Total Quality Management - II
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Lecture - 34
Fractional Replication of the 3^k Factorial Design

A very good morning, good afternoon, good evening all my students and welcome back to this TQM II course under the NPTEL, MOOC series, and this is the 34th lecture and I am Raghunandan Sengupta from IME department, IIT Kharagpur.

So, if you remember we are discussing different designs fractional factorial models depending on the number of variables and attributes which are there, number of levels of significance they are; that means, you are trying to find out 3 to the power 3, 3 to the power 4 or 4 to the power 5. Technically it would be n to the power k and technically and n would be the number of levels of differences which you have for each factors and there are k number factors and there are obviously, different concepts or trying to by and find out the blocks according to which the design can be made, then the folding concept, then trying to basically subsume the relationship with higher orders. So, we can find out the maximum amount of variation.

And as you remember the main emphasis would be on the table which consists of the different variables there are some of the squares of errors, then the degrees of freedom, the F statistic the P value based on which you will comment.

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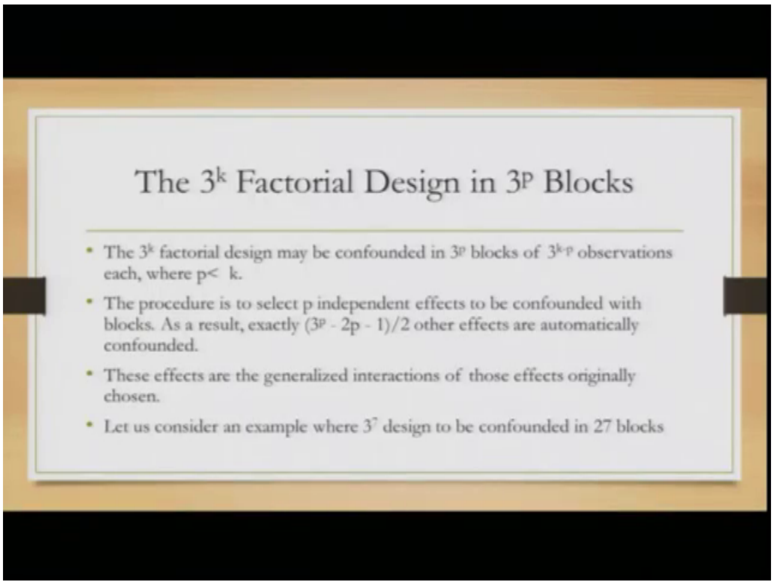

The 3 k fact factorial design of the 3 p blocks, so when here p is less than k and you will do your block design depending on the variable. So, accordingly such that you have the minimum number of blocks to do the maximum amount of such predictions or trying to find out the ANOVA output to the maximum possible level.

The 3 k factorial design may be confounded in a 3 p blocks of 3 k minus p observation. So, you want to basically partial partitioning in 3 to the power p blocks and the partitions of the observations would be done accordingly. And if you remember that for the last example in the in the last class which was in the 33rd class we did mentioned that how the number of observations in each block what exactly 9, and you have 9 blocks, so obviously, those things should be remembered accordingly. So, 9 was a number specific to the example, but obviously, we will design it such that the equal number of variables are there. The 3 to the power k factorial design may be confounded in 3 p blocks of 3 k minus p observations each where p is less than k.

The procedure is to select be independently effects to be confounded with the blocks and find out their effects accordingly, as a result so technically you will basically have effects which are automatically come to unfounded would be given by 3 to the power p minus 2 p. So, 3 to the power p blocks are there, and you are trying to and minus 1 would be basically for the loss of degrees of freedom and the total combination which you are doing would be 3 to the power p minus 2 p minus 1 and divided by 2. So, that would be the total other effects which are automatically confounded. These effects are the generalized interaction of those effects originally chosen.

So, let us consider an example with 3 7 designs to be confounded in 27 blocks or so, 27 would be 3 to the power 3 into 3 into 3 9, 9 into 3 27, so obviously, total number of variables is 7 and blocks would be basically designed 3 to the power 3.

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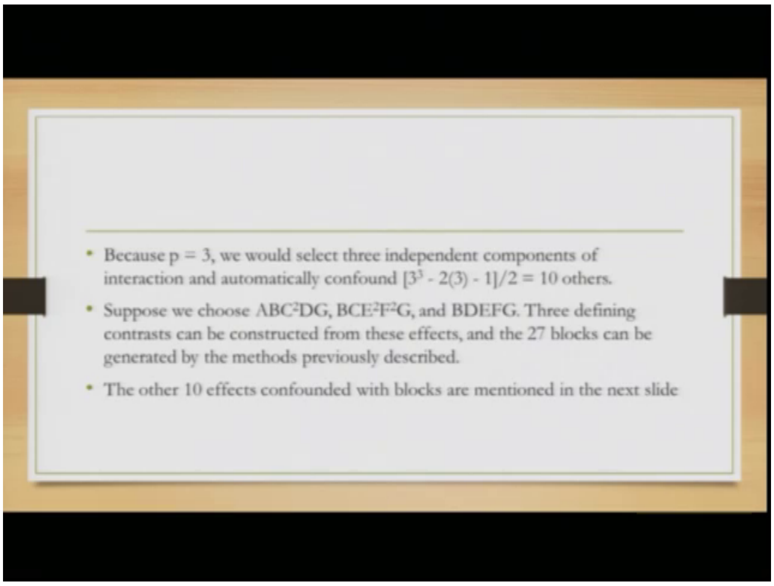

Because p, so obviously, k is 7 p is 3 because p is 3 we should select 3 independent components of the interaction, now, I have to automatically confound it. So, the others one which would not which would confounded would be 3 to the power 3 minus the same formula 2 p. So, 2 into p is basically 2 into 3 minus 1 divided by 2 would we give a 10. So, the total number of 10 confounded automatically and suppose consider hypothetically we choose them as the blocks as ABC squared DG or BCE square F square G and so on and so forth.

So, 3 defining contrast can be constructed from this effects and the 27 blocks can be generated by the matrix we will be described and we can do the partitioning likewise. The other 10 effects confounded with blocks and mentioned in the next slide and we will see that how can be done for this example where there are 7 variables and number of such blocks would be designed is 27. So, p is basically 3.

