Total Quality Management - II
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Lecture - 35
Factorials with Mixed Levels

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you and this is the TQM II lecture series under the NPTEL MOOC. And this is the 35th lecture and I am Raghunandan Sengupta from IME department IIT Kanpur.

So, if you remember we are discussing about the factorial models and their concepts and how the folds could be done, how we can divide into blocks, blocks would basically consider the combination of the variables and their effects and how the total combinations could also be considered and obviously, the end of the day main emphasis was, have the variables denoted in the leftmost column, then you will have the total sum of the squares of the errors, errors for each variable then you will have basically of the degrees of freedom and then you have the f test; f test based on that you have the last column this level of significance. This is basically the overall view point of ANOVA model.

So, each time you will basically test in hypothesis; h naught. Say for example, being such that beta 1 is equal to beta 2 or h alternative being beta 1 not equal to beta 2 or it can be say for example, the standard deviations are same, sigma 1 square is equal to sigma 2 square or the standard deviation being different or the error being equal to 0, error being not equal to 0 basically you will have two alternative or opposing theories or concepts you will try to test them using the analysis of variance models tables. And for that there are different methodologies based on which you trying to find out the minimum number of factors, minimum number of variables, minimum number of attributes which gives the maximum amount of relationship.

So, we will slowly see that how we are considered that we have considered the factorial factor fractional model for of two variables of two levels. Levels means a and b being the variables, and a and b can view at two levels plus minus. Then it could be 3 to the power k or 3 to the power k minus p depending on number combinations you do, and it would be of the third or a fourth or a fifth or a the design set up which we have. So, these were the general discussion we are trying to look at these type of problems into from different angles we will continue the discussions so on.

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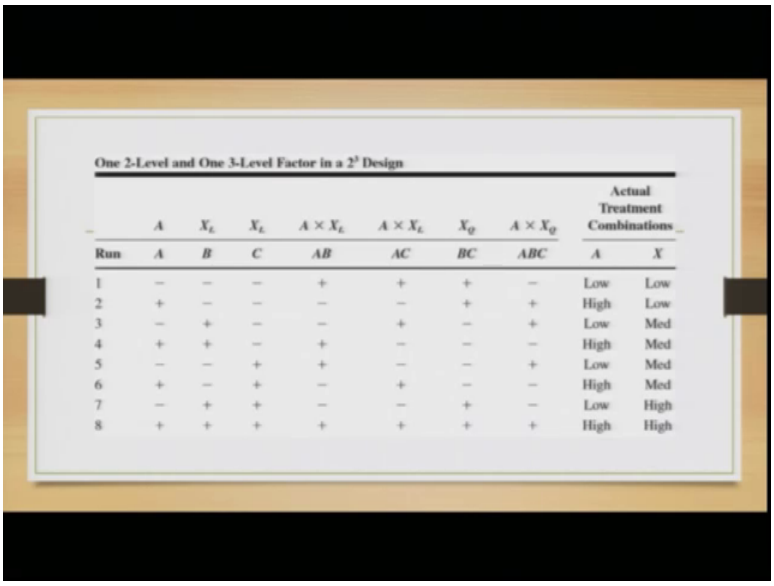

One 2-Level and One 3-Level Factor in a 2^3 Design

	A	X_L	X_L	$A \times X_L$	$A \times X_L$	X_Q	$A \times X_Q$	Actual Treatment Combinations	
Run	A	B	C	AB	AC	BC	ABC	A	X
1	−	−	−	+	+	+	−	Low	Low
2	+	−	−	−	−	+	+	High	Low
3	−	+	−	−	+	−	+	Low	Med
4	+	+	−	+	−	−	−	High	Med
5	−	−	+	+	−	−	+	Low	Med
6	+	−	+	−	+	−	−	High	Med
7	−	+	+	−	−	+	−	Low	High
8	+	+	+	+	+	+	+	High	High

So, we have basically the second level or the 2-level and one of the 3-level factors into 2 to the power 3 design. So, you so, if we if I consider, so you will basically have the runs; runs would also be given, because before you prepare the ANOVA table the runs would be there. So, the runs are given in the leftmost column. Then you the variables A, B, C so there are a three variables and considered they can be at two levels of importance. A can be plus minus, B can be plus minus, C can be plus minus.

So, the total combination would be 2 into 2 into 2 that is 2 to the power 3 what we are considering. So, if you see the top most part of this table; it is 2 to the power 3. So, these 3 concept will be coming up and obviously, it can be 3 to the power 3, 3 to the power 4 or 4 to the power 3 whatever depending on the levels. So, if you I am not digressing just mentioning that if you have basically 3 variables at three levels. So, it will be 3 to the power 3 such that A, B, C; A can be at 0 1 two-levels; B can be at 0 1 two-levels; C can be at 0 1 two-levels. Say for example, if I have A, B, C, D each being at five levels. So, it would be 4 to the power 5.

