Total Quality Management - II
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Lecture - 29
Two level Fractional Factorial Design – VII

Welcome back my dear students. A very good morning good afternoon and good evening to all of you, this is the TQM II subject lecture on the course and another NPTEL, MOOC. And we are in the 29th lecture; that means, we are almost at the fagend of the 6th week. So, we will have the 30th week, 30th lecture it will end the 6th week and then we will basically have another 10; 10 lectures to wrap up the course, anyway.

If you if you remember we were discussing and as you know I am Raghunandan Sengupta from the IME department, IIT, Kanpur. So, as you know we were discussing the factor fractional factors models. So, basically you want to do it combining different factors, basic variables would be there and then you basically combine them depending on their overall effect or robustness on the model they have their final combined effect which is there.

And we saw that for different examples, different concepts how you can basically do the folding; that means, you go one after the other try to basically encompass as much as an information related to the variables are possible. And take the number of variables to be the least such that you are able to predict the highest. That means, you are trying to use in a very simple laymen and terms, least amount of resources to get the maximum amount of output that is what is actually designed.

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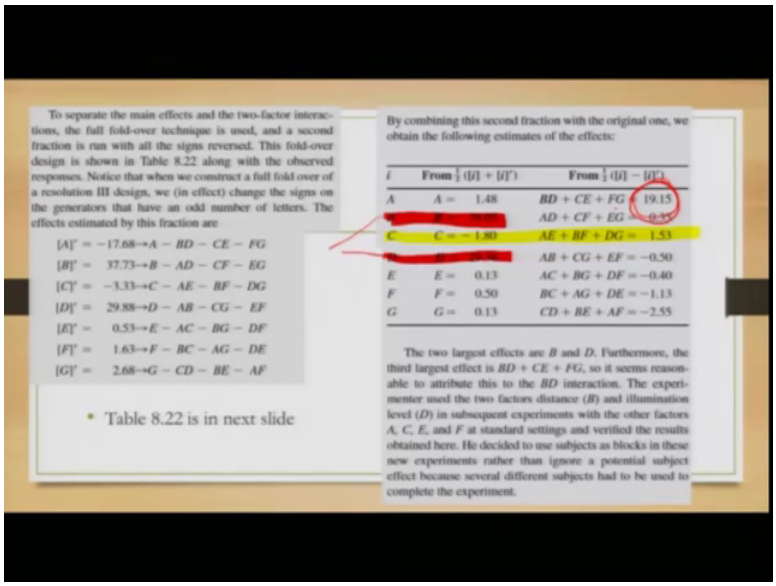

Now, considering the example. So, to separate the main effects and the two factor interaction model the full fold over techniques is used. So, you basically go one fold it like as you are wrapping up cloth or wrapping up bed sheet. So, how you do that, you are trying to basically on encompass all the characteristics of the previous set of variables.

So, to separate the main effects and the two factor interaction the full fold over technique is used and a second fraction to is run with all the signs reversed. So, basically you are trying to take the overall effect. So, in a one case you have to taken the some combinations of plus and minus and the next stream case you will basically reverse there, signs such that all the combinations are taken. Plus and minus basically saying that you are taking at the highest level influence and lowest level influence; obviously, if there more than two you will do it accordingly this fold over design along with the observed responses are shown in figure 8.2.

Now obviously, you will you will put equal weightages. If you remember that if the folds were basically two in number you will take the average half and then do your studies if it was you are doing a combinations of 3 different concepts of folds. Concepts of fold means the total number which you are getting you will take one-third; that means, you are putting equal weightage in the first case half then you are putting equal weightages in the second case one-third, one-third, one-third then it will go into one-fourth, one-fourth, one-fourth, one-fourth and continue.

Notice that when we in construct of full fold over the resolution of 3 design we in effect change the science, on the generators that have an odd numbers of effects and then you do it for the even number. The effects estimated for this factions which we have seen in the overall values for the resolution of the problem on the concept the problem which your taking depending on the optometrist of the ophthalmologist for the eye resolution how far the object is what is the resolution what is the size of the object, so and hence forth.