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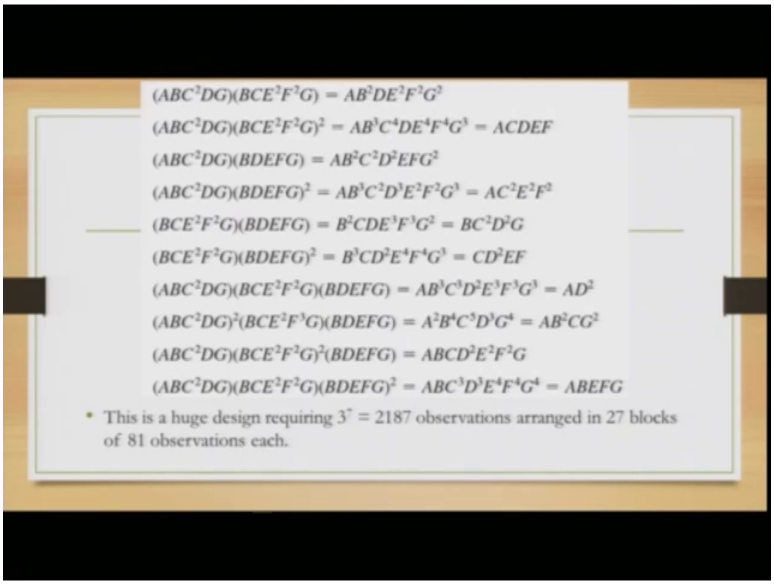

So, if we consider the blocking, so you would basically have the blockings as given as I will just mention one of them is A B C squared into D G.

So obviously, the blocking had been done with the effects of few of them would be considered at the higher level or lower level depending on the levels at which they are interacting with each other. So, it is A B C square D G into B C E square F square G and if you continue accordingly do the calculations it comes out to be A B square. So, A D are at order one B E F G are order 2 depending on the level of significance.

So, if we consider accordingly you would have I am just using the last one, so it will be A B E F G. So, this is a huge design problem which is about 3 to the power 7 which basically is 2187 observations, they should be arranged in 24 blocks of 81 observation each. So, basically you will have 81 observations in block 1 block 2 block 3 so on and so forth or such 27 blocks. Such the total number of observation which you have is basically 3 to the power 7 which is 2187. So, what you are doing? You are partitioning them into blocks such that the effect of each block considering the number of observations which are there which is 81 in number in each block would give you the maximum amount of effect which you would be discernible.

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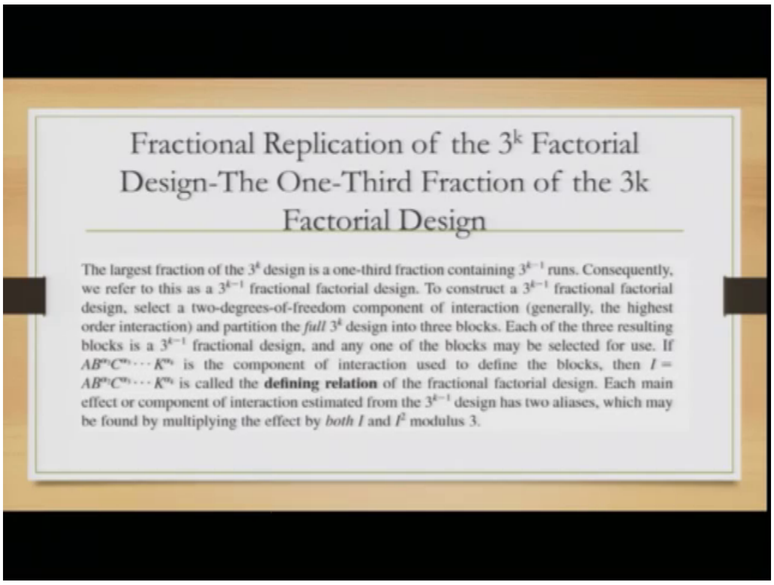

The fractional replication of the 3 k factorial design and the one-third fraction of the 3 k factorial design of the blocks the largest fraction of the 3 k design is a one-third fraction containing 3 to the power k minus 1, k minus 1 because there is a degree of freedom concept.

Consequently we refer to this as a 3 k to the power minus 1 fractional factorial design. To construct a 3 to the power k minus 1 fractional factorial design select 2 degrees of freedom component of interaction, so obviously, that would generally be the highest order interaction which are planning and partition the full 3 k design in 3 blocks. So, basic and obviously, you should remember that any block the number of observations should be the same. Each of these 3 resulting blocks is a 3 k to the power k minus 1 fractional design and any one block may be selected for use and correspondingly the calculations can be done.

Now, if the if you have say for example, the component of interactions as A B to the power alpha 2, so obviously, that a power would which is 2 3 4 depending on the level of significance technically that variable would have. Now, I am using the what level of significance in a very English form of sense which would basically give you the level of effect with this variables have on the total ANOVA output. So, if A B to the power alpha 2 which is C to the power alpha 3 obviously, A to the power is 1 where alpha 1 is 1 and the last one is k to the power alpha k is the component interaction and using the block we define using this we define the blocks.

So, and then obviously, the indicator such I will not use the word variable the indicator factor would be given as A, I will B to the power alpha 2 C to the power alpha 3 till k to the power alpha k, and it is called the defining relationship of the fractional factorial design. So, each main effect or component of interaction estimated from the 3 2 power k minus 1 design has to aliases which may be found by multiplying the effect of both i, i square in the mod 3 model. So obviously, why mod 3? Because you are trying to find out 3 to the power k factorial, if it is 5 to the power k factorial it would basically mod modulus on mod 5 and accordingly the calculations can be done.

