Total Quality Management - II

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Lecture - 32
Confounding in the 3^k Factorial Design – I

Welcome back my dear friends. A very good morning, good afternoon, and good evening to all the participants and the students and this is the TQM II lecture under the NPTEL, MOOC series and this is the 30 second lecture; that means, we are in the almost at the fagend of this course because as you know this is each day is a half an hour or 30 minutes lecture for 5 days in a week, and that is 20 hours, so it is 40 lectures. So, we are in the last week. And I am Raghunandan Sengupta from IME department IIT, Kanpur.

So, if you remember the la the last class which was the 31st one we did start actually the concept of 3 factor models fractional factorial models that means, basically trying to find out the overall combination of 3 to the power k and I did mention about this initially when we are doing the 2 to the power k factor with levels of dependence being on the third order or forth order or fifth order. Those are the subscripts which we are giving, so welcome to that once and more.

Now, we are doing the frothing of the syrup and there was nozzle dimensions or the diameter or the overall area of the nozzle is important, pressure was important and nobe and the temperature was important. So, all these factors were important. So, based this we were basically starting the problem.

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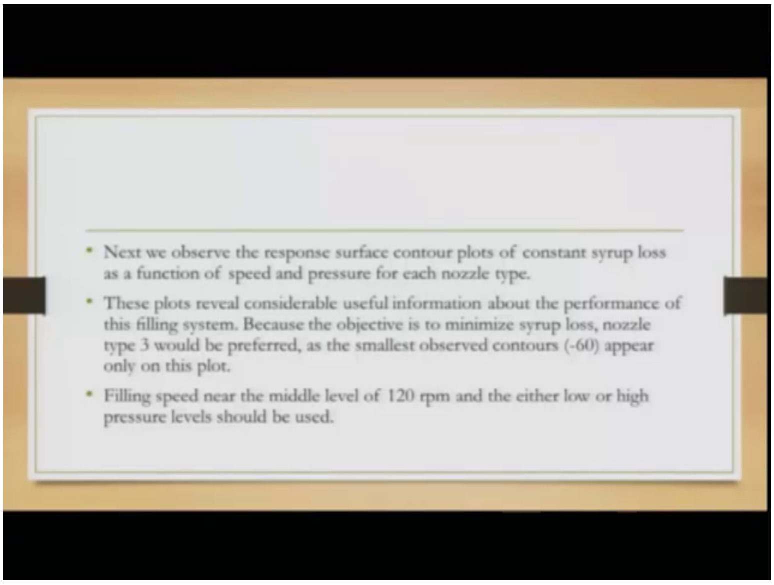

So, next we observe the response surface count contour plots of constant syrup loss for thing was basically there was loss. So, now, you want to find it out as a spare fresh speed and pressure ok, my apologies it was basically pressure and speed of is nozzle now obviously, nozzle diameter would be fixed later on after the speed and pressure are adjust. Obviously, it will mean that you want to basically adjust the pressure adjust the speed and then finally, decide on nozzle dimension because once nozzle nozzle dimension is fixed it cannot be chain because the manage chain process we have to order it there is cause for that all the things are there.

So, these contour plots reveal the considerable useful information about the performance of this filling system because the objective is to minimize syrup loss nozzle type 3 would be preferred. So, they were basically 3 levels, 1 2 3 pressure was are 3 levels could be general than speed could be a 3 levels and so on and so forth, that is why it was 3 to the power k. So, continuing reading it because the objective is to minimize syrup loss I am due to frothing, nozzle type 3 would be preferred as the smaller observed contour of minus 60 appear only on this plot and they can be analyzed accordingly.

Filling speed near middle level of 120 revolutions per minute or rpm and the either low or high pressure levels should be used in order to basically minimize the frothing or the syrup loss.

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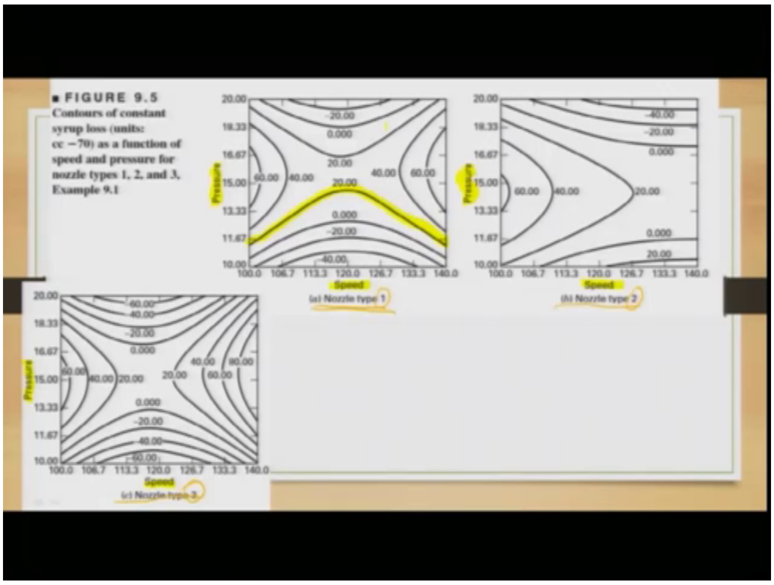

So, the contours or the constant syrup loss see units are given in C C cubic centimeters minus 70 as a function of speed and pressure for nozzle of type 1 2 3. So, if you see the diagrams here. So, this was basically an nozzle of type 1 or at a certain dimension this is are nozzle type 2 certain dimension and this was an nozzle of type 3 of certain dimension and the contour basically were your plotting pressure and speed. So, let me highlight the pressure and speed once more. So, this was basically the pressure along the y axis the speed this is for nozzle 1 type 1 dimension pressure along y axis speed and along x axis for nozzle of type 2 dimension this was pressure and speed for nozzle of type 3 dimension.

