Total Quality Management-II

Prof. Raghunandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Confidence Interval III and the introduction to Hypothesis Testing

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you and I am Raghunandan Sengupta from the IME department, IIT, Kanpur and this is the TQM two lecture series again as we all know, I do repeat it, but you will understand, why I do that and why I mention the lecture number because it makes easy for you to understand and for me also to basically maintain the continuity.

So, this is under the NPTEL MOOC series and the this is lecture number 6; obviously, you will and this is the first class for the second week. So, because we will have 5 lectures of each half an hour for a 1 week, then another 5 which is 6, 7, 8, 9, 10 and it continues for such 8 weeks. So, we are discussing the concept of interval estimation and I did discuss the difference between mean given the population variance is known not known and how we can use the z distribution, t distribution with degrees of freedom reducing by 2 which is m plus n minus 2 is the new degrees of freedom.

So, let us continue this. So, now, I consider the same set of example which we considered in the last few minutes for the fifth lecture, it was something to do with the difference of the sample the population mean so; obviously, they are not known. So, we replace them consider that the population variance is what known unknown first they were known. So, hence we use the z distribution, hence, they were they were unknown and equal. Now, we will consider them be unknown as well as unequal. So, again we follow the same procedure x is distributed normal with a certain mean and certain standard deviation on the variance we know what are they are they are mu suffix 1 and sigma square suffix 1 for Y, it is sigma mu suffix 2 sigma square suffix 2.

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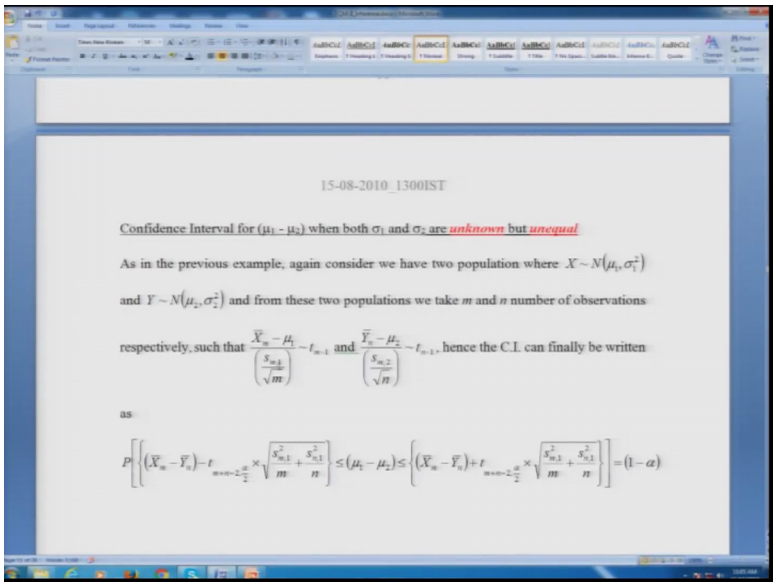

So, and; obviously, you will have the concepts that we will use S without the dash in both the cases because S dash would not be used because the population means respected means are not known. So, once you replace them, you have the same formula in the initial case you had S suffix P with the sample the pooled sample. Now S square suffix m comma 1 and S square suffix n comma n comma 1 are the respective means of a variance variances of the, this sample which is the square of the standard data. So, you replace in the formula the same pictures comes out same formula with just a small change.

So, where the similarities are again on the left hand side, if you see the lower limit is difference with the sample mean the t distribution has a minus because is on the left hand side, you see the suffixes are exactly the same m plus n minus 2 comma alpha by 2 and this case rather than S P which is the sample pooled with corresponding factor being multiplied these would be square root of S square m comma 1 divided by m plus S square suffix n comma 1 divided by comma 2 suffix n. So, these 1 and 2 would basically mean my mistake this should be n so; obviously, this which is n comma 2 and then you solve the problems. So, that would be if you go onto the right hand side the upper values confidence would be exactly the same with the plus sign before t.

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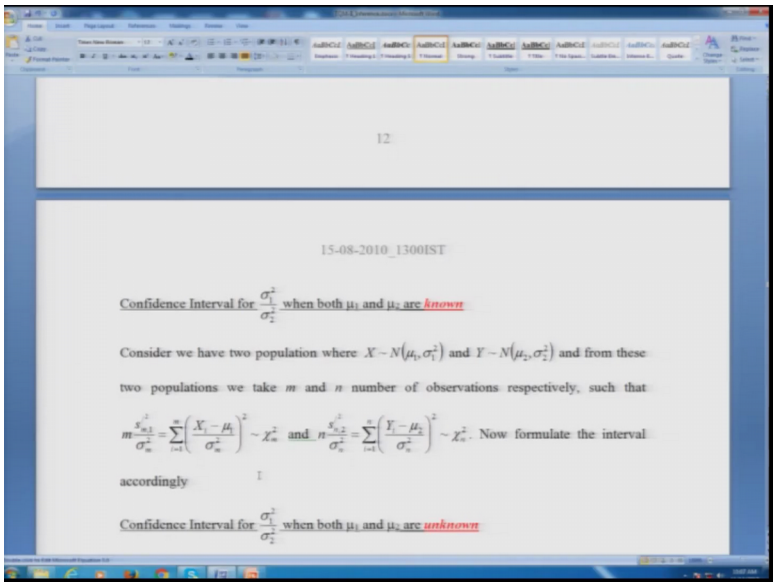

Now, I want to find out something to do with the, this population variances concepts. So, I will I will discuss the concept do not go into the details, but you will see that when we come to the hypothesis testing, we will be able to tackle it much more smoothly and much more efficiently.

So, in the confidence interval, for sigma one square divided by sigma 2 square when both sigma 1, mu 1 and mu 2 are known. So, if they are known; obviously, you will safely use S dashes in the both the cases, if they are using S dashes; obviously, you will know that you would not have any lose of degrees of freedom for chi square. So, it will be chi square suffix n m and chi square suffix n for population 1 and population 2 from when from where you have taken the samples of size m from the one and sample of size n from the second.