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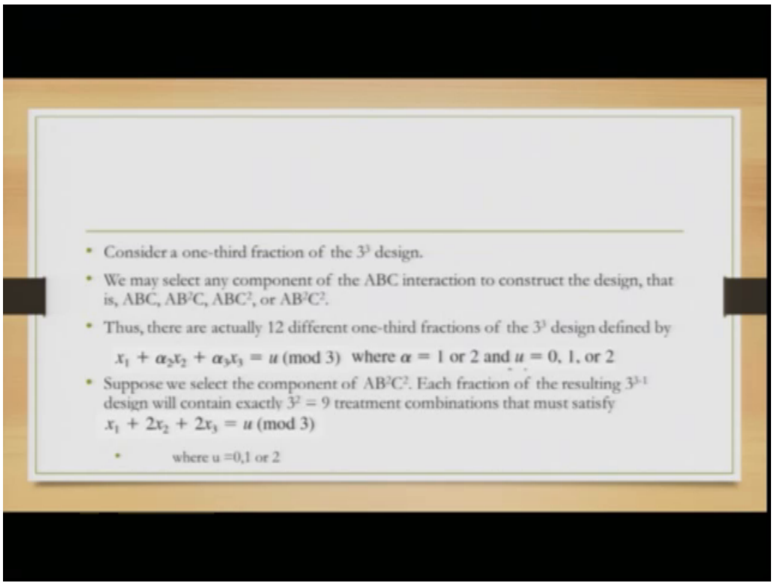

Consider a one-third fraction of the 3 to the power 3 design, we may select any component of A B C interaction to consider the design that would basically given by A B C, A B square C, A B C square or A B square C square based on that we can do the calculation. Thus they are actually 12 different one-third fractions of the 3 to the power 3 design defined by. So, you will basically have x 1, so obviously, a factor is alpha 1, but will considering as 1, then and then x 2 multiplied by his corresponding factor alpha 2 plus x 3 multiplied by corresponding factor alpha 3. So obviously, alpha 1 alpha 2 alpha 3 can take any values, here in this case alpha 1 is 1 which is a mod 3, where alpha 1 is equal alpha is equal to 1 or 2 and the u values or the mod modulus for 3 result of this combinations can be 0 1 2.

Suppose we select the component of A B square C square each fraction on the resulting 3 to the power 3 minus 1 design will contain 9 treatment combination, thus must satisfy the equation x 1 plus 2 x 2 plus 2 x 3 is equal to u and that was the mod 3 level.

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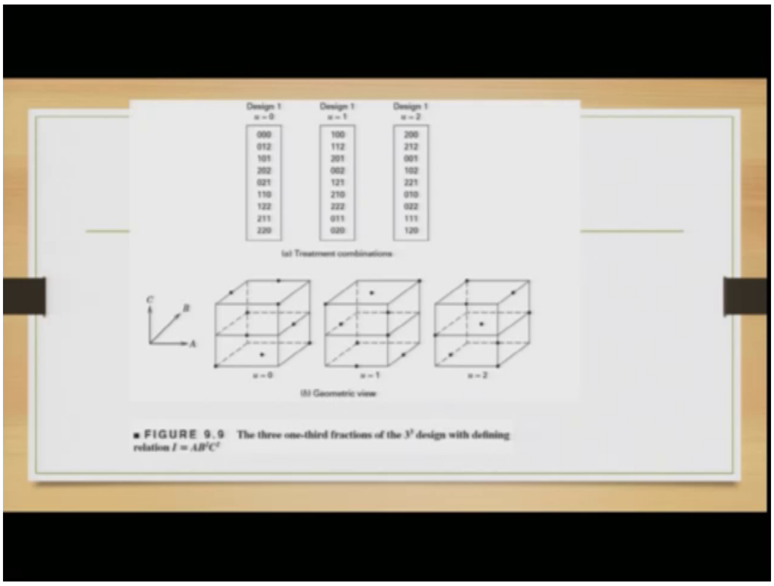

So, you had basically 3 blocks, each blocks if you remember that they have the number of some variables in each would be one would be 000 which is at the lowest level for A B C that is low low low, then you have 012, 101, 202, 021, 110, 122, 211 and 220. So, 220 obviously, I am only mentioning the last first on the last one. The 220 means A has the highest level; that means, there are 3 levels 012, A at the highest level, B at the highest level, C has the lowest level which is 0.

Similarly you have designed where u is equal to 1 for block 2. So, that starts from 100 and goes to 0 to 0. The block 3 which is for u 2 starts from 200 and ends at 120 and you can find on the levels. So, if you again take the same I am repeating it please forgive me if you are considering along my right arm; the axis a as shown in this slide and axis C, C means for C only axis A means for A is along the left arm vertical up and the arrow going towards you basically is for factor C for factor B which is the y axis.

So, if you basically find it and plot the points so, that will give you and this is important things to note. So, if for u is equal to 0, u is equal to 1 and u is equal to 2 you have the so called cuboids which will give you the corner points which is significant to that block only which means that that combination of those 3 blocks in the directions as mentioned by the corner points would give you the best effect or the considering that error would be minimized with the maximum possible extent.

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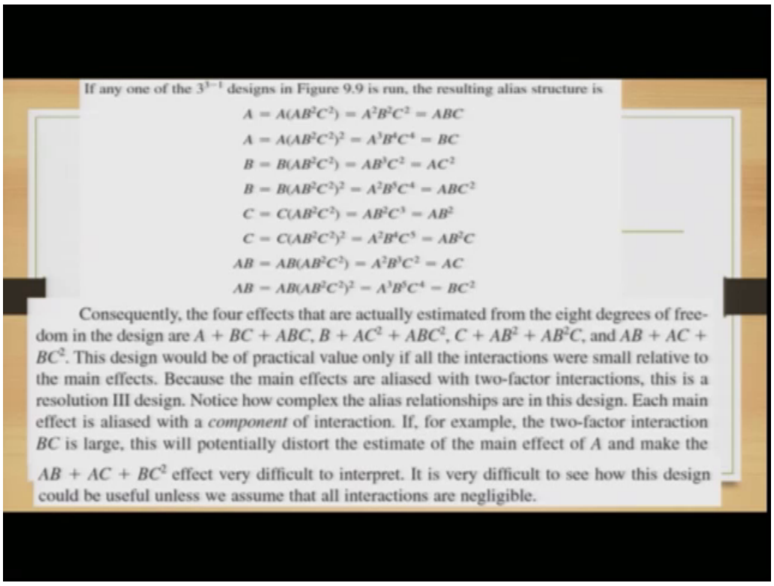

If any one of the 3^{3-1} designs in Figure 9.9 is run, the resulting alias structure is

$$A = A(AB^2C^2) = A^2B^2C^2 = ABC$$
$$A = A(AB^2C^2)^2 = A^3B^4C^4 = BC$$
$$B = B(AB^2C^2) = AB^3C^2 = AC^2$$
$$B = B(AB^2C^2)^2 = A^2B^5C^4 = ABC^2$$
$$C = C(AB^2C^2) = AB^2C^3 = AB^2$$
$$C = C(AB^2C^2)^2 = A^2B^4C^5 = AB^2C$$
$$AB = AB(AB^2C^2) = A^2B^3C^2 = AC$$
$$AB = AB(AB^2C^2)^2 = A^3B^5C^4 = BC^2$$

Consequently, the four effects that are actually estimated from the eight degrees of freedom in the design are $A + BC + ABC$, $B + AC^2 + ABC^2$, $C + AB^2 + AB^2C$, and $AB + AC + BC^2$. This design would be of practical value only if all the interactions were small relative to the main effects. Because the main effects are aliased with two-factor interactions, this is a resolution III design. Notice how complex the alias relationships are in this design. Each main effect is aliased with a *component* of interaction. If, for example, the two-factor interaction BC is large, this will potentially distort the estimate of the main effect of A and make the $AB + AC + BC^2$ effect very difficult to interpret. It is very difficult to see how this design could be useful unless we assume that all interactions are negligible.

