Microeconomics: Theory & Applications
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Lecture – 11
Indiffrence Curves (Part -1)

Today we are going to start with the Hicks Elaine indifference curve analysis or ordinal approach to consumer theory.

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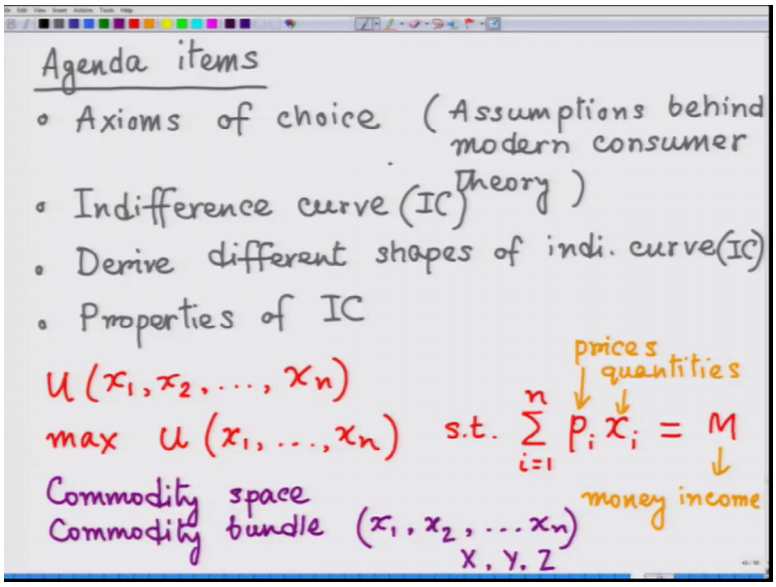

First, we are going to state what are known as axioms of choice. These are basically assumptions behind modern consumer theory. Then we are going to look at a concept which emerges from this set of assumptions. And, this concept is known as Indifference curve. The third agenda will be to derive different shapes of indifference curve and you know let us introduce the abbreviation IC for indifference curve.

Then we are going to talk about properties of indifference curve. This indifference curves are the building blocks of modern consumer theory. So, let us start with the axioms the very, but you know before we start with the axioms let us assume that the consumer has utility function defined over n number of commodities. So, the utilities are interdependent and this utility function is to be maximized subject to some constraint. So, the consumer faces a constraint optimization problem.

Now, what is the constraint? So, this is the postulate part of the consumer theory. So, the so, consumer wishes to maximize his or her utility subject to a thing called budget constraint. So, the consumer starts with some money income M and this money income he has to spend on different goods. So, the expenditure on various goods has to equal the money income. Here the p i is the prices of commodities which are given to the consumer and x i's are the quantities of these commodities and the aim as I told is money income right.

So, that is a consumers utility maximization problem and the consumer as he or she derives utility from this n number of commodities, we can assume that there will be something called a commodity plain, a space and there will be something called commodity bundle. So, this commodity bundle is basically the quantities of commodity 1 consumed quantities a commodity 2 consumed and so on and so forth. And, these are broadly denoted by a X Y Z. So, these are different commodity bundles that a consumer faces to choose from.

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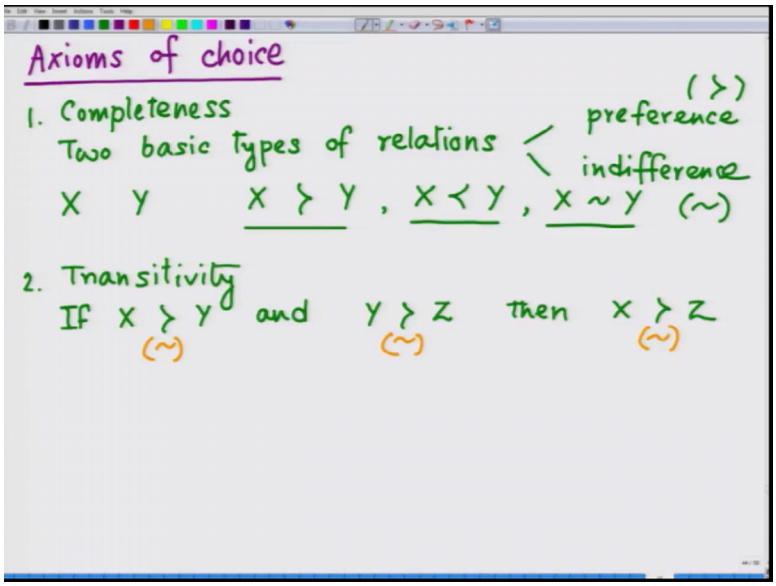