So, we get the values of the combine factors which will mention as a prime to G prime the values are coming as minus 17.68 to 2.68. So, by combining this second fractional the original ones we obtain the estimates of the effect. So, we had already go got the first set of effects which we discuss on the last end last minute of the class which was the 28th one and the 29th one we discuss the other fold depending on sign change. And when we considered that we take the averages of this values if you remember the average values are coming out to be. So, consider them both plus and minus the values are I will just read it. So, for a for the first row a it will be 1.48 when it is positive and one 19.15 it is negative still the last value which we considered as G it is 0.13 for positive minus 2.55 it is negative.

Now, if we consider any one of these any one of these like you are considering the third row. So, in effect we are we are considering that the combine effect on both on the positive and negative side considering more positive effect and less positive effect the values or the factors were like this like if we consider C as the factor of the variable then the so called so called the opposite side effect which is coming from the combination of not combining C would be considered as A E plus B F plus D G.

So, this basically you are trying to co have a complimentary sets such that the effects are not equally divided, but the effects are categorized according to the effects where, so for example, the overall effect from one set of factors I am not using C. One set of factors decreases, so other one increases and in the other case when you are basically taking the fold I like two complementary sets the other set will decrease then the first one will increase. If they are 3, so the combinations would be done accordingly and we proceed.

The two largest effects are basically B and D as we see. So, I will just mark it accordingly. So, B the values are B is 38.05 and D is 29.3 8. So, these are the values to

largest effects are B and D where my cursor is now hovering. Further the third largest effects which we see is basically B D C and F G with comes out to be 19.15. So, this is the third largest value. So, it seems reasonable to attribute this to the B and D effects. So, considering that the ranking of ranking means what are the effects based on which we consider the overall values.

The experimenter use the two factors distance which if you remember and go back to the original slide where the distance resolution sharpness size of the objects were basically considered. So, here the experimenter use the two factors distance B and illumination D in the subsequent experiment with other factors which your A C E F and found on the effects accordingly.

He or she decided to use subjects as blocks in this nee experiment rather than ignore a potential subject effect because several different subjects had to be used to complete the experiment. So, basically rather than considering them an on a single ma individual level they were considering blocks and then found the effect depending on the different type of experiments available data say for their because people are coming or be objects were been taken to the study. So, basically divided in to B and D and other set of variables accordingly and did the so called combined effect and found out there effect in the in the combines level.

So, again coming back to the fold over design, but if you remember we had 7 factors which was key was 7 and hence it is 2 to the powers k minus p that is 7 minus 4 and the third level the design model for the eye focus experiment.

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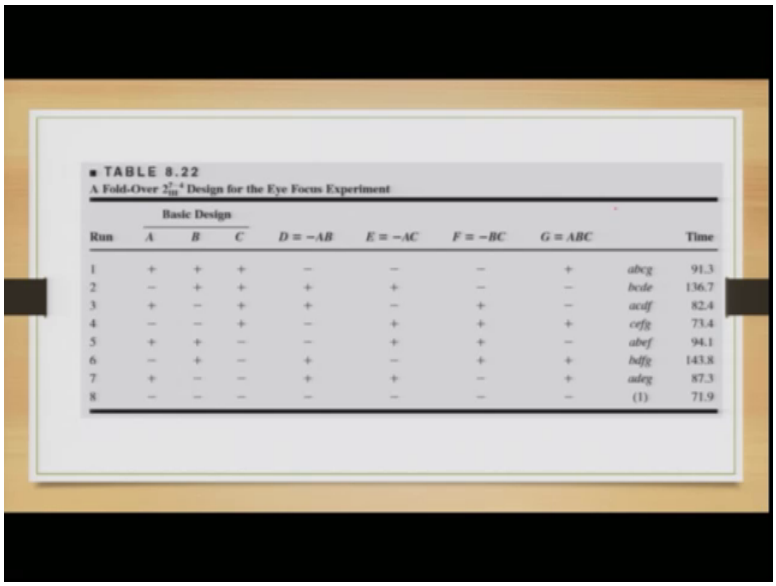