If any one of the 3 to the power 3 minus 1 designs is run the resulting alias structure would be like this you will basically have, I am only reading the right hand column would be A B C, B C, A C square, A B C square, A B square, A B square C, A C and B C square, based on that you can find out the cork combinations of blocks accordingly.

Consequently the 4 effects that are actually estimated from the 8 degrees of freedoms on the design are A plus B plus A B C B plus A C square plus A B C squared, the third one is C plus A B square plus A B square C and the last one is A B plus A C plus B C square based on that you can do the calculations. This design would be a practical value only if the interactions who are very small relative to the main effects and we can find out the effects accordingly because the main effects are aliased with 2 factor interaction this is a resolution of third degree; that means, we are trying to find out the F x on to the third level such that it gives you the maximum contrast.

Notice how complex the alias structures are in this design each main effect is aliased with the component of interaction if for example, the 2 factor interaction B C is large they will potentially distort the a stand estimate the estimate of the main effect of A and make the effect corresponding to A B plus A C plus B C square if very difficult to interpret. So, you are basically placing all your bet on one and trying to minimize the effects of the combination are that which should not be because if you want to find out the effect it should be partitioned in such a way the effects comes out actually.

So, consider you have 100 rupees and you want to basically apportion them into 2 investment and get the maximum benefit. So the apportion can be done accordingly the returns can be found accordingly, but if say for example, the variances are too high and then obviously, you will face a risk or a loss. So, you want to basically minimize that corresponding to the fact that the a portion of your effects are such that you get the best output by considering the ANOVA model whatever you are trying to predict.

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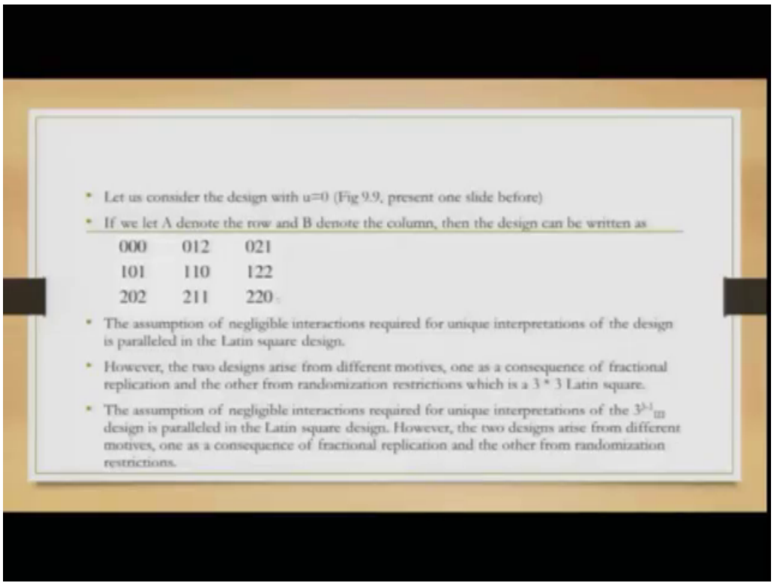

So, let us consider the design for u is equal to 0. See if you if you let A denote the row and B tuned on the column then the design can be written as follows. So, you have basically 000 where they are not the lowest effect, then 012 which is A low B mid may middle or my average and C is high level. 0 to 1 is I will I will just repeat it is low, high, medium that is corresponding to A B C then 101, 110, 122, 202, 211 and 220.

The assumptions of negligible interaction, is required for unique interpretation of the design is parallel and in the Latin square design structure which we had done. However, the 2 design arises from 2 different motives. One is a consequence of the factorial replication and the other from the randomization restriction with a 3 square 3 Latin structure.

The assumption of a negligible interaction is required for unique interpretation of 3 to the power 3 minus 1 or the third design concept which is a parallel to the Latin square. If you remember in the Latin square with the 3 cross 3 square and it was replicated accordingly. However the 2 design arise from different motives, one is consequence of the fractional replication another from the randomization restriction which has been put to the problem.

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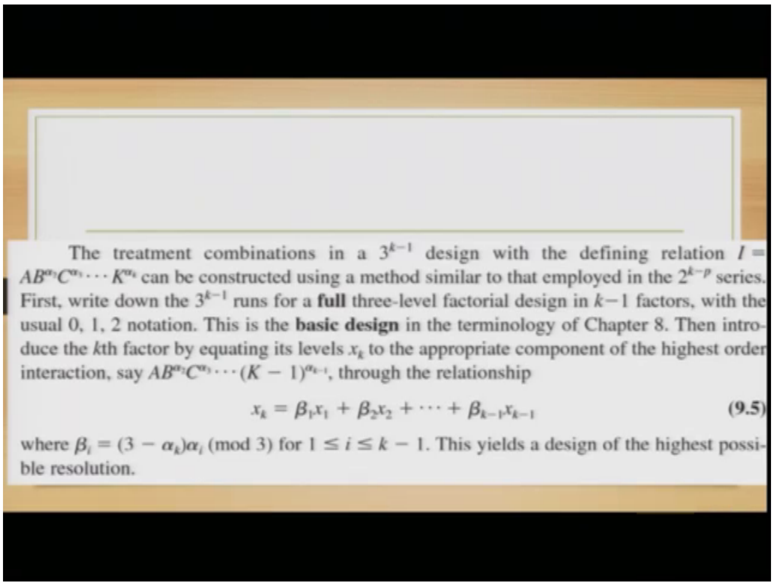

The treatment combination in a 3 to the power k minus 1 design with a defining relationship is. So, basically the indicator variable on the indicator function is again a to the power 1 B to the power alpha 2 C to the power alpha 3 till the case of k to the power alpha k. So, they can be constructed using a method similar to that employed in 2 to the power k minus p series and the work can be done accordingly.