So, now we are going to start with this axioms of choice and the first axiom of choice would be completeness. Now, under this completeness choice we assume that all commodity bundles can be compared in terms of either indifference or preference. So, here we are talking about two basic types of relations between consumer bundles and

they are preference or it is indifference. Now, the preference is given by this mathematical symbol and indifference is denoted by this mathematical symbol.

Now, using these two symbols we can say that suppose, we have two different consumption bundles X and Y. If I write X is preferred over Y, I can similarly also write X is less preferred than Y or I can also write X is indifferent to Y. The axioms completeness says that any two bundle in the commodity space can be compared using either these two relations. Now, let us look at the second axiom which is the axiom of transitivity. Now, the axiom of transitivity says that if X is referred over Y and Y is preferred over Z, then X is definitely preferred over Z. Now, we can very easily replace this symbol with this indifference symbol also. So, any preference relation can be adopted to define transitivity.

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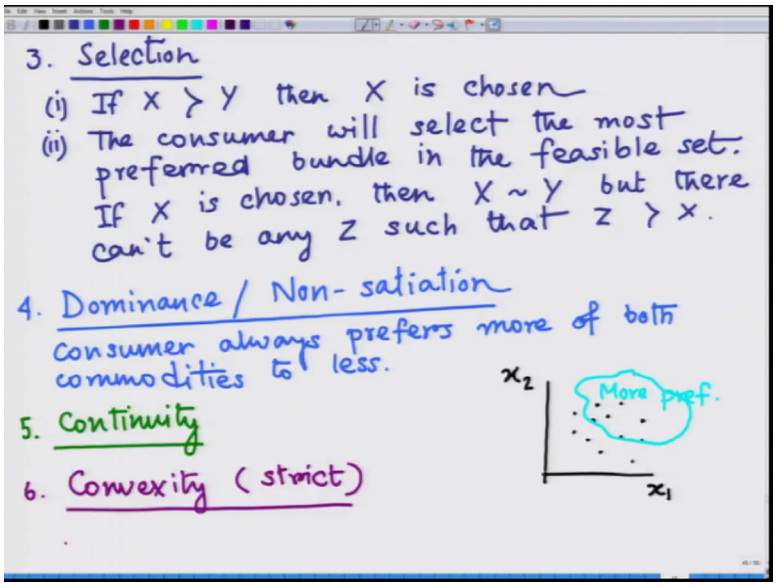

The third axiom will be the axiom of selection. Now, what is that? This in axiom has two parts; if X is preferred over Y then definitely X is chosen by the consumer. And, number 2 the consumer will select the most preferred bundle in the feasible set. What do I mean by feasible set? So, we are working in the commodity space, but there can be some constraints that do not allow the consumer to exploit the entire commodities space. In that case the consumer is working within or optimizing within a constrained subset of the commodity space and then you know we call that subset of the commodity space as feasible set.

So, if X is chosen then there can be some Y where we can say that X is indifferent to that Y, but there cannot be any Z such that Z is preferred over X. Now, we move on to a very important axiom which is known as the axiom of dominance or non satiation. We will see that if we tweak this axiom we can get various shapes of indifference curve. So, this is a very important axiom in the consumer theory. It means in words that consumer always prefers more of both commodities to less.

This axiom can be widened and it can also be said that in two commodity plane, if a bundle X contains more of one commodity and same amount of the other commodity then the bundle Y, then X dominates Y. So, it means the saturation point with respect to each commodity has not yet been reached. So, this point is very important, the saturation point and we will come back to this point again. The last axiom which we are going to not the second last not the last. The second last axiom that we are going to deal with is the assumption of continuity and it means continuity of preferences. So, it means that there exists a set of points on the boundary which divides the commodity space into less preferred and more preferred areas such that these points on this boundary are indifferent to each other.