■ TABLE 8.22
A Fold-Over 2^{7-4}_{III} Design for the Eye Focus Experiment

	Basic Design								
Run	A	B	C	$D = -AB$	$E = -AC$	$F = -BC$	$G = ABC$		Time
1	+	+	+	−	−	−	+	*abcg*	91.3
2	−	+	+	+	+	−	−	*bcde*	136.7
3	+	−	+	+	−	+	−	*acdf*	82.4
4	−	−	+	−	+	+	+	*cefg*	73.4
5	+	+	−	−	+	+	−	*abef*	94.1
6	−	+	−	+	−	+	+	*bdfg*	143.8
7	+	−	−	+	+	−	+	*adeg*	87.3
8	−	−	−	−	−	−	−	(1)	71.9

If you consider the first this is same concept if you consider the first second third 4th row accordingly there are the run numbers then you have the basic designs as A B C and the and the combined him factors which we are considering are D E F and G. And then you find out the time which is given in the last, time means time to in milliseconds if you if you consider though time for this factors and then you come ah and then the times are coming out to be 91.31, 36.7 till the second last one is 87.3 and 71.9. And the combined factors were coming out to be let me read it is A B C G for the first one B C D E for the second one and so on and hence forth for the last one was A D E G.

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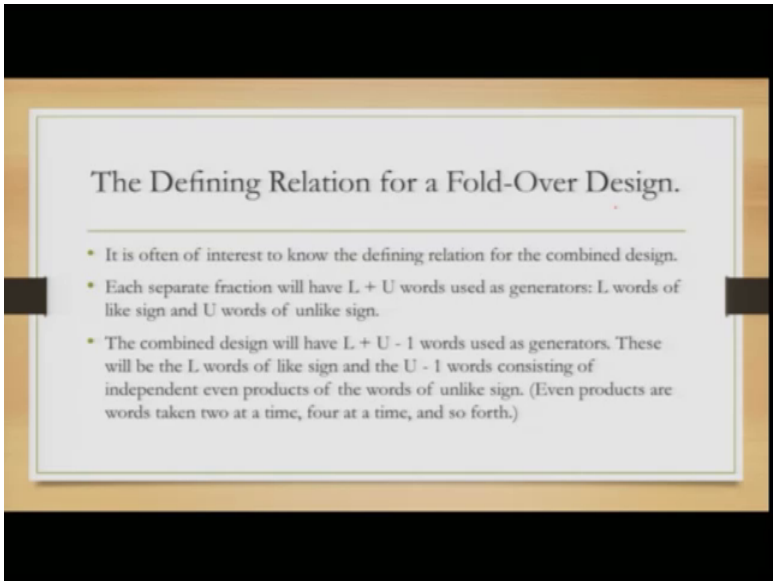

The defining relationship for a fold over design. So, we will just go into that it is often of interest to know the defining relationship of the combined design factors and what is the overall effect each separate fractions will have the words L plus U words used as generators. So, L words would basically words of the look like the sign and U words would be used for the unlike science so, so called, so called opposite science; that means, it may not be plus minus it can be other science also, but they are normal the science would be used of the symbols U and L would be used for those sets where you have high level of similarity and for the sets were there low level of similarities.

The combined design will have L plus U minus one words used as generator, but; obviously, we know that it is N minus capital N minus 1 based on the experimental design which we have. This will be the L words of like sign and U minus words consisting of independent even products of the words of unlike signs. So, even products are words taken 2 at a time, 4 at a time and so on and so forth and we will basically find out the combinations accordingly.