Now, if you have something to do with the ratios of the variances of the population, then the next question coming is that are the population mean value is known we have already discussed, they are known hence we are use chi square with without any lose of degrees of freedom. Now as they are unknown; obviously, you would be using and very intuitively you have or a you would have understood that we are using S without the dashes in the both the cases if you are using S without the dashes in both the cases you will basically have chi square with one degrees of freedom being lost both for m as well as for n. So, it will be chi square suffix m minus 1 and chi square suffix n minus 1.

So, once you do that you can find out the intervals accordingly now; obviously, that the f distribution I did mentioned in the fifth lecture, I did not come to that I am going to come that please have the patience.

Now, I will try to go into the hypothesis testing which is a very important concepts and it will be utilized time and again when you are trying to do some design of experiments; obviously, the background of the theoretical proofs using the maximum estimate the general methods of movements and all those thing which I did not discuss because this is not a part of part of this course would be coming up in pine estimation and interval estimation concepts so, but the concepts for hypothesis testing would be follow up from this point and interval estimation, we will utilize that, but first there is separate concept related to the hypothesis testing which we will try to discuss.

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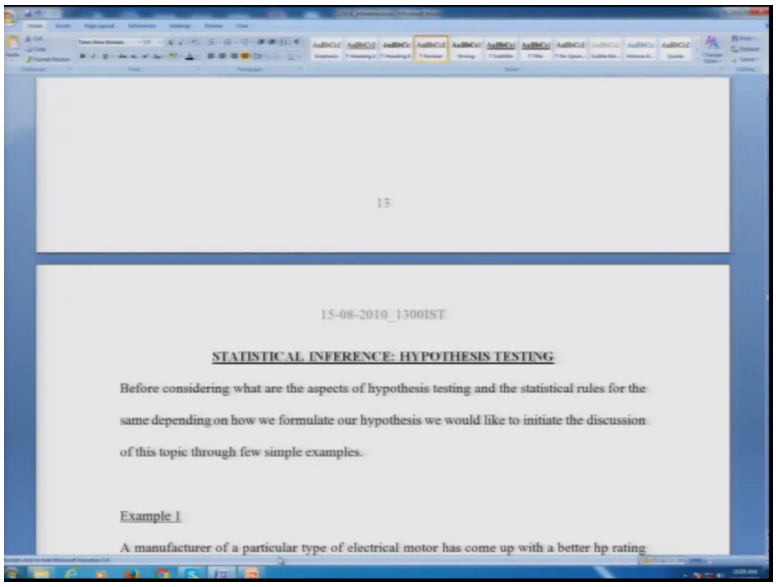

So, before considering; what are the aspects of hypothesis testing? So, I will read few things it will become clear to you and I will share four different scenarios of the example. So, that will make things much easier for you to appreciate as we proceed. So, before considering what are the aspects of hypothesis testing and the testing rules for the same depending on how we formulate a hypothesis we would like to initiate the discussion of this one topic through few very simple example. So, what are those examples? Let us go one by one example one.

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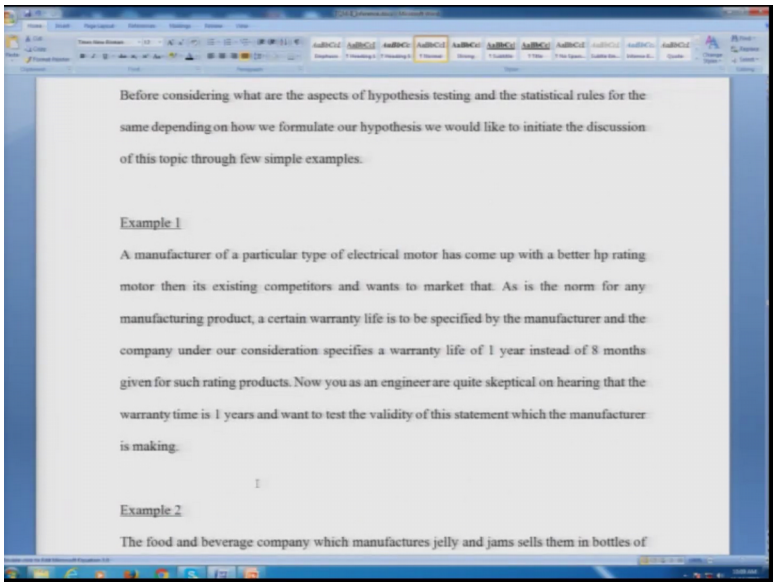

A manufacture of a particular type of electrical motor has come up with a better horse power rating motor, then it is existing competitors and wants to market that as is the norm for any manufacturing product or certain warranty life is to be specified by the manufacture for the new product and the company under our consideration specifies a warranty life of one year instead of eight months for the all the other existing competitor motors which are there in the market.

Now, you as an engineer are quite sceptical on hearing that because none of the motors till now had a warranty life one year. So, you want to basically test the validity of the statement which the manufacturer has made. So, this is the background of the problem based on which we will try to discuss hypothesis testing.