Now, continuing the discussions you have the combinations of AB, AC and B C and finally, combination of all the factors take taken together. So, if we consider the first row, so the effects of A, B, C are minus and we are considering AB, AC and BC at different levels of significance. So, hence the actual treatment combinations would be com if I find to try to find out, it would be A at low high; I am just reading the second most second last column. A can be low, then high low high and the corresponding values of x which is the variable based on which we are trying to study and we low medium. So, obviously what now we are trying to bring the concept of low medium, high also into the picture by this different categories categorization.

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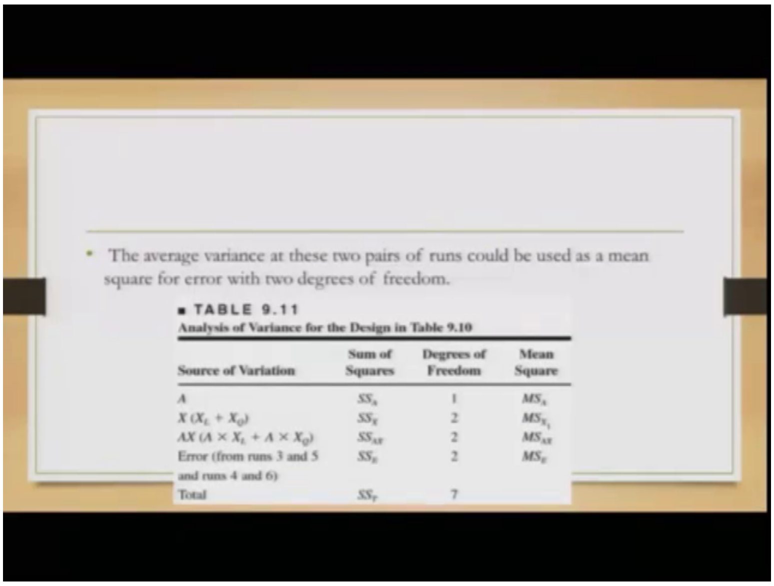

So, the average variance at these two pairs of runs could be used as a mean square of the errors. So obviously, if you remember I did mention in some of the classes in details, in some of the classes though fleetingly that our main aim was basically to minimize the variance; that the square of the variance we take the square of the difference of the actual value and the predicted value which is the error or the actual value and the average value which is again the average error, square in them up, sum them up and that is basically the sum of the square of the errors.

Now, if I read table 9.11 unless the variance for design done. So, you have the source of variations for A or combinations of X A X and so on and so forth, which is the sources and I want to find out the sum of the squares. So, the sum of the squares are S S suffix A; that is from A, S S suffix X which is basically from X the combinations of A B C taken in different directions. Then you have A into X which is and the last one being the error; error being e. The degrees of freedom is given by I am repeat I am reading it 1, 2, 2, 4 and obviously, the sum of all the degrees of freedom should be 7 and similarly if I check at the last most lat most last most row in this ANOVA table, you have S S suffix t which is the sum of the squares of the total errors which would be the sum due to all the variables plus the error term.

And then if I divide the corresponding values of the second column with their corresponding value of the third column which is the sum of the squares and I am does the particular value divide by degrees of freedom in the second last column. So now, you have the mean square errors, mean square errors would actually give you the values of f; f statistic based on that you can come and the level of significance.

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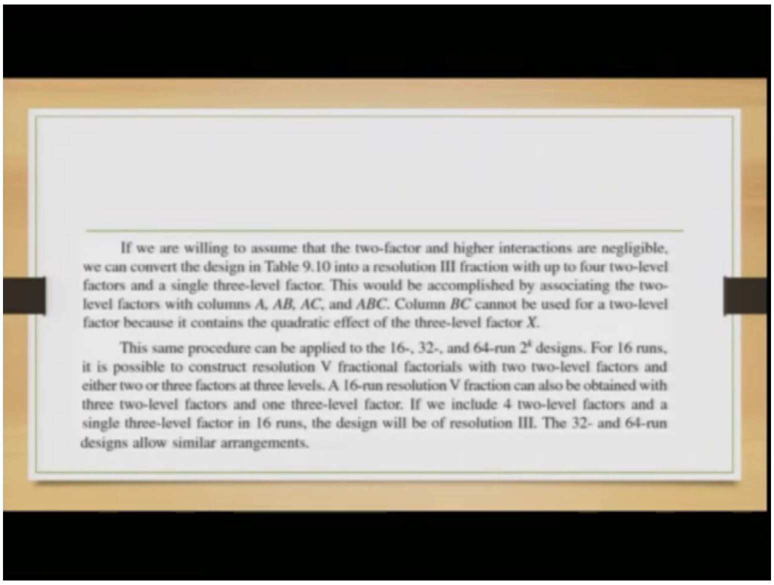

If you are willing to assume that the two factor and higher interactions and negligible, we can convert the design in 9.10 into resolution of three fractions, with up to four-level factors and a single three-level factor. This would be accomplished by as associating the two-level factors with two columns which are basically A, AB, AC and ABC. The column BC cannot be used for a two-level factor because it contains the quadratic effect of the three-level factors which is coming out from X. This same procedure can be applied to the 16 combination, 32 combination and 64 combination runs for the 2 to the power k design. So obviously, there are k number of variables each at two level. So, hence we are k to the power 2 to the power k design concepts.

