Microeconomics: Theory & Applications
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Lecture – 10
Practice Session (Tutorial)

So, far we have covered some basic mathematical tools useful for microeconomic analysis namely differential calculus partial differentiation and unconstrained and constraint optimization problems. Now it is time to look at some exercises as illustration, so that the idea of partial differentiation and optimization becomes clear to you. Now we are going to discuss 2 illustrations one on unconstrained optimization and other one on constrained optimization, we will start with a problem on unconstrained optimization.

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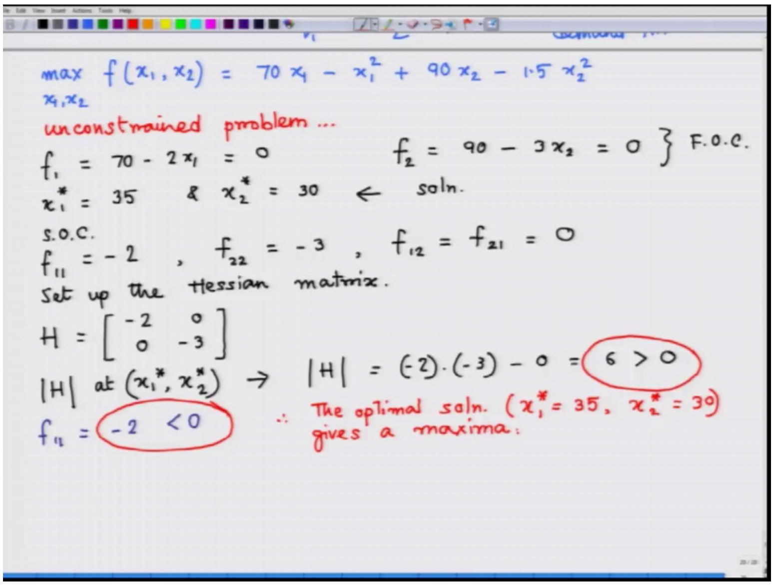

Let us think about a maximization problem where the function is defined over 2 variables x 1 and x 2 and this is the form of the function. So, let us assume functional form and of course the decision variables are x 1 and x 2, so this is an unconstrained problem. So, the first step would be to take the first order derivatives in order to from the first order conditions.

So, f 1 will be 70 minus 2 x1 and we have to set that equal to 0, then the second first order condition will be with respect partial derivative of function with respect to the variable x 2 and that will be equal to 90 minus 3 x 2, we have to set that to equal to 0.

Now of course there are 2 decision variables to solve and 2 equations, so we can get the solution x 1 star equal to 35 and x 2 star equal to 30.

Now, this is the first order condition and this is the solution to the problem, but we have to test whether this is giving indeed a maxima or minima. So, now we have to look for the second order condition, now second order condition we will depend on the higher order partials right. So, we need to now construct f 11 and that will be minus 2 then we need f 22 that will be equal to minus 3, then we need f 12 and we know that will be equal to f 21 and that equals to 0 and then the next step would be to set up the Hessian matrix right.

So, the Hessian will take the form minus 2 0 0 minus 3 and then we need to evaluate the determinant of the Hessian matrix and we need to evaluate that at x 1 star x 2 star right. So, if we do so then we get the Hessian as minus 2 times minus 3 minus 0 and that is equal to 6 that is greater than equal to 0. So, we see the determinant value is a positive one then let us also note that f 11 is equal to minus 2 which is less than 0.

So, thus we can infer that the optimal solution gives a maxima. Now we are going to look at a constrained maximization problem, now we are going to assume a power function because power functions are very common in micro economic analysis and I hope that you know an example or illustration with respect to the power function we will help you in this course.