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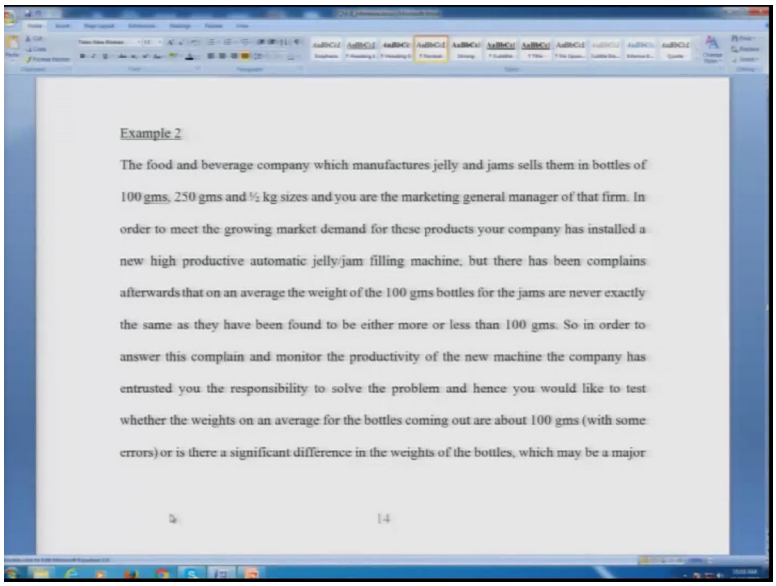

So, this is example one example 2 the food and beverages company which manufactures jelly and jams bottles, sells them in bottles of size 100 grams, 250 grams and half k g sizes and you are the marketing general manager of that form in order to meet the growing demand, market demand for this products, your company has installed a new high productive automatic jelly jam filling machine, but there has been complaints afterwards that on average the weight of the 100 gram bottles of the jams are never exactly the same as they have been found to be either more or less than 100 grams so; obviously, there is complaint people want to find it out.

So, in order to answer this, company and monitor the productivity of the new machine the company entrusted, you the responsibility to solve the problem and hence you would like to test whether the weights on an average for the bottles coming out are about 100 gram with some errors.

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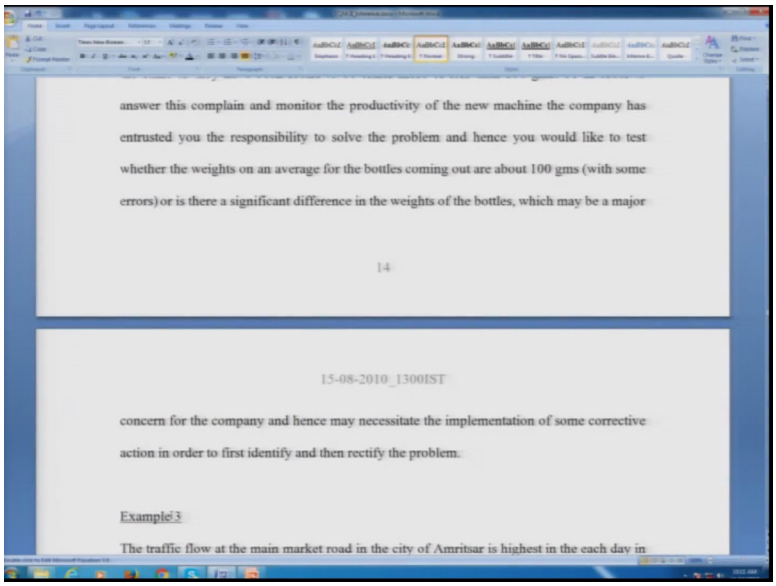

Obviously, there would be errors or is there significant difference between the weights of the bottle which might may be a major concern from the companies and; obviously, people can sue them and there can be litigation cases.

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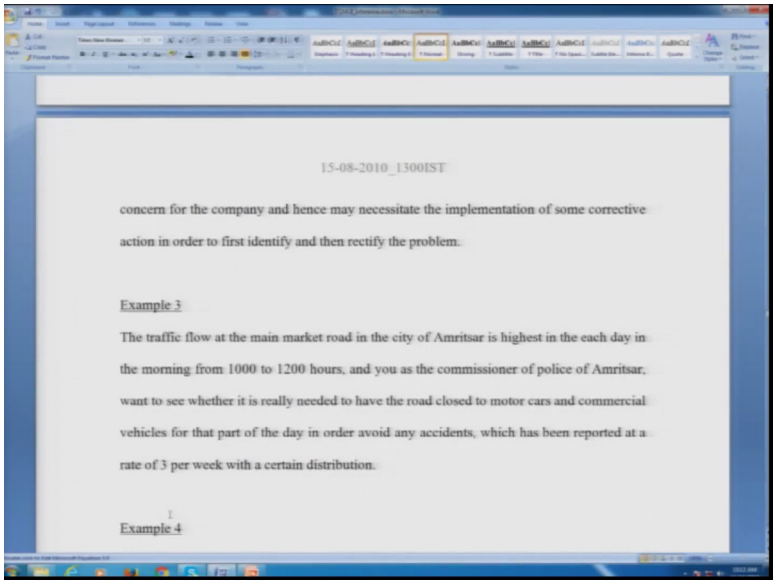

Example three; third example the traffic flow at the main market is road in the city of Amritsar is highest in each day in the morning from 10 hours, 12 hours Indian standard time and you are as the commissioner of police of Amritsar wants to see whether, it is really needed to help the road closed to motor cars and commercial vehicles for that part

of the day in order to avoid any accidents which has been reported at a rate of three per week with a certain distribution. So, you have to verify the fact take action because we close the road traffic; obviously, there would be inconvenience for people if you do not close there would be accidents. So, you have to take a decisions accordingly fourth example in the city of Guwahati a new internet service provided has opened its services.