First we write down 3 to the power k minus 1 runs for a full 3 level factorial design in a k minus 4 factors with the usual 012 notations as should be. So, 012 more notations are basically the level of interaction each variable has, but in medium low medium high. This is the basic design in the terminology then in there. So, so those we in true the scale factor for equating it to the variables or levels x k to the appropriate component of the highest order. So, based on that we have the equation is like this. So, we want to find out the effect.

So, in one orthogonal direction if you remember we are mentioned that one was l 1, it is not the norm not the l 1 norm, l 1 or l 2 and again l 2 is not the norm for the Cartesian coordinate. So, l 1 was basically given by alpha 1 x 1 plus alpha 2 x 2 plus still the case of alpha k x k and similarly l 2 would be given by beta 1 y 1 plus beta 2 y 2 plus till the case of beta l y l. So, l and k are not the same because the level of significance based on which you are trying to do the calculations would be different for 2 different combinations of orthogonal sets, consider is any very simple terms orthogonal sets. So, that we are able to predict it maximum.

So, in this case we have x k is equal to beta 1 x 1 plus beta 2 x 2 plus beta k minus 1 x to the power k minus 1. So, from this we find out beta i and which would of the mod level 3 and this yields a design on the highest possibility resolution such that we are able to predict to the maximum possible extent.

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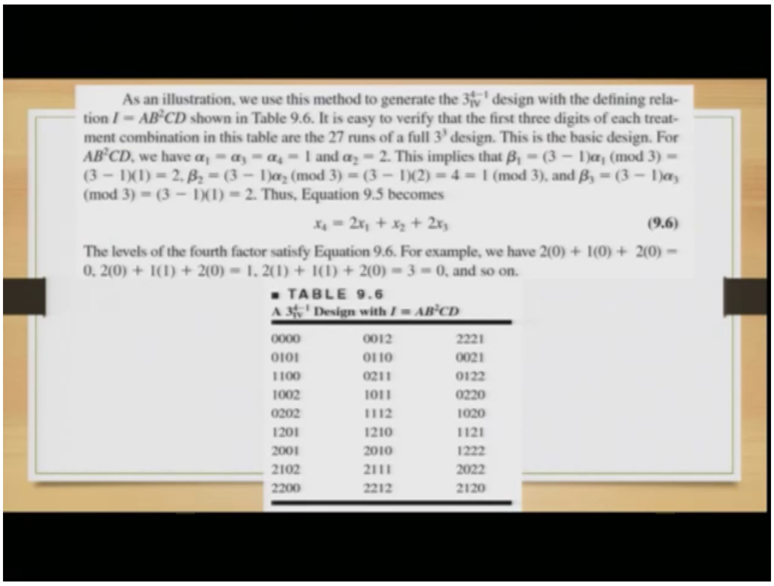

As an illustration, we use this method to generate the 3^{4-1}_{IV} design with the defining relation $I = AB^2CD$ shown in Table 9.6. It is easy to verify that the first three digits of each treatment combination in this table are the 27 runs of a full 3^3 design. This is the basic design. For AB^2CD, we have $\alpha_1 = \alpha_3 = \alpha_4 = 1$ and $\alpha_2 = 2$. This implies that $\beta_1 = (3-1)\alpha_1 \pmod 3 = (3-1)(1) = 2$, $\beta_2 = (3-1)\alpha_2 \pmod 3 = (3-1)(2) = 4 = 1 \pmod 3$, and $\beta_3 = (3-1)\alpha_3 \pmod 3 = (3-1)(1) = 2$. Thus, Equation 9.5 becomes

$$x_4 = 2x_1 + x_2 + 2x_3 \qquad (9.6)$$

The levels of the fourth factor satisfy Equation 9.6. For example, we have $2(0) + 1(0) + 2(0) = 0$, $2(0) + 1(1) + 2(0) = 1$, $2(1) + 1(1) + 2(0) = 3 = 0$, and so on.

■ TABLE 9.6
A 3^{4-1}_{IV} Design with $I = AB^2CD$

0000	0012	2221
0101	0110	0021
1100	0211	0122
1002	1011	0220
0202	1112	1020
1201	1210	1121
2001	2010	1222
2102	2111	2022
2200	2212	2120

As an illustration we use this method to generate the 3 to the power 4 minus 1, fourth design with the defining relationship as the indicator function as A B square C D. It is easy to verify the first 3 digits of each treatment combination instable are 27 runs or the full 3 to the power 3 design problem based on the, so based on that we are trying to get the maximum throughput in order to help us in predictive predict us at the highest level.

This is the basic design for A B square C D we have accordingly as obviously, alpha 1 A to the power it is basically alpha 1. So, alpha 1 would be 1. Then C is to the power alpha 3, so alpha 3 is 1. D is 2 to the power alpha 4, so alpha 4 is 1 and B is basically 2 to the power alpha 2 where alpha 2 is 2.

This implies that we want to find out that beta 1 is given by 2 alpha 1, similarly at the mod 3 modulus 3 level and based on that we can calculate. So thus the equation comes out to be x 4 into 2 x 1 plus x 2 plus 2 x 3. And once we design the overall design levels, so on the leftmost column if you see you have the values it is basically A B C D, so obviously, the combinations would be what one extreme 0000 where it is low low low low and the other case it would be 2222 which is high high high high for A B C D. So, if you see it the combinations were design for indicator A B square C D are coming out to be as given. So, it starts from 000 and it goes to the highest level of 2120. So, combinations would be found accordingly and you can do the prediction likewise.

