you know brand loyalty, you know there is no difference between a bottle of Thumbs-up or a bottle of Coke or a bottle of Pepsi. So, these you know bottle soft drinks are perfect substitutes to each other. So, if that is the case, then what type of shape indifference curve will take place? Note as the marginal utilities are almost same from you know these substitute goods, the MRS the marginal rate of substitution will not decline as one consumes more and more of the 1 commodity.

So, MRS here in this case will be constant. So, if MRS becomes constant, then we are expecting a straight line of course, because MRS is the negative of the slope of the indifference curve. So, let us draw. So, in the case of perfect substitutes, the indifference curve will be a downward sloping straight line and how to write such a preference in terms of utility function. So, this kind of preferences will be presented through utility function of this form, where a and b are some positive parameters and these are the values of these commodities 1 and 2 respectively to the consumer. Now, what will be the MRS? As I have said that MRS will be a constant in this case and the MRS value will be the negative of the slope which is in this case going to be a over b .

Now, let us take an example of perfect substitute goods. Let us assume a consumer who has 1, 5 rupee coin and 5, 1 rupee coins. Now, in that case the consumer will be indifferent between 5, rupee 1 coins and 1, 5 rupee coins. So, of course, we are going to have a straight line indifference curve in the consumption plane. Now, if we draw how that graph is going to look like and what will be the MRS? So, in that case if we plot a number of rupee 1 coins along the X axis and if we measure the number of 5 rupee coins along the Y axis, then we are going to have intercept 1 along the Y axis and intercept of 5 along the X axis. So, in that case MRS is going to be 1 over 5. Now, study the case of compliments. Which axiom is going to be violated if we are going to experience compliment goods?

The axiom of dominance is going to be violated in the case of compliments. So again, we will try to understand compliments through examples. Compliments are commodities which are used in fixed proportions. So, let us consider 2 examples. You consider left shoes and right shoes. So, these are used in fixed proportion 1 to 1 and that is why these 2 commodities are perfect complements. You can also consider your morning cup of tea or evening cup of tea. Suppose there is a consumer who has this preference that you know he or she adds 2 tins teaspoons full of sugar to every cup of tea. So, in that case there is a fixed proportion of half between cup of tea and teaspoons full of sugar, in this case tea and sugar will be perfectly complementary goods.

So, if 2 goods are perfect complements, how to draw indifference curve in this case? Again. So, suppose there is a fixed proportion that the consumer uses to mix these 2 goods to consume them together and if we know that proportion we can draw a straight line whose slope is going to give us that proportion. So, in that case, all these points along this straight line will give us the consumption bundles right. So, if now the consumer is given some more units of X 2 given the unit of X 1, these extra units of X 2 does not have any utility or satisfaction to the consumer because you know he has extra units of X 2 compared to the units of X 1.

So, he will be indifferent between this point that we have already drawn here A and say a point B here. Similarly, if the consumer is given some extra units of X 1, but he or she is not given extra units of X 2, then the consumer can be here say A community bundle C. So, in that case also the consumers indifferent between this bundle A and C because he cannot use these additional units of X 1 as he does not have those many number of X 2.

So, in this case the indifference curve, we will take this kind of right angled L shape and this corner point will be the point of consumption for the consumer. So, in this case of complements perfect complements, we observed that indifference curves are going to be right angle curves.

Now, how to represent perfect complements kind of goods or preferences through utility function? Well, in general one can write this type of utility function. So, now, let us consider the case of this tea and sugar. So, I said that there is a consumer who mixes 2 teaspoon fulls of sugar to 1 cup of tea. Let us assume X_1 denotes the cups of tea and X_2 denotes teaspoons spoons of sugar. So, X_1 over X_2 is 1 over 2. So, now, note from here we can write, we can rewrite this in 2 ways; we one can write $2X_1$ equals to X_2 or one can also write X_1 equal to X_2 over 2. So, if we follow the expression 1, the utility function becomes. If I follow the expression 2, then the utility function becomes. So, this is the type of utility function that is used in the case of the perfect complements.

So, what will be the marginal rate of substitution MRS in the case of fixed proportion utility function, where we see L shaped indifference curves? Now, it is very interesting to note that for this kind of indifference curves, we have a kink point. So, there is you know kink that is at the corner of this L. Now, if you want to draw a straight line through that point, you know you can draw multiple straight lines. So, basically the slope is indefinite. So, that is why the MRS at this corner point or the kink point is also undefined. But, what happened to the arm of this L which is parallel to the X_2 axis? For that of course, the slope is infinite. So, MRS is also infinite. What happens to the arm of this L shaped indifference curve which is parallel to X_1 axis. The slope here in this case is 0. So, MRS is also 0 in this case.

We stop here for the moment; we will continue this discussion in the next lecture.