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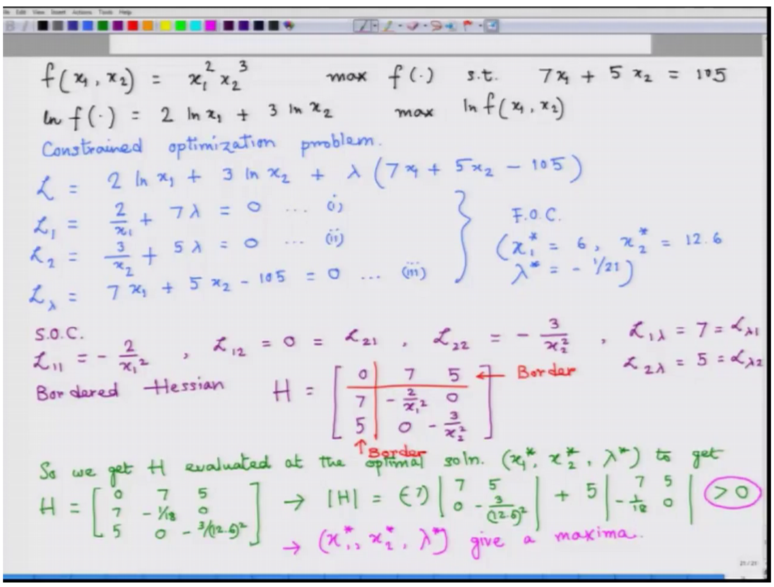

Here I am going to assume a function again defined over 2 variables x 1 and x 2 as x 1 square and x 2 cube. Now, we want to maximize this function f subject to subject to a constraint 7 x 1 plus 5 x 2 equal to 105. Now if we have a power function it is convenient to express that power function in a log form because, logarithm is a monotonic transformation it will not change our solution. So, let us take log and we get log of x 1 equal to 2 log, so we are going to maximize log f x in place of f x.

So, by nature this is a constraint optimization problem and we can say it Lagrangian equal to 2 log x1 plus 3 log x 2 plus we have this Lagrangian multiplier lambda. So, now we know that we need to differentiate the Lagrangian function with respect to 3 variables, to decision or control variables x1 and x 2 and the Lagrange multiplier lambda.

So, let us do this one by one, so first we take differentiation with respect to the variable x 1. We get our first order condition, then we differentiate the Lagrangian partially differentiate with respect to the control variable x 2 and that will sorry that will give me my second first order condition and then finally I have to again differentiate the Lagrangian with respect to this time the Lagrange multiplier variable and that would give me basically the constant.

Now there are 3 variables control variables or decision variables and 3 equations, so of course we can find solution to the set of equations and if we find out solution we can get these values x 1 star equal to 6, x 2 star equal to 12.6 and lambda star equal to minus 1 over 21 is very small number I am skipping the steps. Now whether this set of values x 1 star x 2 star and lambda star indeed give maxima or not for that we have to check the second order condition right.

So, now we are going to look at the second order condition. So, in the second order condition we know that we have to use the concept of bordered Hessian right. So, the elements of the border Hessian will come from the higher order partial. So, let us first construct them one by one and finally so we can now construct our bordered Hessian and it will take the following shape 0 7 5 7 5 I am writing the border elements first right.

So, the border element is basically so these gives you the border and this one as well. So, we get Hessian matrix evaluated at the optimal solution x1 star x 2 star and lambda star to get new H matrix, 0 7 5 7 5 minus 1 over 18 0 0 minus 3 12.6 square.

Note that I am not interested in deriving the particular values because, actually we do not need to derive the particular value of the determinant of the border Hessian, what we need to see we need to look at the sign and we can judge from the sign. So, these matrix will lead to this particular determinant and we can see here that the sign of the determinant will be a positive one.

We do not have to calculate the values of the determinants we need to see the sign of the left hand side determinant and we get a positive sign of the bordered Hessian determinant and we can then say that x 1 star x 2 star and lambda star indeed generate a give a maxima to this optimization problem. So, with these illustrations our discussion on basic mathematical tools are done, now we are going to start the modern consumer theory and we will start with the Hicks Elaine indifference curve approach or ordinal utility approach in the next lecture.