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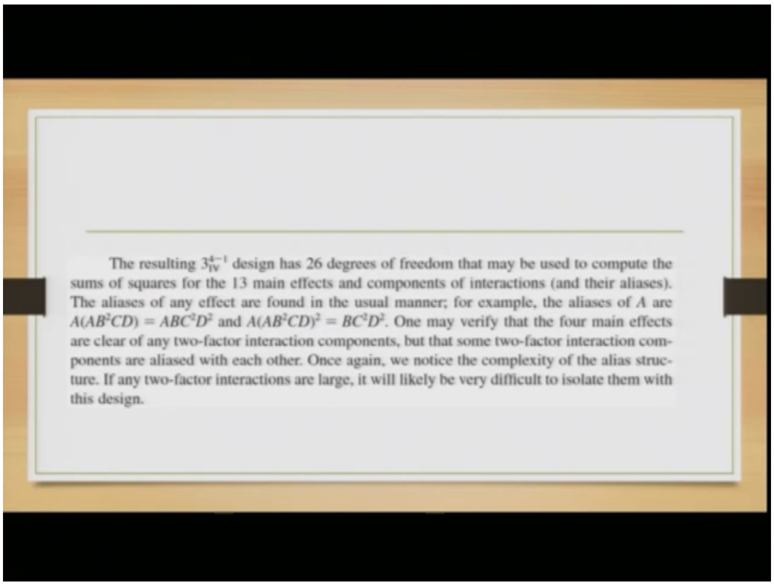

The resulting 3 to the power 4 minus 1, fourth level design has 26 10; 26 degrees of freedom that may be used to compute the sum of the squares for the 13 main effects which you are trying to find out of the interaction.

The aliases of easy effects are found in the usual manner. For example, the aliases of A are given as A into A B square C D which is basically found out accordingly and for A into A B square C D whole square root coming out to be B C square D square. One may verify that the phone main effect are clear of any 2 factor interaction component, but that too bad that some 2 factor interaction components analyze each other. Once again we notice the complexity of the alias structures such that if any 2 factor interactions are large it will be likely to be very difficult to isolate them with this design and do the calculations accordingly.

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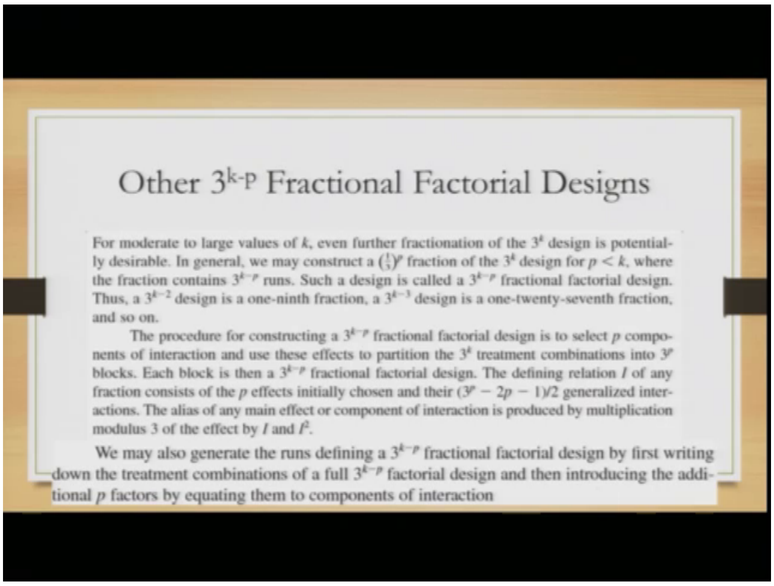

So, other 3 to the power k minus p fractional factorial designs are for moderate to large to large values of k even for the fraction fractionalization of fractional nations of the 3 k designs is potentially desirable. In general we may construct a power of 1 3 to the power p factors 3 to the power k designs for p less than k and do it accordingly.

The procedure for constructing the 3 k to the power minus p factorial fractions it designed to select p components of the interaction level and use these effects to partitioned the 3 k into block such that the effects comes out to be the maximum. The defining relationship of the indicator effect of any fraction consists of p effects initially is chosen in such a way, so you had basically 3 to the power p, p 2 minus 2 p minus 1 that whole thing divided by 2. So, you can find out how many blocks and how many such variables are there such that number of blocks multiplied by the variables in each yeah from the number of variables in each block would give you the total number of such observation which we have.

We may also generate the runs defining 3 to the power k minus p fractional factors by first writing down the treatment combinations of a full 3 to the power k minus v fractional design and do the work accordingly.

We illustrate the procedure by constructing a 3 to the power 4 minus 2 design that is one-ninth from the fraction of the 3 to the power 4 design. So let us let us see in that A B square C and B C D we have the 2 component the interaction chosen to construct the design.

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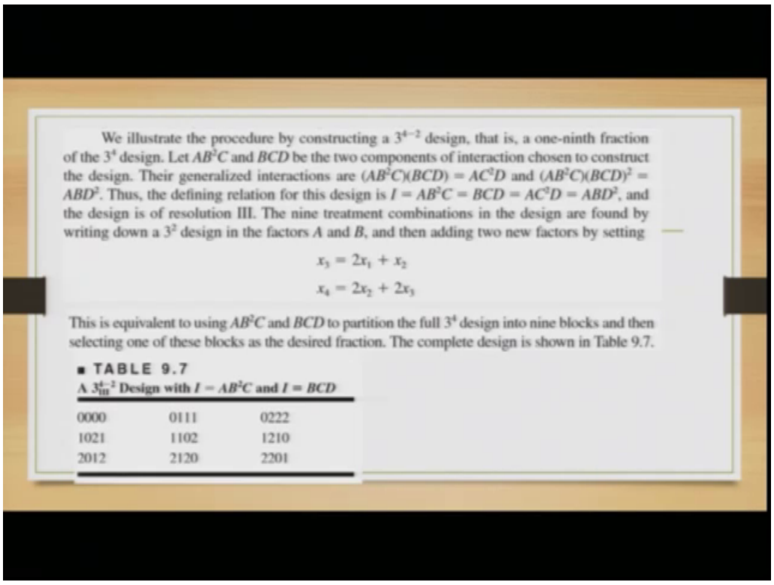

We illustrate the procedure by constructing a 3^{4-2} design, that is, a one-ninth fraction of the 3^4 design. Let AB^2C and BCD be the two components of interaction chosen to construct the design. Their generalized interactions are $(AB^2C)(BCD) = AC^2D$ and $(AB^2C)(BCD)^2 = ABD^2$. Thus, the defining relation for this design is $I = AB^2C = BCD = AC^2D = ABD^2$, and the design is of resolution III. The nine treatment combinations in the design are found by writing down a 3^2 design in the factors A and B, and then adding two new factors by setting

$$x_3 = 2x_1 + x_2$$
$$x_4 = 2x_2 + 2x_3$$

This is equivalent to using AB^2C and BCD to partition the full 3^4 design into nine blocks and then selecting one of these blocks as the desired fraction. The complete design is shown in Table 9.7.