Now, from this we will go into analyzing and the, what are importance of this. So, basically it is giving a plot of how the pressure and speed change for one dimension. So, in case is if you see look here I will just. So, considering that the pressure is slowly increasing the nozzle speed also speed also varies then after reaching a peak it starts decreasing. So, they would be different this is for the dimensions being 20. So obviously, you have dimensions of 60 40, so 20 would basically mean this is the contour lots of the overall fluctuation of speed and pressure for a type of nozzle which you are trying to utilize.

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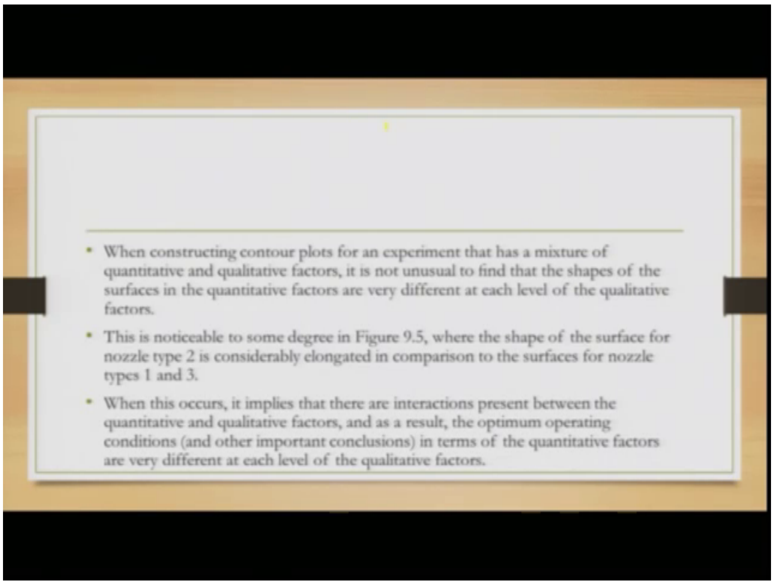

When constructing contour plots for an experiment that is a mixture of quantitative as well as qualitative factors or variables or attributes it is not unusual to find that the shape of the surface it is the quantitative factors are very different type at each level of qualitative factors.

So obviously, qualitative and quantitative inter relationship would be such that find trying find out the contours would be would be quite interesting in this shape. This is noticeable to some degree in figure where the shape of the surface for nozzle type 2 is considerably elongated in comparison to the surfaces for nozzle type 1 and type 2, because in there the combination in nozzle type 2 the overall counter is much more elongated or in one direction this is basically stretched. When this occurs it implies that there are interaction present between the quantitative and qualitative factors and as a result optimum operating conditions and other important conclusions in terms of the quantitative factors are very different at each level of qualitative factors combinations.

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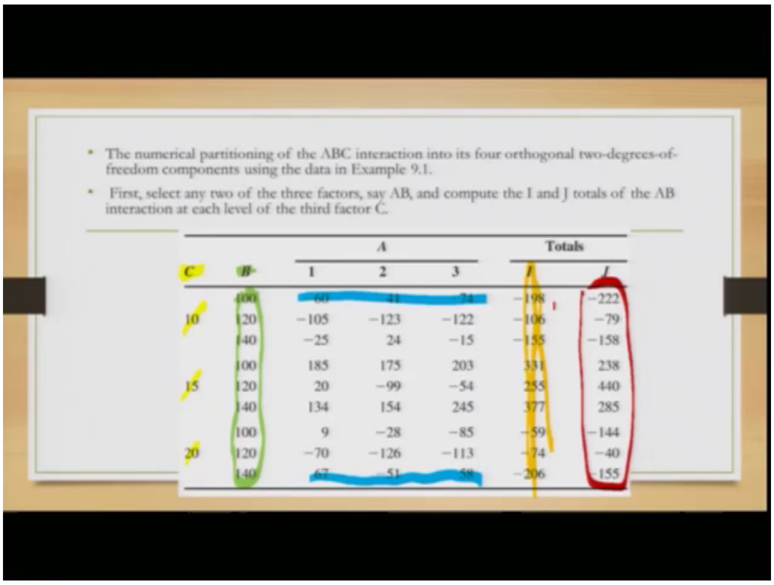

The numerical partition of A B C interaction into its 4 orthogonal 2 degrees of freedom components using the data of 9.1 which is basically speed pressure and trying to find out on nozzle dimension and then trying to basically find out the frothing and the loss of syrup.

First select any 2 of the 3 factors, so basically we have pressure speed and nozzle dimension. So, 3 factors A and B you consider and compute basically the values of I and J and I and J if you remember I the over the combine effects which were happening and how you could find it out it could be A B square C, A B C square and so on so forth combinations. So, compute I and J that is the total of A B interaction at each level of the third factor of C.

So, you are basically have the you plot the values of C along the leftmost columns. So, these are basically 10, 15 and 20. So, these are basically the dimensions, which were talking about and the values of A and B, so which was corresponding to the fact they were the speed and the pressure. So, the factor B, so I should use a different colour, it would be. So, this is for B and the values are I will just highlight then this basically starts from 100 goes on to 140. The combinations are given, so for 10 for a nozzle dimension of 10 you have to 10, 100, 120, 140, then for dimensions of 50 you have again the combinations of 100 120 140 and final be for a nozzle dimension of 20 the combinations are 10 100 120 140.