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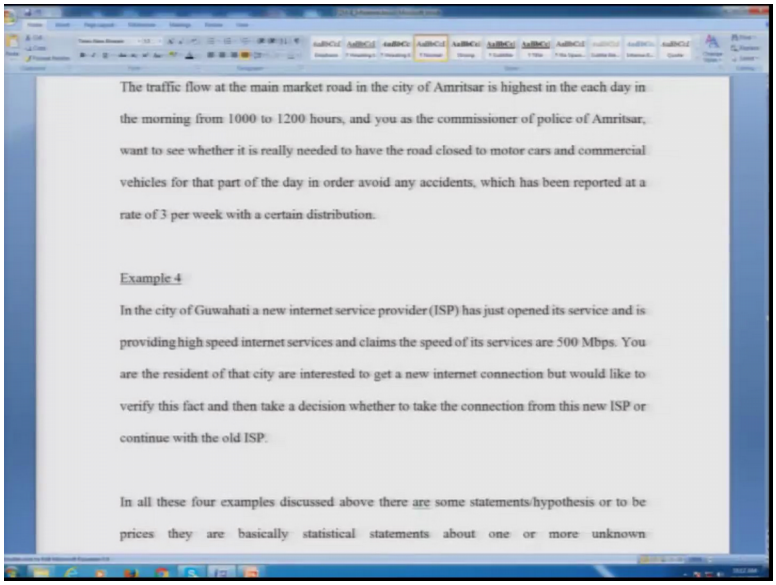

And is providing high speed internet services and claim the speed of its services are 500 MBPS, you as the you are the resident of the city and you are interested to get a new internet connect connection, but would like to verify this fact and then take a decision whether to take the connection from this new service provided or basically continue with the existing service provider because this is a cost involved.

So, these are the four examples in different perspective it can be also see for example, you have farmer in Andhra Pradesh or a sugarcane manufacture in Maharashtra and you want to basically test a new fertilizer and when you are trying to basically test a new fertilizer, the existing output is given in some metric transfer acre or of or kilometre square of area and you want to verify whether the new acreage will be more or less based on that you will basically waste your money in the new fertilizer or it can be some farming equipment also. So, these are the four examples already considered and the fifth one which is just mentioned verbally.

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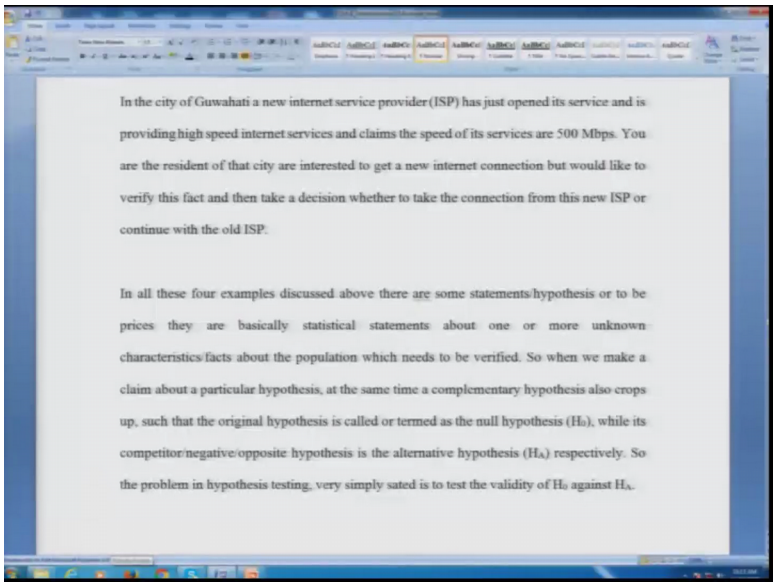

In all this four examples discussed about, there are some statement of hypothesis or or some words have been mentioned something has to be verified nullified agreed upon so and so forth or to be or hypothesis has been mentioned. So, they are basically some statistical statements I have been made about one or more unknown characteristics or facts about the population when needs to be verified or refuted.

So, when we make a claim about a particular hypothesis at the same time a complimentary hypothesis also crops up which basically will it be the other part. So, if you believe a statement one they would be complementary statement one would basically complete the whole scenario which is basically whole scenario means whatever the different type of scenarios you can have. So, when we make a claim about a particular hypothesis, let me read it again, at the same time, a complementary hypothesis also crops up such that the original hypothesis is termed as the null hypothesis which is H suffix naught or o or 0 while its complementary or negative counterpart or the opposite hypothesis is the alternative hypothesis which is given by the symbol H suffix A.

So, the problem is to state the validity of H naught verses A; H A that is the overall essence and how you do that that is what it basically this part would be coming from the concept we have already discussed how you formulate and understand the problem is basically the real part in hypothesis testing.

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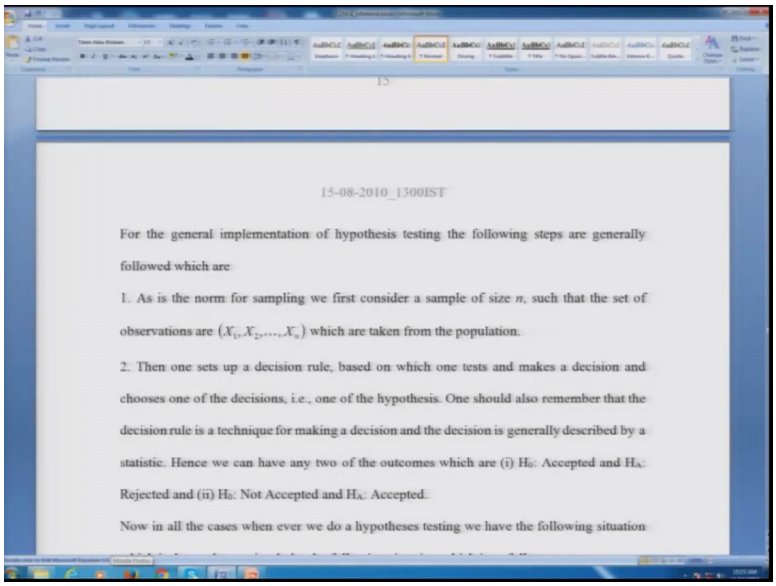

Now, generally how you do that for general implementation of hypothesis testing we follow the following steps which are as follows as the norm in the sampling we first consider sample size of size n pick up sample observations capital X 1 to capital X suffix n. So, the 1 and 2 and 3, whatever I mentioning as you can see on the slide are the suffixes so; obviously, if I pick up a sample size of n and few pick up the sample size of n. So, initially before we find out what are the exact values of those random variables they would be denoted by capital X suffix 1 capital X suffix 2 and so on and so forth.