So, basically these are community bundles right. So, let us look at a simple graph to explain this thing a very simple one. So, suppose we have only two commodities to deal with and this is the commodity plane. So, x 1 is measured along the horizontal axis and x 2 is measured along the vertical axis. So, these are the points in the consumption plane or community plane. Now, the continuity assumption is saying that there will be some region which is like more preferred and there will be some region which will be less preferred. So, suppose let me write that this is more preferred and it is saying that that the boundary between this more preferred and less preferred region will be a continuous one.

We will again come back to this issue later. Now, comes the sixth and final axiom and that is known as convexity axiom. And, we are going to talk about only the strict convexity because that will finally, give us the indifference curve which is the key tool of modern consumer theory. Now, convexity means I there can be two different types of expositions of convexity and we are going to see both of them.

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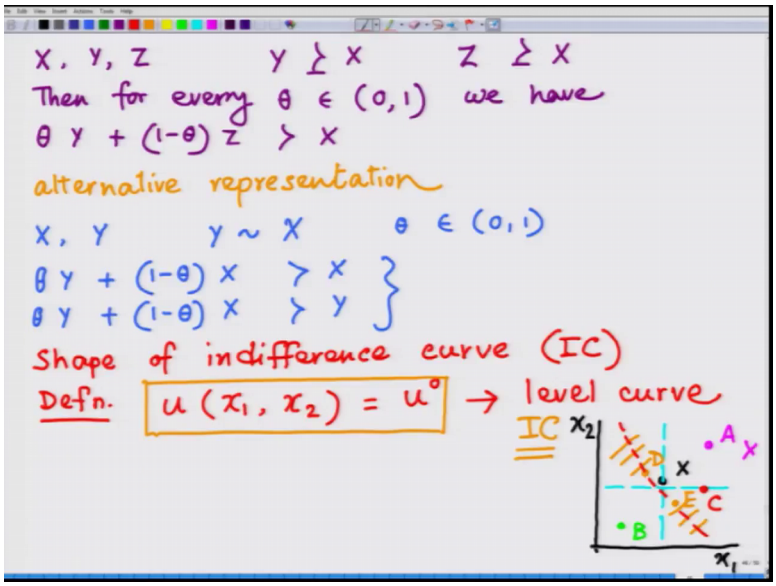

So, let us first start with three consumption bundles X, Y and Z. Now, I also assume that Y is at least as good as X and Z is also at least as good as X. Then for every theta belonging to open interval 0 1, we have theta Y plus 1 minus theta Z is strictly preferred over X. It implies if two distinct bundles that are treated as at least as good over a third bundle, a weighted average of the two bundles is viewed as being better than the third bundle. Now, there can be an alternative representation of convexity and that is what we are going to study next.

Here we are going to assume two consumption bundles X and Y and we are also going to assume that Y is indifferent to X and we are going to assume that theta belongs to this open interval 0 1. So, if strict convexity holds then we have theta Y plus 1 minus theta X. So, this is the convex combination of X and Y we are writing that is strictly greater than or better than X right. So, if we want to put these two mathematical statements in words, you know we can say that for two bundles that are being equivalent in this case that will be Y and X; a weighted average of the two bundles is better than each of these bundles. So, consumer wants or prefers you know a convex combination of two different bundles over these individual bundles.

So, now with this axiom stated we can look at the shape of the indifference curve, how to derive indifference curve. As indifference curve IC is the key tool of modern consumer behavior analysis, let us first understand what do we mean by indifference curve. So, we

will first start with the definition of indifference curve and then we are going to look at the shape of the indifference curve, how to derive indifference curve. So, we will start with the utility function itself u of x 1 and x 2, let us assume that the consumer is consuming only two goods. So, we are working with a two-dimensional commodity plane. Now, you can see that different combinations of x 1 and x 2 are possible and different combination of x 1 and x 2 can lead to same level of utility.

So, if I now fix utility levels at u naught then we get what is known as a level curve in mathematics. So, along this level curve we see different combinations of x 1 and x 2 which leads to the same functional value which is u naught in this case and this level curve or locus of these combinations of x 1 and x 2 is called indifference curve in economics. So, indifference curve is defined by this equation. Now, we are going to derive an indifference curve from the axioms that we have stated before. So, for that let us draw a simple diagram again assume a two commodity world. So, the consumer consumes only commodity 1 and community 2.