For the 16th runs it is possible to construct a resolution of 5 fractional factorials with two levels of factors. Similarly your 16 run resolution of fifth fraction can be obtained with three of the 2-level factors and one at the 3-level factor combination. So, if we combine 4 two-level factors and a single three-level factors in 16 runs, the design could be a resolution 3 and we can change the resolution of 3 to 4; 4 to 3 and corresponding be such; such that the overall implication or overall information coming out from the study would be maximum.

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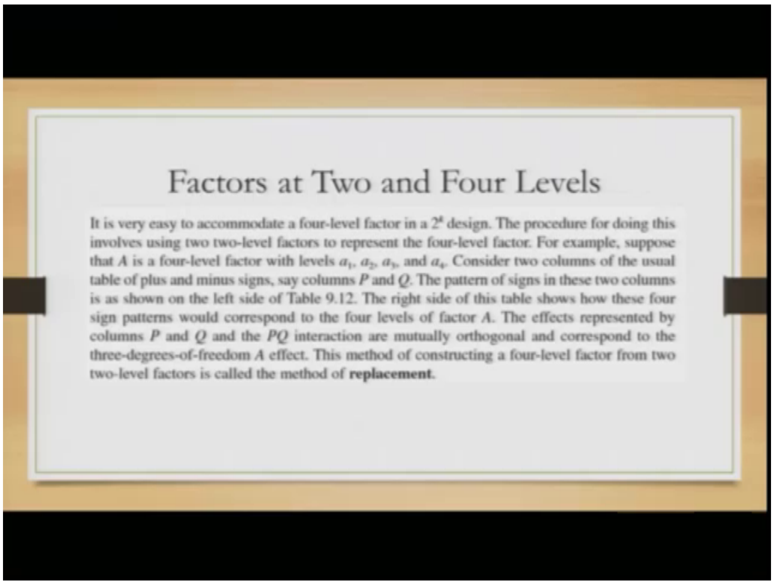

Now we consider factors at two and four-levels. It is very easy to accommodate two four-level fact factor in a 2 to the power k design. The procedure for doing this involves using a two-level factors to present the four-level factor module. So, what you are doing trying to do is that, you in when you are trying to basically change from us 2 to a 4 or 4 to a 2 or 2 to 3, 3 to 5 or come let come down; it is like this your overall concept. You are trying to basically divide your total errors. So, errors would basically consists of two parts; one is from the error white noise which you cannot predict, and another would be from the variability or the from the various or from the variables; variables or attributes whatever it is.

Now, my main concern is to basically find out the errors to the minimum extent possible using the minimum number of variables. So, if I am able to use only two factors or two variables or two attributes and basically give me give me or give the experiment on the maximum amount of information nothing like that.

But it also maybe possible that in each of the combinations of the variables we would like to basically divide into subclasses such that the category of subdivision basically gives us the maximum set of information's. So, in that case we will try to basically convert from a two factor to a three factor; three fact to a four factor or the you must be such that our study gives us the maximum amount of information; given us the maximum efficiency.

So, if you remember if at all that we want to find out the efficiency and integration we basically have the r square, which is the sum of the squares based on which we are proceeding. So, we will try to basically predict the r square which is 70 72 percent or 80 percent or 90 percent whatever it is, it gives us to what level that we are able to predict. It can be the 72 percent or 80 percent or 0.8 or 90 percent 0.9, such that we are confident that r answered depending on the number of variables we have consider and what are the variables; you are able to predict either 72 percent or 80 percent or 90 percent of the times in a prediction that is to what is the level of accuracy.

So, is very easy to accommodate a four-level factoring in a 2 to the power k design. The preceding for doing this involves using the two two-level factor. So, there are groups of that the and combine them and try to find the four-level factor design. For example, suppose that that A is a four-level factor and you have basically levels at the at given by the values of A A; 1 small a1 suffix 1, small a 2 suffix 2, small a 3 suffix 3, small a a suffix 4. So, it is a 1, a 2, a 3, a 4 with all the numbers 1, 2, 3, 4 coming in the suffix.

So, consider two columns of the usual table of plus and minus signs, say they are columns P and Q. The patterns of signs in these two columns is shown to the left of the table 9.12. The right side of this table shows how these four sign patterns would corresponding to the four-level of the factors A. The effects represented by column P and Q and PQ interactions are mutually orthogonal. So, what you are trying to do is that? Trying to basically break them into orthogonal direction such that, whatever effect is coming from A should only be assigned to A. If B is orthogonal to A, then whatever effect we are assigning to B should remain there only.

So, if we are able to break them into factors, factors are orthogonal such that are prediction would be of the atmost importance considering that we can assign a level of significance on A or B or C or D whatever they are depending on the fact that they are orthogonal to each other. The effects represented by columns P, Q and PQ interactions are mutually orthogonal and corresponding to the three-degrees-of-freedom of the effect.

This method of constructing a four-level factor from 2-level two; two-level factors is called the method replacement. So, you are trying to basically replace the form four of one to two of two type so that means, you are trying to break them into more subcategories in order to do a better analysis of the prediction or analysis of the of the overall assignment of the errors.