■ TABLE 9.7

A 3_{III}^{4-2} Design with $I = AB^2C$ and $I = BCD$

0000	0111	0222
1021	1102	1210
2012	2120	2201

This generous interactions are accordingly found out as A C squared D and A B D square. So, if you remember here alpha 1 is 1 in the first case, alpha 2 is not there because B is not there which is 0 alpha 3 would be 2 and alpha in the second case it would be alpha 1 is 1, alpha 2 is 1, alpha 3 is 0, alpha 4 is 2.

So, based on that you can find out what are the best combinations; that is the defining factor relationship of the design would be found not such that it will be equal to and reading the last one only it would be A B D squared which is alpha 1 1, alpha 2 2, alpha 3 0, alpha 4 2. By writing the design of the resolution the 9 treatment combinations in the design are found by writing down a 3 square design in the factors A and B and those factors are given as a 3 x 3 is equal to 2 x 1 plus x 2 and x 4 is equal to 2 x 2 plus x 3.

This is equivalent to using the A B square C and B C D in power in partition to the full 3 4 design into 9 blocks and then selecting one of these 3 blocks as the design factors thus the complete design is shown in this figure. So, they are basically we start at level of 0000; that means, all at the lower level and go up till I am not reading in the whole sequence the last one would be 2201 which is A B at the highest level, C at the lowest level and D at the medium level, average level.

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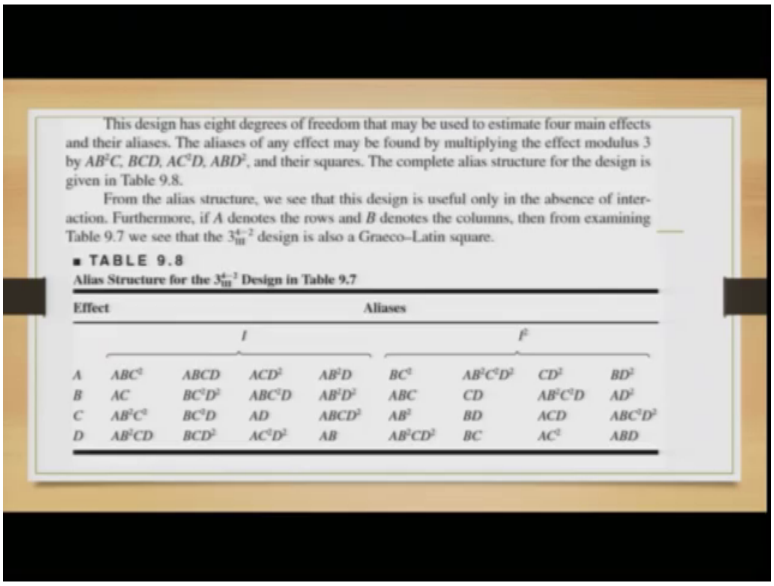

This design has eight degrees of freedom that may be used to estimate four main effects and their aliases. The aliases of any effect may be found by multiplying the effect modulus 3 by AB^2C, BCD, AC^2D, ABD^2, and their squares. The complete alias structure for the design is given in Table 9.8.

From the alias structure, we see that this design is useful only in the absence of interaction. Furthermore, if A denotes the rows and B denotes the columns, then from examining Table 9.7 we see that the 3^{4-2}_{III} design is also a Graeco–Latin square.

TABLE 9.8
Alias Structure for the 3^{4-2}_{III} Design in Table 9.7

Effect	Aliases							
	I				I^2			
A	ABC^2	$ABCD$	ACD^2	AB^2D	BC^2	$AB^2C^2D^2$	CD^2	BD^2
B	AC	BC^2D^2	ABC^2D	AB^2D^2	ABC	CD	AB^2C^2D	AD^2
C	AB^2C^2	BC^2D	AD	$ABCD^2$	AB^2	BD	ACD	ABC^2D^2
D	AB^2CD	BCD^2	AC^2D^2	AB	AB^2CD^2	BC	AC^2	ABD

This design this design would basically have 8 degrees of freedom that may be used to estimate 4 main effects which are there. So, you are basically dividing them into blocks and multiplying the blocks in there with respect to their number of observations such that we will get the aliases to the maximum level. The aliases of any effect can be found out by multiplying the effect of the modulus 3. The complete alias structure for the design is given in table 9.8.

From the alias structure, so we can find out we see that design is useful only in absence of interaction. So, if there are interactions are there or not there that will also have an effect. Furthermore if A denotes the row and B denotes the column then for examining from this table we see that we have basically the 3 to the power 4 minus 2 design of third design and they would be of the gray code Latin square. So, the square would be truly found according to the concept of a Latin square from the concept of mathematics of which is Greco Latin square. And the combinations have how you are partitioning in the blocks would exactly be the same way how you basically made them into the square, so that will give you the maximum effect.

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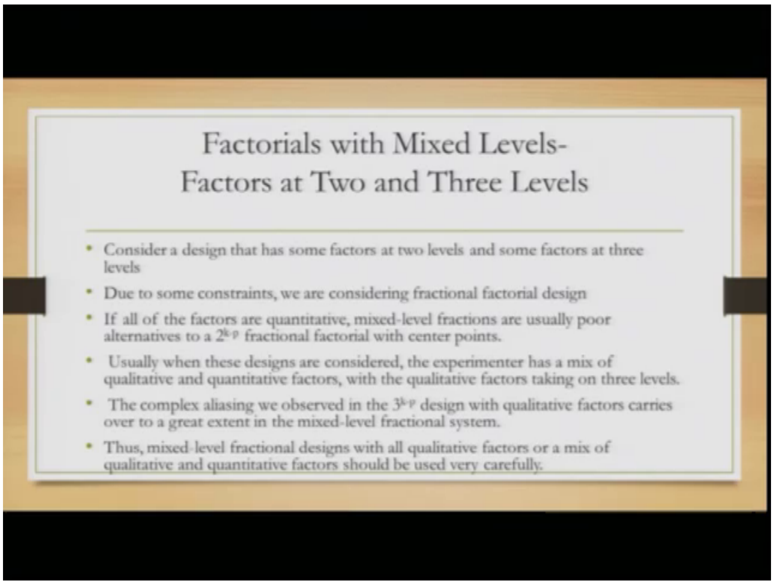

Factorial with mixed levels, factors are two, two and three levels you will finally, take out accordingly. So, considered a design that has some factors at two levels and some factors at three levels, due to some constraints we are considering fractional factorial design. If all of the factors are quantitative mixed level fractions are usually poor alternatives 2 or 2 to the power k minus p fractional factorial with centre points. Usually when these designs are considered the experiment has a mix of qualitative and quantitative factors with the quality factors taking on 3 levels and all corresponding with the quantitative values could referred also we determined accordingly.