So, we plot x 2 along the vertical axis and x 1 along the horizontal axis and let us start with one commodity bundle say X. Now, let us draw some quadrants along this commodity bundle X. So, now we have to find out a point a commodity bundle Y and another commodity bundle say Z such that X is indifferent to Y and X is indifferent to Z. Now, where will these commodity bundles be located. So, let us play a trial and error game. So, let us assume that we get a commodity bundle here. Will it be a part of the indifference curve? The answer is no, because if we denote this commodity bundle by A, you can see that in commodity bundle A both the commodities have more number of units. So, of course, by the axiom of dominance A is strictly preferred over X.

So, A cannot be indifferent to X. So, this is ruled out. Now, let us take another point say point B or community bundle B in this point our commodity bundle B, both of these commodities have a less number of units compared to the initial bundle X. So, that is why X is strictly preferred over B. So, there is no way B point can be on the indifference curve. Let us take another point say here and let us call that this is the point C. Now, here you note although in the bundle C we have same number of units of x 2, but it has more units of the commodity 1. So, of course, that by the dominance C is preferred over X. So, C cannot also be on the indifference curve.

Now, note at these regions if we take a point in this region say here we have more of commodity 2 compared to the bundle X, but we have less units of commodity 1 in this bundle. Let us name this bundle D. Let us have another bundle here and we name this commodity bundle E. In commodity bundle E we have more number of more units of commodity 1 compared to bundle X, but we have less units of commodity 2. So, we cannot say whether E is preferred over X or not. So, this bundles D and E and X we cannot say which one is preferred over the other.

So, maybe there indifferent, so, these dashed regions are basically the regions of indifference and one can draw the potential boundary between the less preferred or the more preferred zone. And, that boundary basically will be giving us the boundary of indifference and this is the way we can logically derive an indifference curve from the commodity plane.

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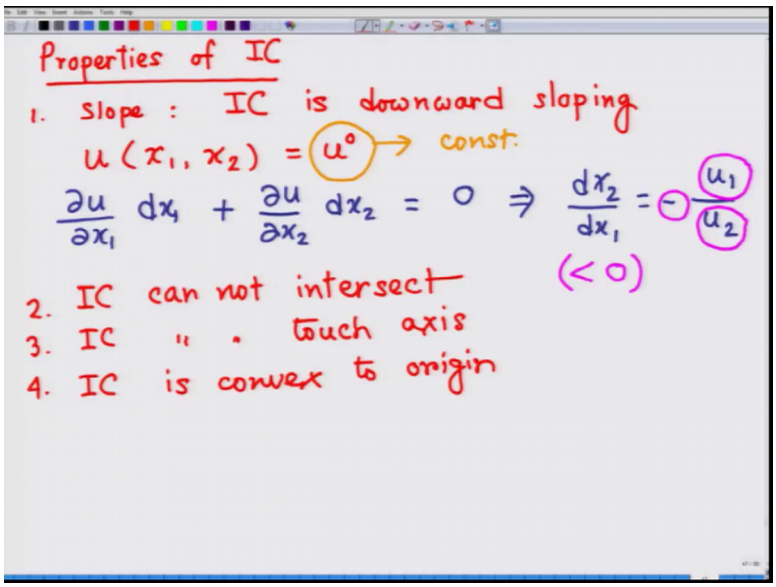

Now, let us move to the properties of indifference curve. Let us first talk about the slope property. IC is downward sloping in general cases. How can we prove that? Let us start with an utility function defined over two commodities x 1 and x 2 and let us assume that for indifference curve we set the value at u naught. So, this becomes a constant. Now, let us take the derivative of this function. So, of course we are going to have 0 and then we get that is the partial first partial, partial derivative of which will be function with respect

to commodity 1. And, this will be the partial derivative of utility function with respect to the commodity 2.

So, we know that no the basic assumption says that the marginal utility is positive. So, u 1 and u 2 both are u 2 both are positive. So, as we have this negative sign in front of the ratio of two positive numbers, the slope becomes negative. Let us talk about the second property of indifference curve. Two indifference curves cannot intersect because then there will be some logical violations. IC cannot touch axis also and then the most important property of indifference curve that in general case indifference curve is convex to origin. In the next lecture, we are going to continue with the properties of indifference curve and we will explore more on the convexity of indifference curves.