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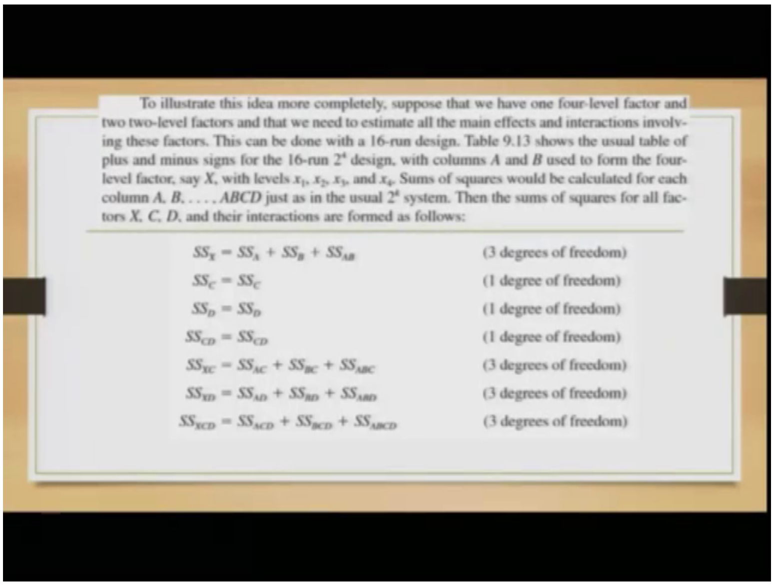

To illustrate this idea more completely suppose that we have a one four-level factor and two two-level factors, and that we need to estimate all the main effects and interaction involving these two factors. So, this can be done with 16 run design. So, consider the table 9.13 shows the usual table of plus and minus signs for the 16 run 2 to the power 4 design with columns A and B used to form the four-level combination which we have. Sum of the squares would be calculated for each of the column like A, B so on and so forth till AB C D; just as usual in a 2 to the power k system, then the sum of the squares of all the factors which we have would basically performed accordingly; like if we have sum of squares of X.

So, X would basically be for A, B and AB combined. So, you will have three-levels; three three I would not use the word level, three different variables. If it is C we are only assigning to C; if it is D we are only assigning to D, and if it is XC obviously, it would be A C, B C and ABC. Similarly for X t it would be AD, BD and ABD. Similarly for XCD; it would be corresponding to that based on that you find out the errors. You have the degrees of freedom, find out the root means average root mean square, mean square values, find ut the f statistic; pass your judgement depending on the level of significance.

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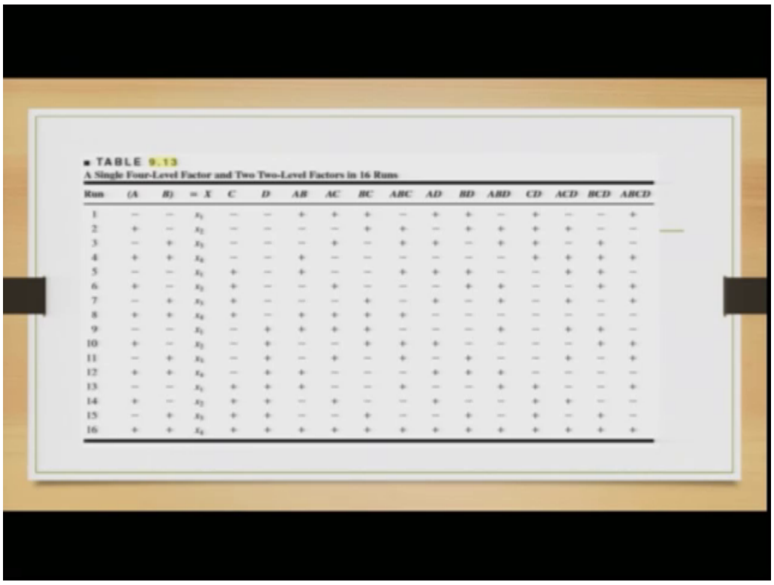

■ TABLE 9.13

A Single Four-Level Factor and Two Two-Level Factors in 16 Runs

Run	(A	B)	= X	C	D	AB	AC	BC	ABC	AD	BD	ABD	CD	ACD	BCD	ABCD
1	−	−	x_1	−	−	+	+	+	−	+	+	−	+	−	−	+
2	+	−	x_2	−	−	−	−	+	+	−	+	+	+	+	−	−
3	−	+	x_3	−	−	−	+	−	+	+	−	+	+	−	+	−
4	+	+	x_4	−	−	+	−	−	−	−	−	−	+	+	+	+
5	−	−	x_1	+	−	+	−	−	+	+	+	−	−	+	+	−
6	+	−	x_2	+	−	−	+	−	−	−	+	+	−	−	+	+
7	−	+	x_3	+	−	−	−	+	−	+	−	+	−	+	−	+
8	+	+	x_4	+	−	+	+	+	+	−	−	−	−	−	−	−
9	−	−	x_1	−	+	+	+	+	−	−	−	+	−	+	+	−
10	+	−	x_2	−	+	−	−	+	+	+	−	−	−	−	+	+
11	−	+	x_3	−	+	−	+	−	+	−	+	−	−	+	−	+
12	+	+	x_4	−	+	+	−	−	−	+	+	+	−	−	−	−
13	−	−	x_1	+	+	+	−	−	+	−	−	+	+	−	−	+
14	+	−	x_2	+	+	−	+	−	−	+	−	−	+	+	−	−
15	−	+	x_3	+	+	−	−	+	−	−	+	−	+	−	+	−
16	+	+	x_4	+	+	+	+	+	+	+	+	+	+	+	+	+