And for similarly for basically for A the values are given as 1 2 3 and based on that you find out the total combinations and on the values are coming. So, I will use another colour. So, the values are given starting from the first row minus 60 41 74 to the last row which this is basically 67 minus 51, 58. So, these are the combination of the value which your getting. The totals which you find out for the values of I and J com corresponding the combinations of A B C which you have, if you remember I J word in different variables as A B square C, A B C square and so on so forth.

So, the values are given the second last and the last column. So, the second last I values are given again I should use a different colour, let me use the (Refer Time: 08:44). So, these are the values of i, and these are the values of J based on that you can do the calculations accordingly.

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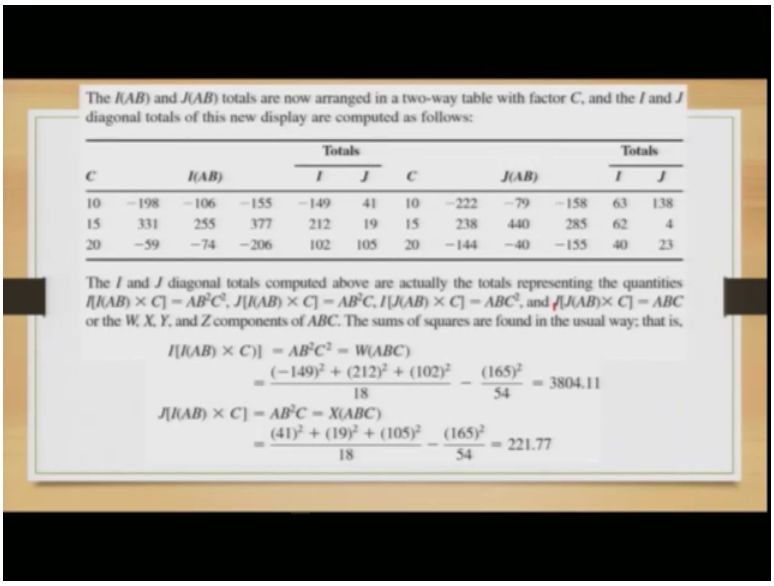
The $I(AB)$ and $J(AB)$ totals are now arranged in a two-way table with factor C, and the I and J diagonal totals of this new display are computed as follows:

				Totals						Totals	
C		$I(AB)$		I	J	C		$J(AB)$		I	J
10	−198	−106	−155	−149	41	10	−222	−79	−158	63	138
15	331	255	377	212	19	15	238	440	285	62	4
20	−59	−74	−206	102	105	20	−144	−40	−155	40	23

The I and J diagonal totals computed above are actually the totals representing the quantities $I[I(AB) \times C] = AB^2C^2$, $J[I(AB) \times C] = AB^2C$, $I[J(AB) \times C] = ABC^2$, and $J[J(AB) \times C] = ABC$ or the W, X, Y, and Z components of ABC. The sums of squares are found in the usual way; that is,

$$I[I(AB) \times C)] = AB^2C^2 = W(ABC) = \frac{(-149)^2 + (212)^2 + (102)^2}{18} - \frac{(165)^2}{54} = 3804.11$$

$$J[I(AB) \times C] = AB^2C = X(ABC) = \frac{(41)^2 + (19)^2 + (105)^2}{18} - \frac{(165)^2}{54} = 221.77$$

The I which is the combinations of A B in different in different factors and J which is also combinations a main different factors totals are now arranged into a table with factor C and the I and J diagonal totals of this new displays are computed as follows. So, you basically plot the only the values of I J and C and try to find out the combinations in such a way such that we are able to get the maximum effect accordingly.

The I and J diagonal total computed above are actually the total totals representing the quantities which are if the remember I was mentioning time and a time and again that I and J can be in different combination. So, I is basically A B square C, and J is basically given by A B C square and based on that we are and on the other combinations I could be A B C square. So, J was see other the combinations A B C which are basically if you remember we generate as w x y and z components. The sum the squares obviously, the main effect you want to find out all the variables and the left most set of columns. Then you basically the sum of squares and also the second last row would be the errors such that the total of all the effects all the factors A B C and all the combinations of A B C and summed up with the sum of the errors would basically give you the total sum and based on that you have the degrees of freedom.

So, the sum of the squares or the squares divided by the errors and squares divided by the degrees of freedom would give you the a factor and a factor based on, whether it was a left hand test or right hand test and also the concept the p value and whether you want to basically subscribe to the fact whether you want to consider that h naught as true or untrue will basically take a decision accordingly.