The complex analysis we observed in the 3 k minus p design with qualitative factors carries over a great extend in mixed level fractional system which you are designing. Thus mixed level fractional design with all qualitative factors or a mix of qualitative and quantitative factors should be used very carefully in order to find out the overall effect.

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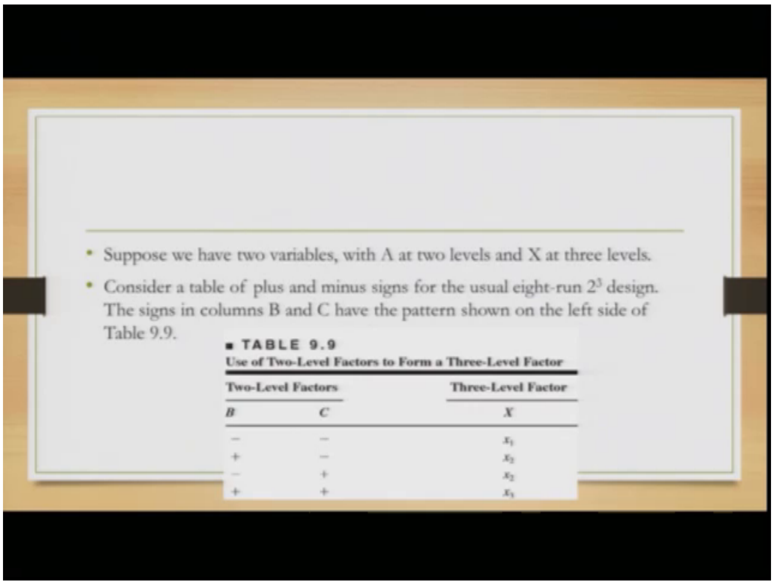

Suppose we have 2 variables with a at 2 level and x at 3 levels. So, we consider a table of plus and minus signs for the usual 8 run 2 the power 3 design. The sign in the columns B and C have the pattern shown on the left side of the table and which is basically 2 factor levels it will be B at minus level C at minus level. Other combinations are B at high C at low and another could be at B at low C at high another combination would be both of them at the higher level. Then effect of the fillet 3 and the effect would be coming out in x 1 x 2 x 3 x 4, so obviously, it will increase depending on more numbers of such B C D and combinations levels also being plus minus or plus plus plus 0 minus minus minus minus 4.

Let the levels of X be represented as I said the x 1 x 2 x 3 x 4. The right side shows that the sine patterns of B and C are combined to form the levels of this 3 level factor model.

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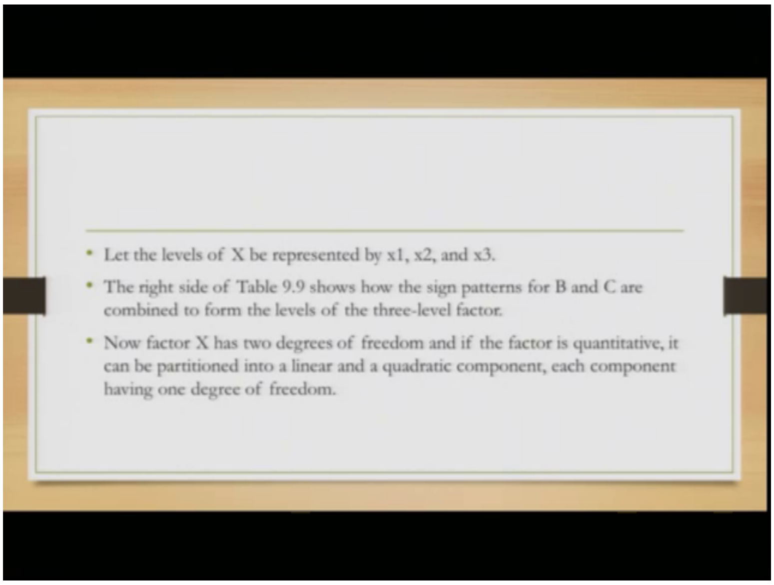

Now, factor access to 2 degrees of freedom and if the factor is quantitative, it can be partitioned into a linear and quadratic component, each component having one degrees of freedom and calculations can be done accordingly.

Table, the next table basically the slide would show basically 2 to the power 3 designs with the columns ables to show the actual effects. Note that the level of effects of X is the sum of 2 effects estimate computed from the column.

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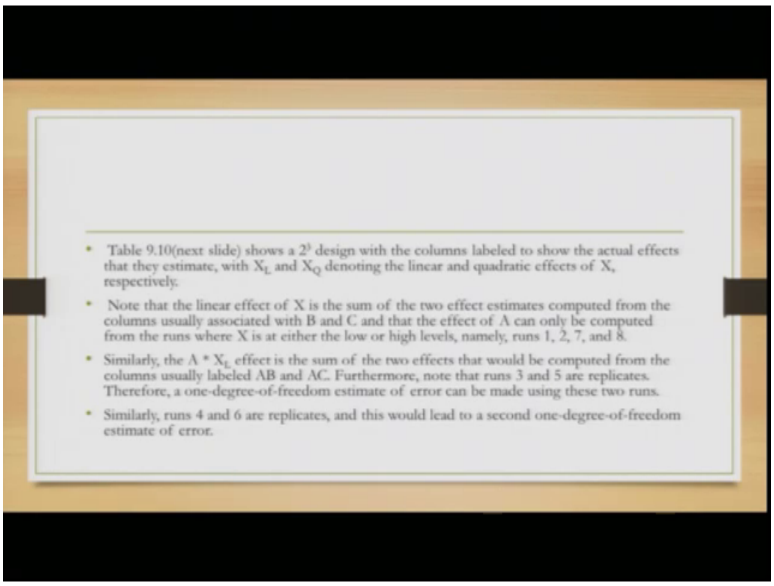

Similarly, you will basically have A to the power X L, A to the power X m into the X n based on which you will basically find out the combinations. I will come to that later on.

So, similarly for runs of 4 applications 6 applications 8 applications can be done and we can find out the partitions accordingly to their aliases, to the folds, to the level of factors, to how many such blocks would be there, what is the number of such observation is block, we will study it more details later on. Have a nice day and with this I will end this lecture.

Thank you very much.