So single factors, four-level factor and two-level factors in 16 runs are given; so you have basically the runs are given the leftmost column and corresponding to that you have AB as the variables, then you find out X, then you have basically consider in the full next level and CD and then combine them in such a way that the last effect which is there is coming from the combination of A, B, C, D and you pass a judgement accordingly.

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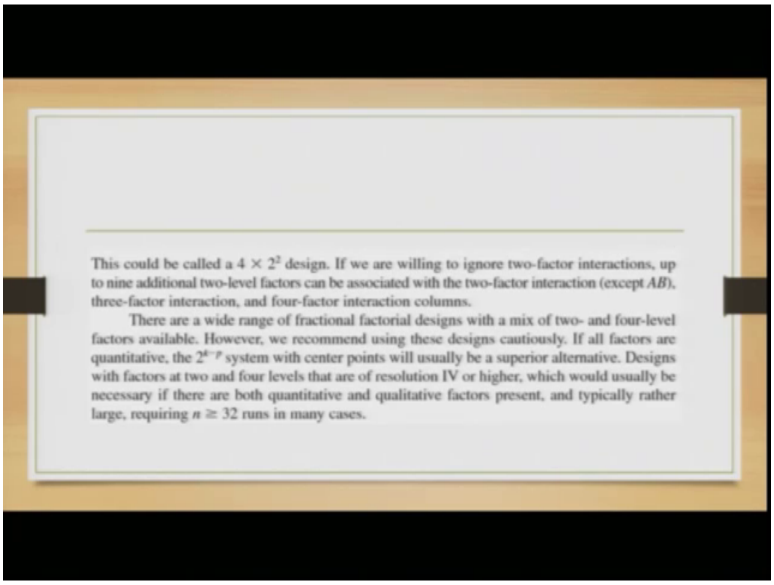

So, thus this could be a 4 into 2 2 design. So, basically you have a 2 2 design and basically have such four different combinations or applications of them. So, if you are willing to ignore two-factor introductions up to nine additional 2-level factors can be associated with the two-factor interaction or three phase factor interaction of the four-factor interaction and they can form can be found out accordingly. There are wide range of fractional factorial designs with the mix of two and four-levels factors available.

However, we recommend using the design cautiously. See if all the factors are quantitative that is and the system is 2 to the power k minus p depending on the levels of importance you want to assign was to or 2 to the power k minus p system with centre points, it will be easy usually be a superior alternative. Design in factor at two and four-levels that are of resolution of fourth or higher order, which would usually be necessary with there are both quantitative as well as qualitative factor. So, we will def define them and design them depending on whether there quantitative or qualitative. Factors present and typically rather large requiring about n greater than 32 such many cases which you have.

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So, now will take go into the realm of trying to find out the fitting the best regression model.

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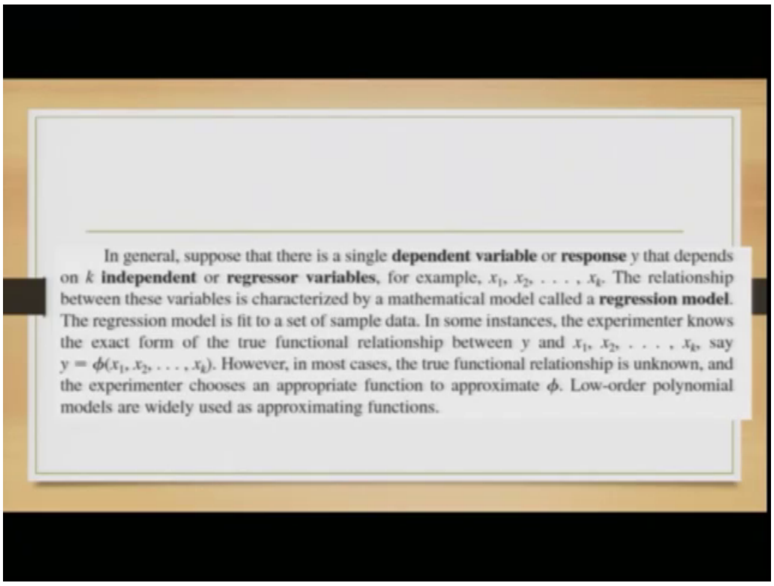

So, in general suppose there are single dependent variables or responses y I have mention about that, but still repeated. And there are k independent variables which is x 1, x 2, x 3 till x k. So, independent factor means that if you bring the into the picture the concept of a matrix. So, this matrix x, which is from x which consist of the variable x 1 to x k and each variable being of n number of durations or timeframes. So obviously, it will be depending on your nomenclature would be n cross k or k cross 1 matrix. So obviously, we should remember the rank of the matrix is k, which basically is the number of independence such variable which we have 0.1, point number 2 in general multiple any regression we consider all the axis are normally distributed. You also consider they are orthogonal or they independent of each other, if they are not independent of each other the rank would not be k; it would be less than k which is not possible. Will also consider the errors are independent of each other, will also consider the covariance existing between the errors and axis values are 0.