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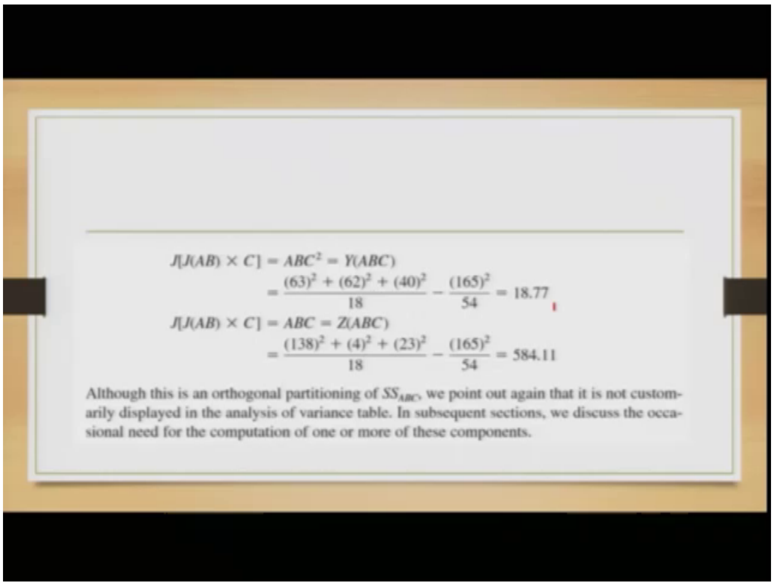

So, although this is an orthogonal partitioning of the SS sum of the squares of A B C combinations we point out again that is not customarily displayed in the analysis of table value of table. So, and you have basically ANOVA and MANOVA. In subsequent sections we discuss the occasional need for the computation of one or more of these components and basically tried to find out how this effect can be maximize effect means that overall effective want to find out for trying to predict the overall effect from the factors, in order to predict what is the decision or what is the so called the y value which is the dependent variable which we have.

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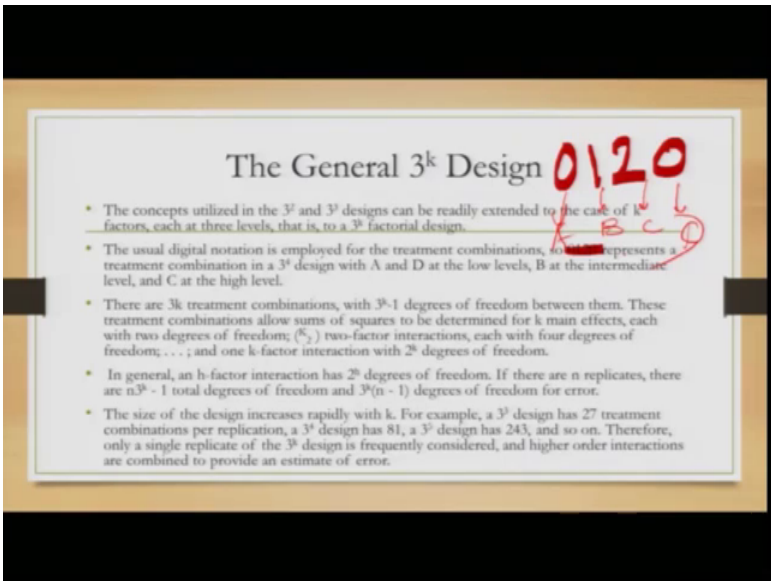

So, now the general 3 to the power k design. So, 3 on the on the levels of each factors which are there. So, the concept utilized in 3 square and 3 cube, so obviously, I am trying to utilize the nomenclature we also always use we basically 3 to the power 2 and 3 to the power 3 design can be readily extended to the case of k factors also. So, it can be 3 to the power k factors also each are 3 levels. So, basically there are k factors it can be a 3 levels it will be 3 to the power k. If there are 4 factors each are at k level it will be 4 to the power k, and so on so forth, but obviously, the degrees of freedom would be calculated accordingly.

The usual digital notation is employed for the treatment combinations so that was 0 1 2 0 represent a treatment combination of 3 to the power 4 designs depending on the different combinations which we have. With A and D at the lowest level, so if you find out this. So, this is basically what you have a 0 1 2 0 they are. So, this is for A, this is for B, this is for C, this is for D.

So, if there are 3 levels, A is at the and the levels are 0 1 2 or see for example, minus 1 0 1. So, consider this minus 1 0 and 1. So, A is at 0 level which is at the normal level and ok, so let me change it my apologies it would be basically be 1 2 3 level. So, if it is 0 level a would be the minimum level if B is at 1 it is a normal level average, C is at 2 is basically the high level at the high highest effect which we can find out for for C and the value of D it is basically 0 means it is the lowest level. So, in case say for example, you have the factors are A B C D and the levels are 3 and the overall effects can be basically given so called weights.

Let me use the words weights. So, there are minus 1 0 1. So, obviously, A would be if it is A is minus it is A lowest level B is basically at 0 it is as middle level or normal level C is basically has plus 1 it is the highest level, and D is basically again plus 1 is the highest level. So, the nomenclature how we use we will basically assign them accordingly.

But again I am repeating which I did mention the example if you remember of the building the dam of example of electrical circuit or the concept that we want the average to be 0, but we also want at the variance to me. So, both of them I have to basically met it is basically something to do with me not be exact the concept of unbiasedness and consistency. The usual digital notation is employed for the treatment combinations. So, 0 1 2 0 represents a treatment combination a 3, 3 to the power 4 combinations where A and D at the lowest level B is the intermediate level and C is the highest level as I mentioned 0 1 2 3 where the levels are basically given as 0 1 2.

So, there are 3 to the 3 into k treatment combinations with 3 to the power k minus 1 degrees of freedom between them. So, this treatment combinations allow sum of squares to be determined for the k mean effects each with 2 degrees of freedom, so obviously, it will be k 2, 2 factor interactions each with 4 degrees and one factor interactions with 2 degree 2 to the power k degrees of freedom depending on the number of factors which we have. In general h factor interaction, so if it will be two to the power k for if at 2 levels if there are n replicates then obviously, it will be n multiplied by 3 to the power k minus 1 it will give you the total degrees of freedom or the degrees of freedom for the errors would be 3 k into 9 minus 1.