But obviously, the real as well use the different so; obviously, what I predict from hypothesis testing would be different from your case so; obviously, neither you are right or wrong neither am I right or wrong because based or the examples of the observation we picked up are answers are different.

So, remember this very carefully, this will become more and more clear as you proceed then one sets a decision rule what is the decision rule, we are going to come to that just understand the steps without going to the actual theoretical concept based on which one tests and make the decisions and chooses one of the decision that is one of the hypothesis one should also remember that the decision rule is a technique for making a decision and the decision is generally described by a statistic; statistic is basically the corresponding characteristics which I am getting from the from the sample.

Hence, we have 2 of the outcomes H naught which is ah the null hypothesis H A which is the alternative hypothesis. So, they can be 2 cases either accept H naught and reject H a or vice versa. So, the words I am using accept reject technically has to be used in a different frame work I will come to that later, but very simply considered either your for H a and do not agree with H A um H naught or vice versa. So, this is the overall general problem scheme.

Now, in all this cases, when you are doing that a certain scenario will come up. So, this scenario would basically a 4 cases and what are the cases if is read and understand the table we were all spend some time you will understand consider the action which you are taking is can be 2 type either you basically reject H naught or you reject H A rejecting H naught; obviously, means accepting H A and vice versa so, but I am using the word reject based on the information which you have you cannot accept untilness your 100 percents are, you can only reject depending on the on the set of information which you have picked up.

Now on other side is mother nature and mother nature's information's would be either H naught is true or H is true. So, now, mark the words true and reject. So, reject you are doing it because your set of information is only once based on that you are passing a judgement and mother nature as all the information based on which it is saying that either H naught is true or H A is true.

Now see the four combinations I would not consider the pink one now first I will consider the cells which are not coloured if H A is true by mother nature and you reject H naught which means you are agreeing with H A so; obviously, there is no error on the other hand if H naught is true and you reject H A which means you are agreeing with H naught which means that is also true there is no loss or there is no so called risk.

Now consider the other 2 cells which are along the principle diagram. So, called principle diagram and they are coloured pink in colour. So, these are the scenario if H naught is true and you reject H naught there is a error or type one which is given by the value alpha and in the other case if you basically H A is true and you reject H A it is basically the error of type 2 which is beta.

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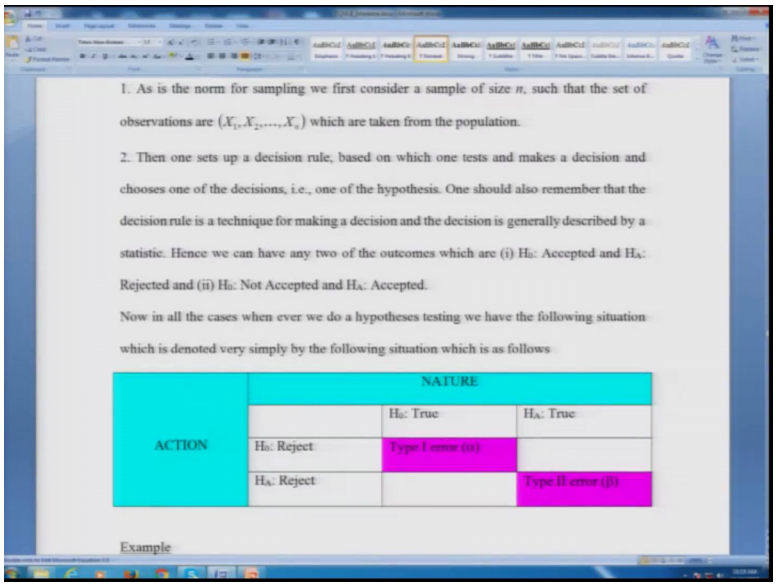

Now, let me give an example in order to for you to make to understand and then scroll down to explain it in more details consider, I will just put the example in the right perspective you are a bank manager and as a bank manager you have been given the business that you should give loans; obviously, loans can be there they can be education loans housing loans personal loan business loans people are coming in your bank to open a fixed deposits they have salary accounts, then they are coming for opening the reckoning deposits, there are many things they have the lockers.

So, consider that you want to give a loan and you are considering on only that and you have a set of question is for people to answer based on the question is the point which you give you will basically you and your team will assign a loan or not assign a loan. So, what can be the question as we which is very very simple to understand? So, whether he or she is a salaried person what is his or her age whether the person has a government job or a private job whether the person has a business or he is not a business whether the person has his own house or does not have a own house whether the person has any liabilities does not have the liabilities whether what is the overall credit rating on the person or there is there is the credit rating on the person is absolute fantastic.

So, based on that you have different question is and you give points consider the total points is 100. So, you have decided if a person gets a score or a set of points of about 50, you will 50 and above you will give that loan and if it is 50 below 50-49 and below;

obviously, and consider they are only coming in integers the sum of the scores you would not you would not give the loan, but they can be 2 scenarios, there can be a set of persons to whom you should not give the loan, but you are giving the loan. So, that is a loss how it is lost because of bad debt. So, is a bad business for you another set of persons who are there who should get the loan because they are able to pay the loan, but you deny them the loan. So, they are opportunistic cost lost.

So, these type of losses bad debts or opportunistic cost can be considered as the concept of alpha and beta which are that discuss this is a very simple example now intuitively if you see and I am when I will come to that diagram you will see that minimising both of them are the same types are not possible why I will explain that.