The point which I just mentioned before that the errors are independent of each other which would also mean the covariance eh covariance existing between the errors is also 0. Will consider the errors of a fixed variance, we will also considered the errors have a mean value of 0 and obviously will we will have the expected value of x 1 to x k would be which would be different values which will be mu 1.

So, that is suffix 1, mu suffix 2, mu suffix 3 till mu suffix k. Similarly the variances of x 1 to x k would be again independent each other, they would be sigma suffix 1 square, sigma suffix 2 square, sigma suffix 3 square so and so forth till the last value sigma suffix k whole square and then obviously, y is a depend y which is dependent variable is dependent on this x1 to x k, being multiplied by the respected values of beta 1 to beta k which the coefficients and obviously, they would be error term which is epsilon.

So, with this beta 1 to beta k are the values, which actually means what is the partial derivative the rate of change of y with respect to x 1, what beta 2 would basically mean, what is the rate of change of y with respect to x 2 and so on and so forth. So, this I had repeated. So, hence add a I am again repeating it without showing or going to the small assumption as statements.

So, in general suppose there is a single dependent variable response y that depends on k independent on regressor variables for example, there are x 1 to x k. The relationship between these variables is characterized by mathematical model called the multiple linear regression or regression model. The regression model is fit to a set sample data points which are n in number. In some instant the experimental knows that the exact form the two functional relationship between y, x 1, x 2 till x k is given, and say there is given by y is equal to some function of x 1 to x k.

However, in most cases the two functional relationship is unknown and the experiment chooses an appropriate function to approximate the function value phi. So, low order polynomial models are utilized to find out the best fit. So, now we are slowly expanding our discussion away from the realm of multiple linear regression.

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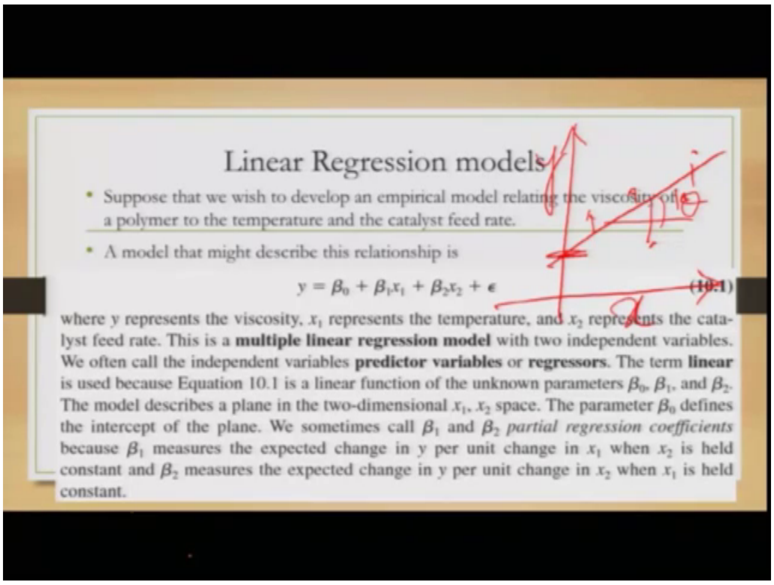

So now, if you come to the linear regression model, suppose that we wish to develop an empirical model relating the viscosity of polymer to the temperature on the catalyst feed rate which is being given. Consider the model is given like this y is equal to beta naught; beta naught is basically the intercept.

So, if you remember in class 10 or 11, we consider y is equal to m x plus c. So, thisc is basically the value of beta naught which we have this is the intercept at where it is cutting the x axis. Now here again, if it is y is equal beta naught plus beta 1 into x1 plus error without any x 2 term. It means that you were able to draw in a Cartesian coordinate or two dimensions. So, along my left arm the y axis, along my right arm you have basically the x axis. So, I plot the values on this Cartesian coordinate and the value with the line, best fit line cuts the y axis basically the value of C or beta naught here.

And this beta; this beta 1 would basically be the rate of change of that function or the tan of this best fit line with respect to which is basically the divide of d y of d x x 1 I would should not be using the what x and then it will give you the solution. So, if I have the best fit here let me draw it. So, these are the points, this is the best fit line, these are the errors, this is the value of beta naught, this is the theta degree, tan of theta is equal to beta 1, this is the y variables which of the x variables, y is dependent, x is independent ok.

Now the question is that why did I mention that consider that you go into three dimension one. Again y you are measuring along my left arm, which is basically going vertical up towards the roof of this room where I am basically giving the lecture. The x 1 values are basically on my right arm going from this point on to the my right and consider nomenclaturally x 2 is there which is basically from the same point and is going towards you. So, that basically these x 1 and x 2 is the floor in the room and the y is basically going to the roof.