So obviously, the sum of the degrees of freedom would basically give you the overall information as the sum of the squares of all the factors errors plus the error of the of the sum of the of the so called sum of the squares of the errors of white noise that being added to the sum of the square of the errors or the variables should basically give you the total errors. The size of the design increases rapidly with k. So, as k is increasing obviously, it becomes 2 to the power k then it becomes, so if it is increasing. So, into if it is 2 to the power k and if there are 2 factors it is 2 to the power 2, 4.

If it is 2 to the power 3 it is 8, if it is 2 to the power 4 it is basically 16. So, is basically increases at the very high rate. And if it is 3 or 4 or 5 different levels for each factors, so obviously, the increases much faster like 5 to the power 2, 5 to the power 3, 5 to the power 4, so obviously, 5 to 5 square would be 25, 5 to the power 3 would be 125 and increasing and it increases accordingly. So, it 25 into 25 it will be 625 of different combinations you have to find out.

So, the obviously, it is tedious, but you have to make a decision that what is the best level of efficiency considering the time constraint which would also mean that there is a cost for doing all this work.

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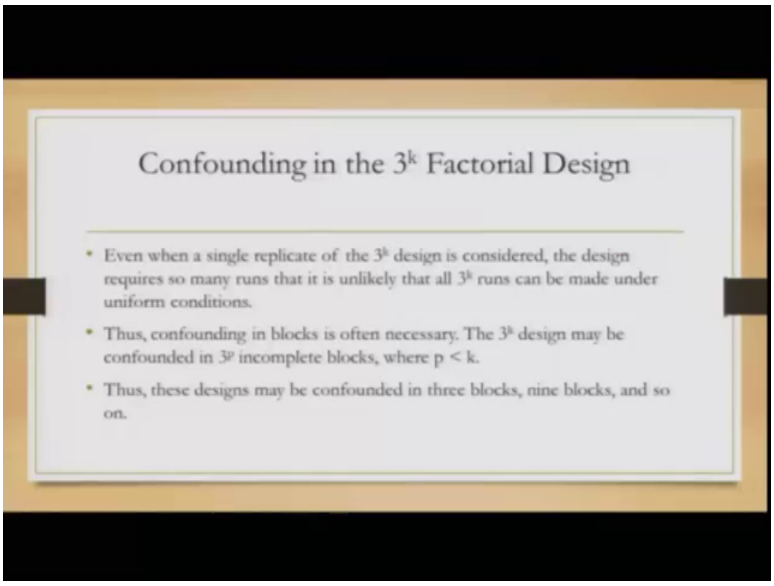

Confounding of the 3 k factorial design, fractional factorial design problems even when a single replicate of 3 to the power k design you consider the design requires so many run that is unlikely that all 3 k runs can be made under uniform conditions.

This compounding in blocks is often necessary thus the 3 k design maybe confounded in a 3 p con incomplete blocks such that p is less than k. So, if our overall, set of information based which you are trying to find out the overall effect it can be done by p such factors with p is less than k is obviously, it should be attempted. Thus this design may be compounded in 3 blocks, 9 blocks so on so forth depending on the on the overall grouping you want to do.

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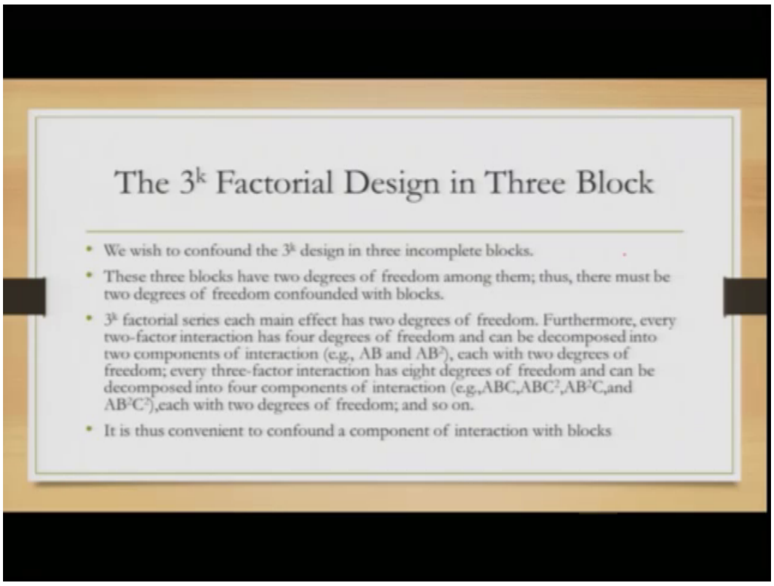

The 3 k factorial design in 3 blocks, so we wish to compound the 3 k design in 3 incomplete blocks, so block 1, block 2, block 3, but they would be done in such a way that the values of factor p is less than k. These 3 blocks have two degrees freedom among themselves thus there may be 2 degrees of freedom confounded with the blocks because there are 3, so obviously, the overall effect would be 2 degrees of freedom would be 2 the effects.

The 3 k factorial series in this series each main effect has 2 degrees of freedom for the more every two factor interaction has 4 degrees of freedom, and can be decomposed into two components of interaction basically A B A B square or it can could have been by a square B also depends on how you are trying to find out the overall effect.

So, each with 2 degrees of freedom, every 3 factor interaction has 8 degrees of freedom and can be decomposed into 4 components. So, the 4 components are basically A B A B C square, A B square C, so obviously, if you are able to basically define A B C accordingly could have been by given as a combination of a square B C or different combinations as required.