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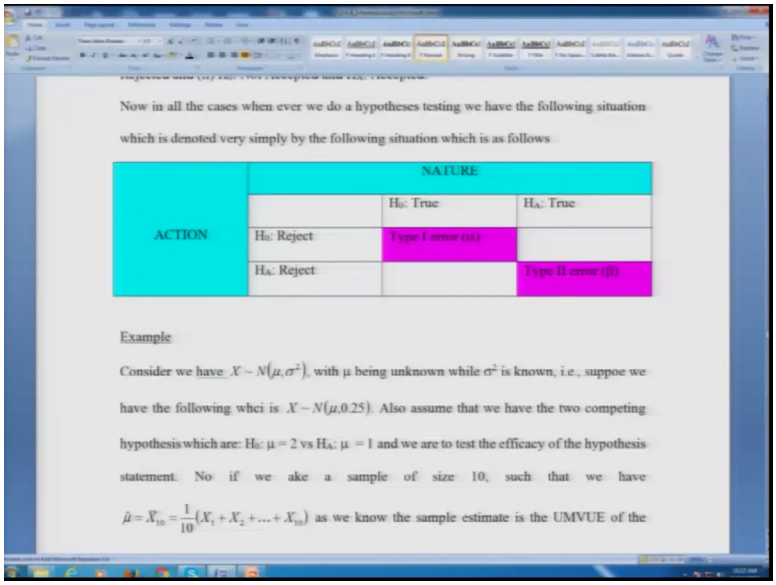

So, example let us consider you have. So, I have basically the example which I am going to consider and the example which I has mentioned are exactly the same, but I gave an example of the bank in order to make you understand very simply consider, we have a some the x which is normally distributed the mean mu and sigma square where mean mu is the mean and sigma square is the variance mu is unknown sigma square is known and suppose it is given as mean as mu and sigma square is 0.25 for the other case.

So, also we assume that we have 2 computing hypothesis. So, this this is the more mathematical formation that is why I give the example beforehand, but the diagram would make sense to you. So, 2 computing hypothesis which are h naught mu being 2

with respect to H A mu being one, it can be not equal to 2 also, it can be greater than 2 less than 2 whatever it is and we have to test the efficacy of the hypothesis now we consider we pick up a sample of size ten if you pick up the sample of size ten; obviously, the sample mean is given by this formula. So, which I will just highlight for this discussion the, this is the sample size. So, n sample size can be n which is 10, 20, 25, 30 whatever it is then; obviously, this would become true which is the overall ah sample mean x bar suffix n being 10.

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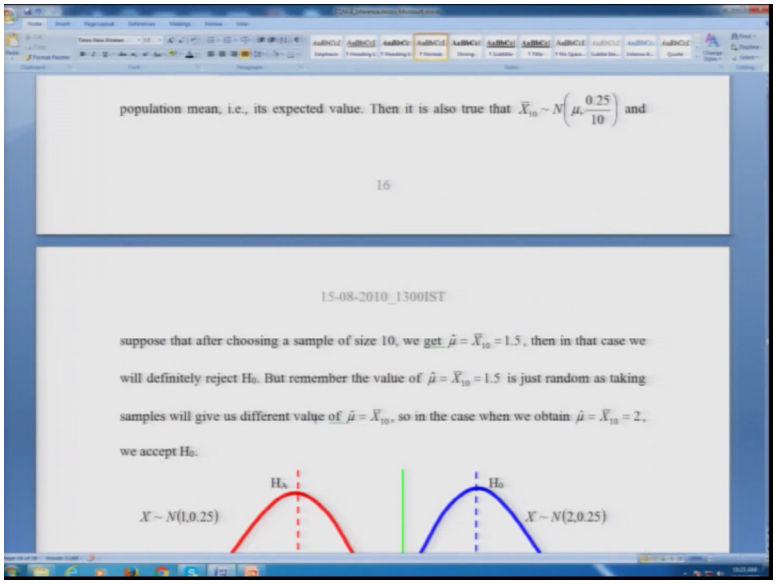

Obviously, would have a distribution as given with mean as mu and the variances 0.25 divided by 10. So, once we are aware the, of the information we move on further and suppose that choosing a sample size of 10, we get mu the mean value comes out to be 1.5. So, you want to basically definitely we are not interested into accepting H naught we will reject the H naught.

But; obviously, if you pick up that set of observation ten in number your value can be greater than 2 so; obviously, in that case, you will accept it or it is to you will basically take a decision accordingly, now I will read it, but consider the example here.

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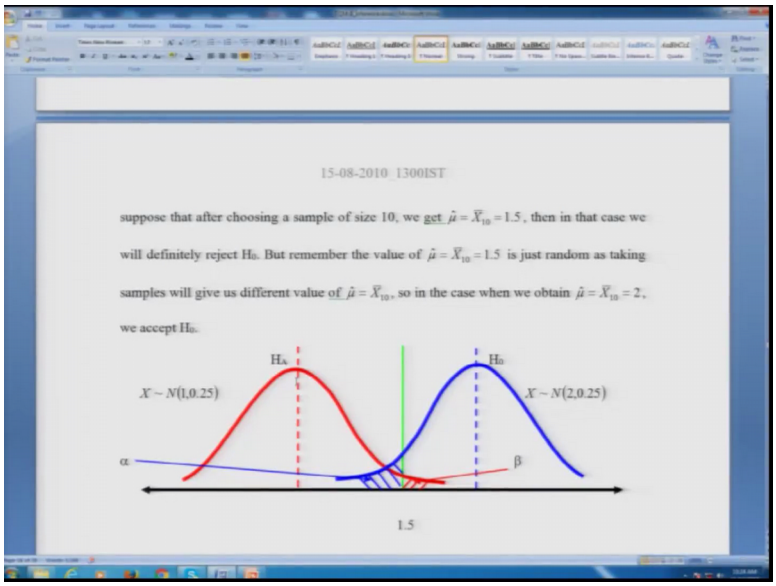

This is the diagram where I want you all of you to pay special attention forget about what are the markings numbers just concentrate on the normal distribution red in colour normal distribution blue in colour the mean values which are vertical dotted lines for the means for the red distribution which is the red on the left hand side where I am where I am placing my left hand and on to the right hand side which basically the blue one and also pay attention on the green vertical line which is the bold one which is here.