Now, if I plot y is equal to beta naught plus beta 1 x 1 plus beta 2 x 2 without the error; error I am not considering. And if it is a linear line, then it will basically be planes in this room. Now if I am looking on the plane from the direction of x 2 or am I looking at the plane of the direction x 1, it will mean that one of them is fixed. So, if you have basically project it onto the wall any wall which I have, then we will have basically respectively the equation corresponding to y is equal to depending on x 1 or y depending on x 2, considering that we are basically trying to fix either x 2 or x 1 depending on which direction we are looking at.

So, now if I have the equation has y is equal beta naught beta plus beta 1 x 1 plus beta 2 x 2 plus epsilon, where y represent the viscosity, x 1 represent the temperature, x 2 represent the catalyst feed, these are the 2 variables then a multiple linear regression with two depend independent variables is fitted. We often called the independent variables or predict variables are regresses. The term linear is used because the equation which I have written is a linear function of the unknown parameter is beta naught, beta 1, beta 2.

The model describes the plane in the two-dimensional as I just mention in the x1 x2 space, the parameter beta naught defines the intercept of the pain plane on to the y axis. We sometimes call beta 1 and beta 2 as the partial degree regression coefficient because beta 1 means the expected change of the rate of change of y per unit change in x 1, while x 2 is basically being held constant, while beta 2 measure the expected change in y per unit time with respect to x 2 given x 1 is constant. Exactly what I mention in the predictor will be it is being mentioned here in the in the slides.

In general that equation can be expanded, now, we have a multiple linear regression with k independent variables y is equal to beta 1 plus beta 1 x 1 plus beta 2 x 2 plus dot dot till beta k x k plus epsilon which is there.

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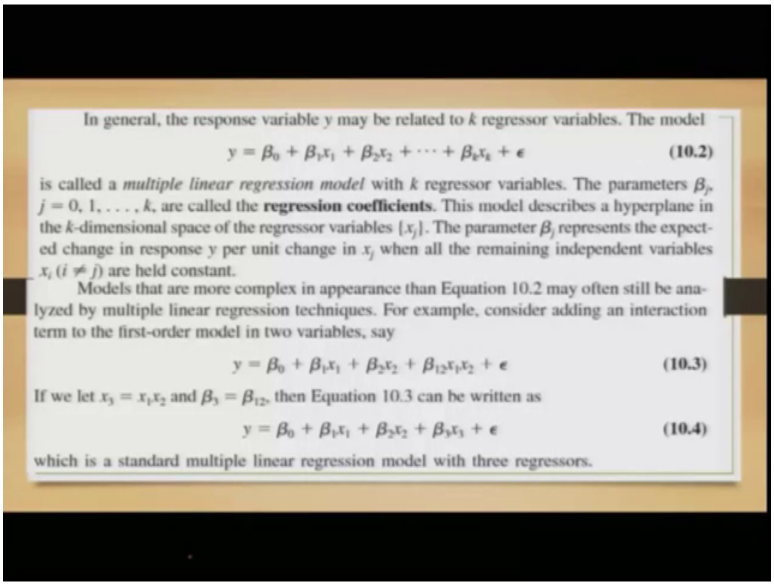

In general, the response variable y may be related to *k* regressor variables. The model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon \quad \textbf{(10.2)}$$

is called a *multiple linear regression model* with *k* regressor variables. The parameters β_j, $j = 0, 1, \ldots, k$, are called the **regression coefficients**. This model describes a hyperplane in the *k*-dimensional space of the regressor variables $\{x_j\}$. The parameter β_j represents the expected change in response y per unit change in x_j when all the remaining independent variables x_i $(i \neq j)$ are held constant.

Models that are more complex in appearance than Equation 10.2 may often still be analyzed by multiple linear regression techniques. For example, consider adding an interaction term to the first-order model in two variables, say

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon \quad \textbf{(10.3)}$$

If we let $x_3 = x_1 x_2$ and $\beta_3 = \beta_{12}$, then Equation 10.3 can be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon \quad \textbf{(10.4)}$$

which is a standard multiple linear regression model with three regressors.

This is the multiple linear regression model with k regression variables. The parameters beta j, which j is equal to 0, 1, 2, 3, 4, till k are call the regression coefficients; beta naught is the value at which it is no independent variable and it is basically cuts the y axis or touches the y axis or the planes which is there the hyper planes in the n dimensional where it touches the y axis. This module disrupts the hyper plane. In the k dimensional space of the regressor variables x j; the parameter b j beta j represents the expected change in response of y per unit change in x j, when all the remaining variables are considered to be independent or they do not change. So, it means that beta 1 is the rate of change of y with respect to x 1 keeping x 2 to x k as constant.