And obviously, you would basically you have the last one is A B square C square each with each with 2 degrees of freedom. So, it will be 2 degrees freedom for any combinations as we find out and as they are the number blocks increases or number of factors increases the combinations would basically also increase accordingly. It is the convenient to compound a component of interaction with the blocks and do the calculations accordingly.

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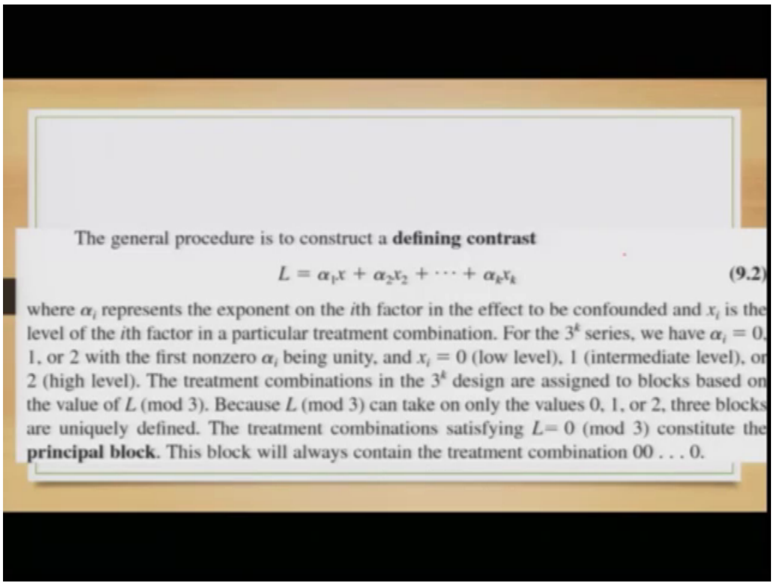
The general procedure is to construct a **defining contrast**

$$L = \alpha_1 x + \alpha_2 x_2 + \cdots + \alpha_k x_k \quad (9.2)$$

where α_i represents the exponent on the ith factor in the effect to be confounded and x_i is the level of the ith factor in a particular treatment combination. For the 3^k series, we have $\alpha_i = 0$, 1, or 2 with the first nonzero α_i being unity, and $x_i = 0$ (low level), 1 (intermediate level), or 2 (high level). The treatment combinations in the 3^k design are assigned to blocks based on the value of L (mod 3). Because L (mod 3) can take on only the values 0, 1, or 2, three blocks are uniquely defined. The treatment combinations satisfying $L = 0$ (mod 3) constitute the **principal block**. This block will always contain the treatment combination 00 . . . 0.

So, the general procedure is to construct a defining contrast. So, with alpha i represent the exponent in the ith factor in the effect to be compounded and x i is the level of the ith factor in a particular treatment combinations. Thus for the 2 3 to the power k series we would basically have alpha i is basically the at the factors would decide the what is level importance you are going to give. So, in in alpha is equal to 0, where i is basically one or two with the first nonzero alpha i become unity and and the lower levels would obviously, be eliminated by considering the factors as 0.

So, or our or, so obviously, if 3 or 4 or 5 or 6 are considered, so obviously, technically the lower values would be cannot considered. The treatment combinations in the 3 to the power k design and assigned to blocks based on the value of L, so obviously, it would be a mod 3 depending on 3 factors mod 4 depend if there are 4 type factor mod 5 depending on 5 factors and so on so forth. Because L which is a mod 3 can take only on the values of 0 1 2 it is higher it is 0 1 2 3 and corresponding to that will increase. So, this 3 blocks are uniquely defined once there uniquely defined you can find out the combinations accordingly.

The treatment combination satisfying L is equal to 0 constitute the principal block, the block will always consider the treatment as at the lowest level 0 0 0 0 0 and then different combinations of 0 1 0 0 or 0 0 1 0 0. So, I am basically repeating it 0s depending on how many such different factors which we have we can find out the best possible combinations or the factors to give you the maximum output. Output obviously, whenever I use the what output it basically means I am able to predict it to the maximum level such that the white noise the sum the squares of the errors is as flow as possible such there are prediction is the highest or the maximum accuracy.

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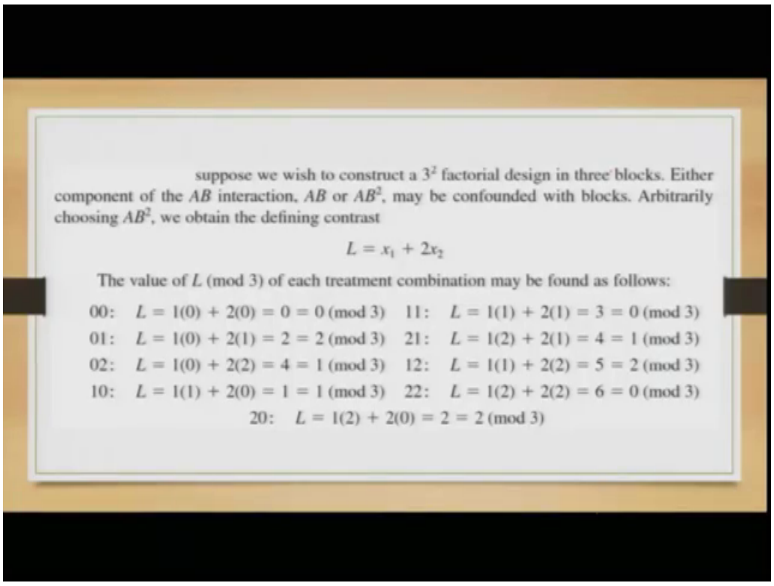

Suppose we wish to construct a 3, 3 to the power 2 factorial design in 3 blocks either component of the A B interactions are A B or A B square or it can be A square B maybe confounded with this blocks also arbitrarily choose A B square it could have been A square B is also. So, the combinations are x 1 into 2, x 2 and basically the factors could be 2 x 1 plus x 2 also depending on the combinations we are going to take. The value of L which is of now you are considering mod 3 of each treatment combinations can be found as fallows.