Now come back to the example which I just discussed if 50 is the marks at the points. So, any person getting less getting less than 50 will be rejected any person getting more than 50 would basically 50 or more would be accepted for the loan. Now the value which is hashed on to the right red are the set of persons who should be denied the loan, but they are getting the loan which is basically bad debt for you if you remember I did mention that and the hash blue one which is there on the left hand side, consider is alpha and the red one is basically beta is the set of person the blue one is the set of persons who should be given the loan, but they had denied the loan which is the opportunity cost lost.

So, now if we consider shifting the green line like trying to reduce it from 50 to 45 or try to increase it from 50 to 55 you will see trying to basically minimise both alpha and beta at the same time is not possible. So, what you will try to do is that you will try to basically minimise the sum of alpha and beta and that is the essence based on which we will try to solve the problem it in the sense that will keep beta fixed and try to play that

and do a problems with considering alpha or you can do the other way around, but we will always consider an alpha because that is one minus alpha will give you the level of confidence based on which we are trying to do that example.

So, in this problem, it can be H naught can be greater or left. So, this values of where you paste h naught and h a are not important for you to understand from the problem perspective the example which I gave. So, if you understand the example and understand the diagram it will be much easier for you to solve the hypothesis could also have been framed in the sense that H H naught which is mu is greater than 2 and H A which is mu for that case is less than 2, again you can formulate the problems accordingly and solve them, but the essence of the diagram remains the same.

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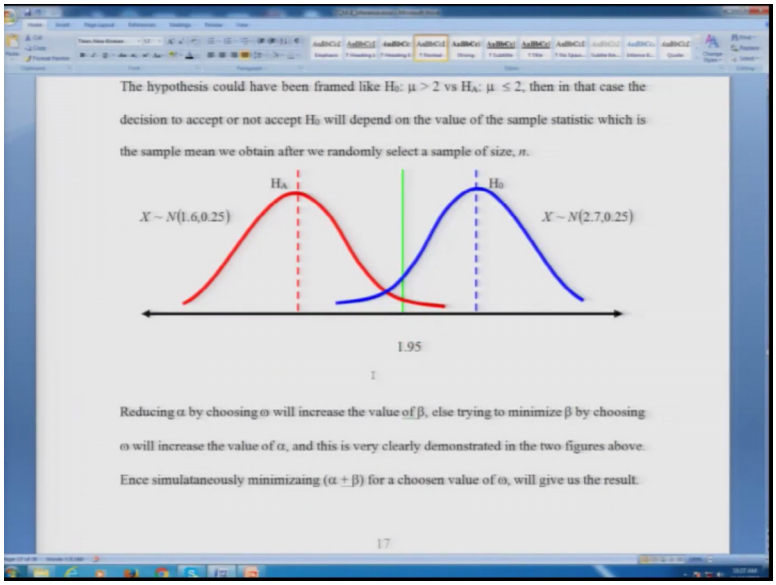

So, choosing any variance of alpha when you choose it basically that you will try to choose it in such a way that your level of confidence or the values which you are trying to pick is basically optimal; optimal in the sense that you are able to cover the maximum area considering the level of confidence which you have placed.

So, with this if you understand the examples and pause and think about that the problems which we did discussed in case of say for example, the pint distribution they can be easily picked up from those scenarios and try to basically put up in the case of the hypothesis testing, but only remember framing the problem perspective from the hypothesis testing the main task based on which we will try to basically proceed. So, I

will come to the general. So, these are not relevant for this problem I did discuss that and if you are aware of the problems for the case of the banking manager giving a loan not giving a loan. So, with that we try to basically solve the problems.

Now, this is the first scenario, suppose we want to test the hypothesis, I will directly come to the hypothesis testing you want to this select the hypothesis as highlighted which is under H naught mu is equal to mu naught. So, this mu suffix naught is basically under the assumption h naught is true verses the case when you want to find out an alternate hypothesis for which the mean value is mu suffix a and you want to test that mu suffix A is less than mu naught. So, in that case you will be concentrating on the distribution on to the left hand side.

So, if you can see that the distribution on the left hand side the problem becomes that the overall area to be covered till is one minus alpha area has been covered is alpha sense in hence you will basically reject or accept H naught based on this what is that. So, this is what is highlighted is here. So, you the rule is reject H naught if you are on the left hand side. So, what is the left hand side that your mean value of the sample is less than mu naught minus z alpha. Now remember very important thing, it is not alpha by 2 this I will be repeating time and again into sigma by square root of n. So, now, if you go back to the interval estimation the problems on the of the left confidence was exactly the same it was x bar n n was the suffix for the sample size minus Z alpha by 2 into sigma by square root of n.

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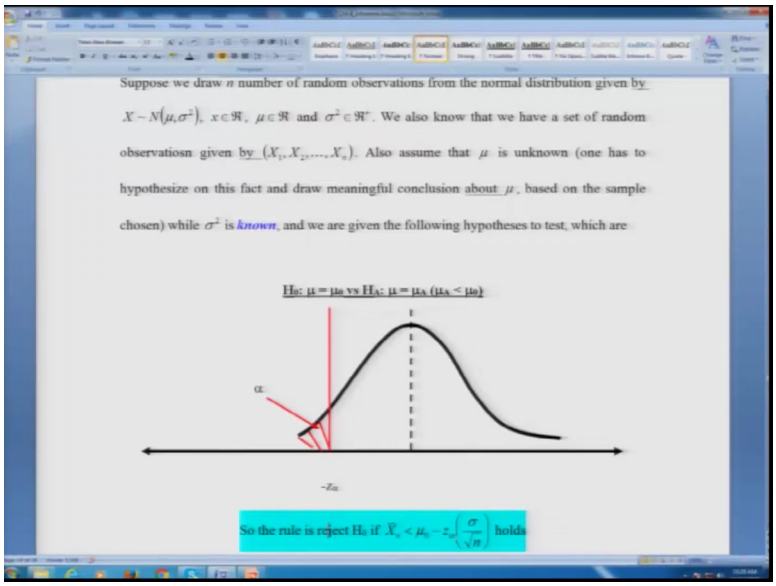

So, because it is only left hand limit all are left tailed, hence, it would basically be consuming all the alpha values which is there still one minus alpha it is to be covered on to the right hand side right hand side means this area. So, if you remember this is the value which I am talking about with this is z minus z alpha by 2 minus z being because it is symmetric one; obviously, on to the left where I am pointing my left hand would be negative values on to the right of the dotted vertical black line would be positive. So, based on that the alpha value is here and you basically do the problems accordingly.