If I consider beta j, it means basically the rate of change of y with respect to x j and keeping x 1 to x j minus 1, and x j plus 2 till x k as constant. So that means, the variables corresponding to the rate of change would only be considered that that variable is changing others are being kept fixed. More models are more complex can be build up and in a appearance say for example, we can consider the model as y is equal to beta naught plus beta 1 x 1 plus beta 2 x 2 plus beta, 1 k x 1 into x 2 plus can be another term can be beta 2 1, x 2 into x 1, but will consider beta 1 2 and beta 2 1 has to be equal because when we try to differentiate the function of delta f with respective x 1, and then I can differentiate that differentiation with respect x 2 it will give me the same value, if we differentiated first we x 2 and then with respect to x 1, which means del 2 f del x 1 del x 2 is exactly equal to del 2 f del x 2 an x 1 that means, the differentiations are symmetric.

So, and the model can be y is equal to beta naught plus beta 1 x 1 plus beta 2 x 2 plus beta 1 k x 1 x 2 epsilon. If we basically put x3 is equal to x 1 x 2 and beta 3 is a x 1 x 2; then the equation can be written as y is equal to beta naught plus beta 1 x 1 plus beta 2 x 2 plus beta 3 x 3 plus epsilon. So, which is a standard multiple linear regression model with three regressors; x 1 x 2 and x 3.

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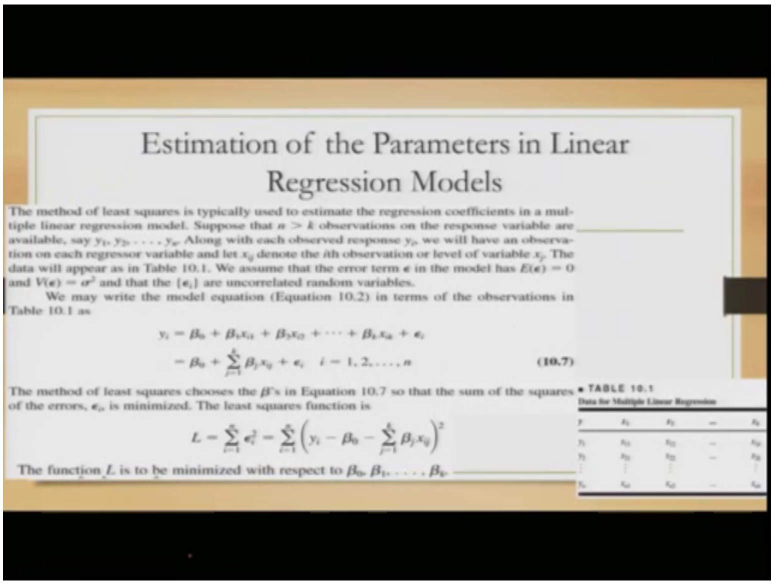

Now, you want to basically estimate the parameters of linear regression model. So, the method of least square is typically used. What is the method of least square, basically we try to find out the errors, square them up, sum them up and try to basically minimize the error with respect of something what we are going to do that? We are going to come that within few minutes. So, the method of least square is typically use to estimate the regression coefficients in a multiple linear regression model. Suppose that n is greater than greater than k; k is the number of variables and n is basically number of readings. So, n is greater than k observations on the response variables are available. Say they are double one y 1, y 2, y 3, till y n and similarly the values of x 1 to x k square given from the corresponding first value to the nth value. So, if I consider x 1; it will be x 11, x 12, x

13, x 14, till x 1 k where the first suffix basically denotes the variable and the second suffix 1, 2, 3, 4, till n or not to scale; till kn would basically mean the reading number.

Similarly, if I have x 21, x 22, x 23, till dot dot till x 2 n; it will basically mean the second variable values are from first to the nth one. Similarly the last variable set vector would be xk 1, xk 2, till xkn. So, the xk1 is basically the first value of the kth variable; xk 2 is the second value of the kth variable, similarly xkn it is the nth value of the kth variable.

Along with each observed response y i we have an observations on each regression variables and let they X ij denote the ith observation at or level or a level of variable x j. The data will would basically given as given in table 10.1 we assume that the error terms epsilon in the model have such that the expected value 0 if you remember that I had mentioned that few minutes back in the last two last slide. And the variance if you also remember I mention that is fixed, at and that and also mention that the covariance existing between two errors is 0 and obviously, the error value is fix which is given here as sigma square. And then the epsilons are uncorrelated random variables, they are uncorrelated or not related to each other. We may write the model as given; that means, y i; y i is equal to beta naught plus beta 1 x i 1 plus beta 2 xi 2 plus dot dot beta kxik plus epsilon. So, you basically put it in the form as this.

Now, what is important is this. I will explain that later. We basically find out the squares of the errors that is y 1 minus y 1 hat, y 2 minus y2 hat square them up add them up. So, when you are adding them up and basically tried for minimize it, you will find out there are variables which are unknown. So what are they, you will ask? So, they are basically beta naught, beta 1, beta 2, till beta k you will basically differentiate partial differential them with respect to beta naught, beta 1, beta 2, till beta k has such k equations, equate them to 0 and find out beta hat, beta 1 hat, beta 2 hat, till beta k hat. So, once you find out the, this estimates you will again use them for the nth plus 1 reading and do the calculations accordingly.

So, with this I will end the 34th lecture, continuing more discussion some of the regression model in the 36th and the corresponding lecture.

Thank you and have a nice day.