So, once we find out it could be 0 0; that means, at their lowest level 0 1 they can be at the first one is the lowest level, second one is the middle level, it can be excuse me it can be 0 2. First as the lowest level, second the second one of the highest level, the combination can be 1 0; that means, the first one as the middle level, second one of the lowest level, it could be 1 1, where both of them the middle level it can be 1 2 when the first one is at middle level, second one is highest level and the combinations could continue.

So, if it is 2 0 the last one which you see, it will give you the level of interaction for factor A as high at like if there are 3 levels is the highest one and the effect of B which is a 0 is just the lowest level because you have considering 0 1 2 as the combinations.

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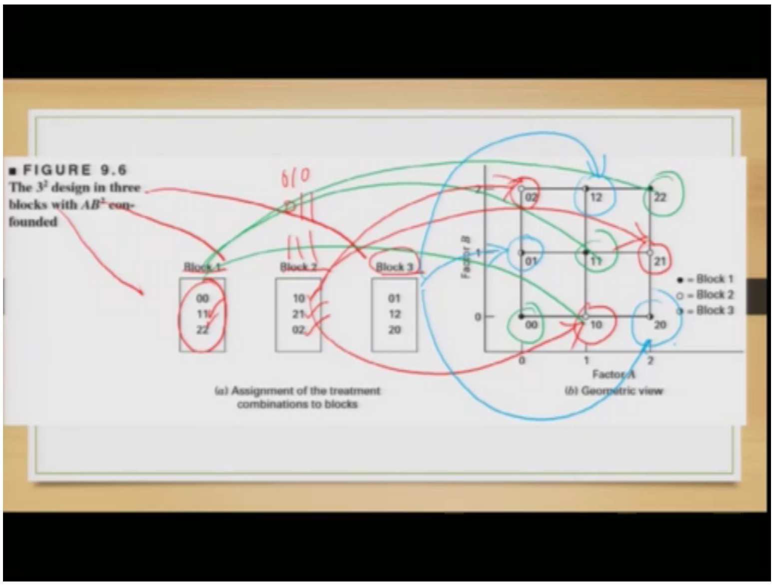

So, 3 square design in 3 blocks with A B square confounded. So, we basically consider 3 blocks. So, the block 1, so let me highlight block 1, block 2 and block 3. The combinations if you see it is 00 which is low low, 11 which is basically middle or normal 22 is high high, basically there are two combinations which are considering. So, these 3 are basically the blocks and there are two factors so that to combinations coming out here. If there are 3 factors it could basically would have been 010, 011, 111 and so on so forth.

So, this value of 10, 21, 02 basically gives the combination where the first one consecutively or respectively are middle value where my highlight it is been highlighted the pointer, high value, low value for the second factor it will be 0 1 2 which is low value middle value and high value and from block 3 which we are considering is 01, 12 and 20.

So, it will be for the first factor it will be low, middle, high and for factor 2 it would be middle, high, low.

So, the combinations we do if you take a geometric view obviously, there are two factors, also as I mentioned along the x axis you have basically you take factor A as it is given it could be have could have been factor B also. Along the horizontal and the vertical line you take the other factor and the grid structure which you have here in the Cartesian coordinates gives you the different combination.

So, the combinations points are if you notice around 00 is there, the which is belongs to block ok, by the way we are trying to basically mark this colours or the dots in same way denoting in which block the belong. So, if you consider that the diagonal one going towards the right hand corner on my right basically 00, 11 and 22 are in block 1, corresponding to block 2 you have basically 10, then you have basically 0 2 1 and basically you have to 02. So, these are for block 2. I am just using the red colour.

Then if I use 00, 11, 22 is basically block 1. So, if you consider here the red one basically for B block 2, block 3 would be 01, 12, 20. So, there they here I am just trying to basically draw the arrows in order to basically depict in which combinations can be done. It could have been done in other combinations also the blocks 1 2 3, but this is just a sample set of the calculations we are trying to do.

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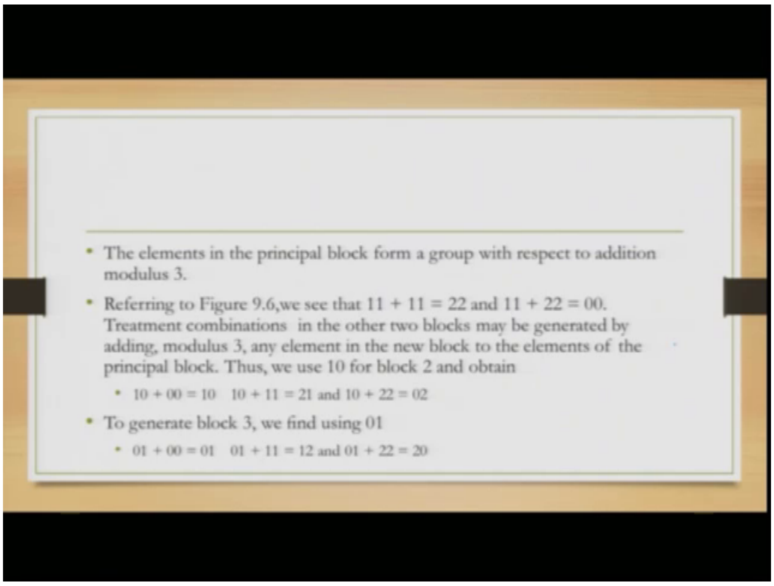