So, if you remember the main task is basically beta is basically given as a fixed level and you are trying to basically minimise the or trying to find out the optimal value of alpha. Now it is something to do with the hypothesis testing where it is on the right hand side again the concepts comes out to be same I will highlight it.

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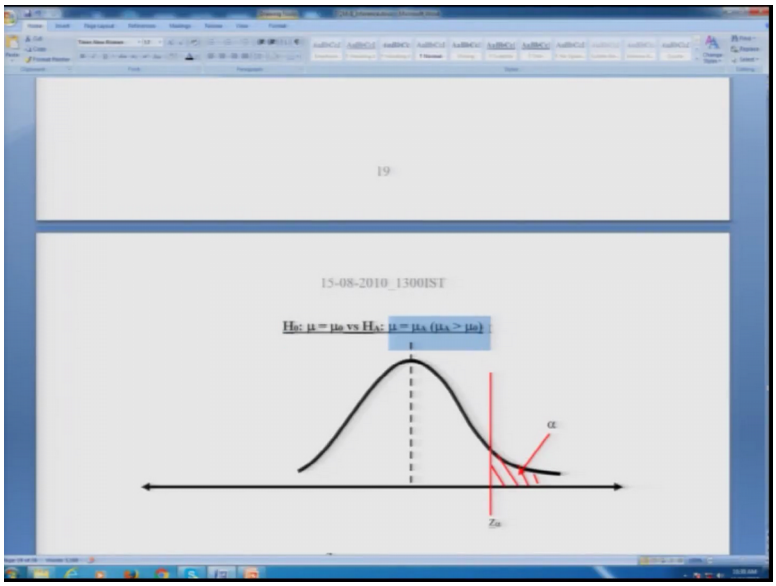

So, here mu naught H naught is case for when mu is equal to mu naught and H A which is alternative hypothesis is basically mu a is greater than mu naught so; obviously, it will be on the right hand side again be careful it becomes that you will reject h naught if mu is x bar n is greater than mu naught plus z alpha in multiplied by sigma by square root of n and; obviously, this z alpha is not alpha by 2, it is coming for the case because you are considering the right hand side of the distribution not the left hand side.

Now the actual case is coming where you are trying to test the case where under h naught mu is equal to mu naught and under H A mu is not equal to H naught which means that you are not do not know whether is on the right left hand side or the right hand side. So, now, the actual essence of the interval estimation will be making sense. So, if we go to this diagram. So, diagram just note the diagram we have tried to make; it as clear as possible, this is the for case when you are considering H naught is equal to mu naught H naught is case for mu is equal to mu naught against H A where mu, mu is a is not equal to mu naught. So, you can consider both on the sides and if they are at the both end sides you will basically reject H naught, if this is true.

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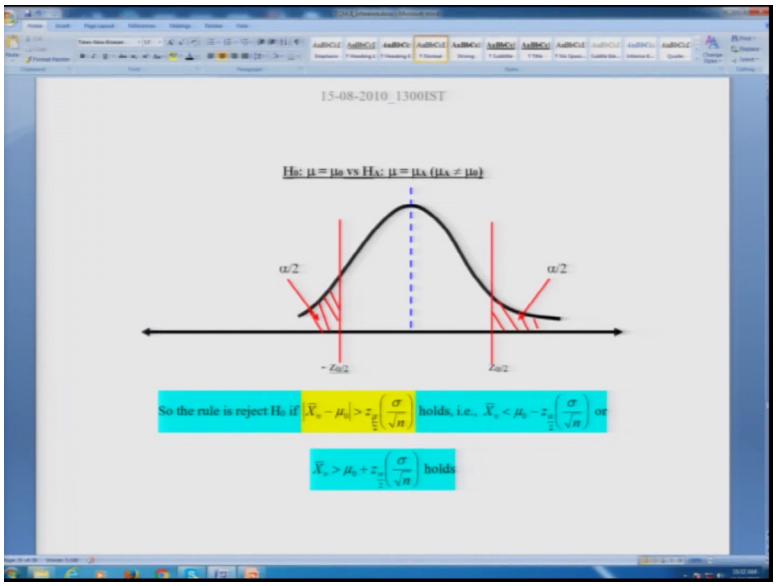

So, this being true, I will try to highlight with the different colour for the time being, now this is the mod now if you basically expand and find out the mod it is basically if it is on the left hand side or right hand side. So, left hand side means this one. So, this is where the highlighter is basically on the left hand side. So, again the same formula as for the left hand concept on the right hand concept which you just discussed, but only remember. Now z is suffix alpha by 2 because it equally proportioning the level of confidence in such a way the middle value is one minus alpha and the left value which is alpha is being equally distributed on the left hand side and the right hand side.

We will continue discussing such topics in related to the mean value when standard deviation is known then come to the case of the difference on the mean value when the standard deviations are known when the difference of the mean value when the standard deviations are unknown when we come to the ratios of the variances when the mean values are known ratios of the variances when the population means are unknown and continue that.

So, we will see that cover both the z distribution t distribution with the certain degrees of freedom pi square with the certain degrees of freedom f distribution with the certain degrees of freedom and then move on to the concepts of more of design of experiments and this would make sense that how we can basically understand and appreciate the concept which you are learning about hypothesis testing more in the design of

experiments frame work. So, with this I will end this lecture and wish you all a very happy day and stay well and.

Thank you very much.