The elements in the principal block form a group with represents to addition modulus of mod 3, referring to this diagram we see that the combinations are basically 22 and 00 which is basically high and low low. Treatment combinations in the other two blocks may be generated by adding modulus 3, and any element in the new block can be addition and subtraction can be done accordingly. So, generate block 3 would basically do the combinations accordingly.

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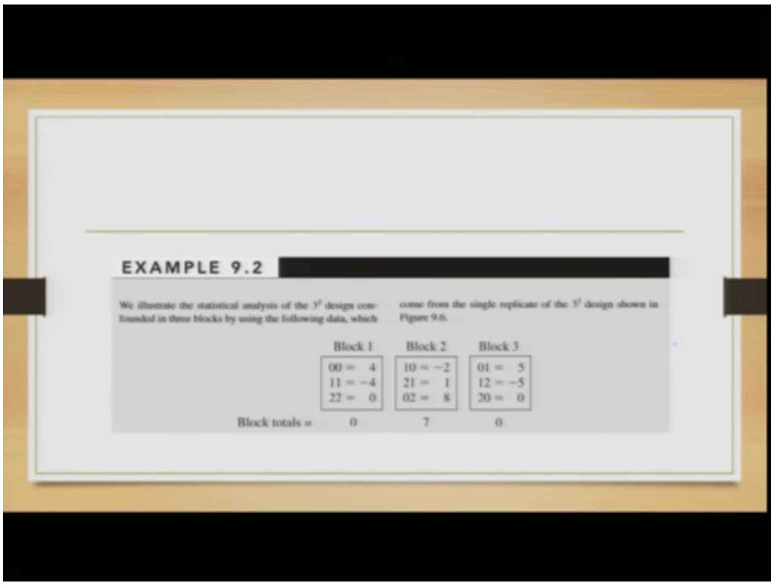

We illustrate the statistical analysis of the 3 square design confounded in 3 blocks by using the following data which comes from the single replicate of 3 square design as shown in figure 9.6. So, the block is are as I mention block 1 2 3, the combinations were 00, 11, 22 for block 1, block 2 was 10, 21, 02 and block 3 was 01, 12 and 10. If you find out the block totals basically comes out to be after finding on the blocks weights comes out to be from block 1 it is 4 minus 5 which is 0, for block 2 it is minus 2 1 and 8 which is 7, and block 3 it is again plus 5 and minus 5 it is 0, so based on that we can give the importance accordingly.

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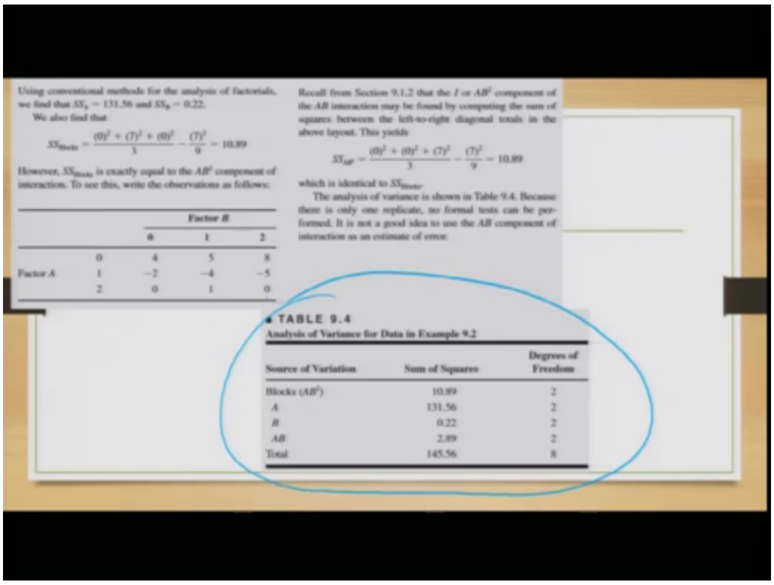

		Factor B 0	1	2
Factor A	0	4	5	8
	1	−2	−4	−5
	2	0	1	0

Source of Variation	Sum of Squares	Degrees of Freedom
Blocks (AB^2)	10.89	2
A	131.56	2
B	0.22	2
AB	2.89	2
Total	145.56	8

Using the conventional method for analysis of factorials we find out SS A which is the sum of the squares for A factors, SS B for sum of the square of the B factors, they are 135.56 and 0.22. So, if we considered the factors and the analysis the analysis of the variance are the data table are given. So, the blocks again I will only go to the final one which I trying to highlight.

On the leftmost column you have the sources of variations which are blocks A B combinations errors and the totals. The sum of the squares are given, the degrees of freedom are given, sum of the squares are given in the second last column, degrees of freedom the last column based on that equal find out f predict whether the p values are high and low and basically pass your judgment accordingly.

So, with this and with this I will end the 32nd lecture, and try to basically continue in more details about this in the 33rd and the few remaining lectures, and also the type of the whole course accordingly. Have a nice day.

Thank you very much.