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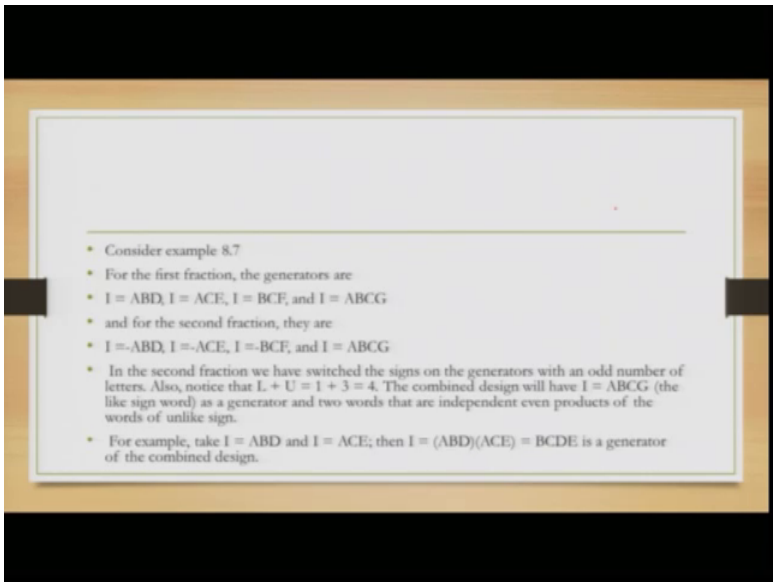

Consider for example 8.7 for the first fraction the generators are as you mention I would be A D B, and then the next generator would be A C E, others would be B C F and A B C G. And for the second fractions they are and; obviously, the negative would be taken when in D if the generators are A B D minus A C E minus is the overall effects which is in the opposite direction.

So, we will basically it consider the L L U combinations which you are talking about just few seconds back. Then other one would we B C F minus and A B C G the in the second fraction we have switched the signs on the generators with an odd number of letters also known note that L plus U is basically given by 1 plus 3 is equal to that 4, so obviously will take 4 minus 1 combinations. The combined effect design will be for A B C G as a generator and two words that are independent even products of the words of unlike signs and unlike signs would be taken for example, if you take the generators as A B D A C E then the combined effect would be B C D E as a generator which can basically give you the full set of information.

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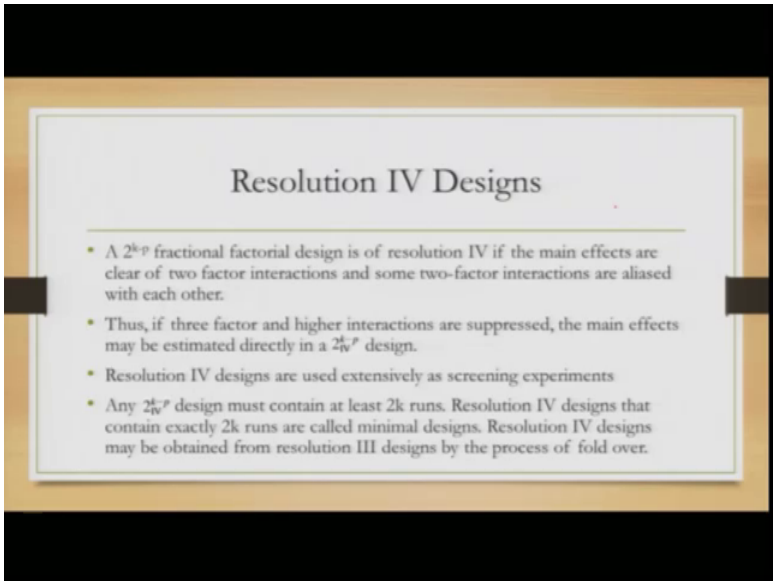

Then resolution of the 4th design. So, consider for the two fact fractional factor model which is 2 to the power k minus p which you have been discussing time and again. So, this fractional factorial design is of resolution 4 which means if the main effects are clear of two factor interaction and some two factor interactions are analyzed with each other and we can solve the problem.

Thus, if you have 3 factor and higher interactions are suppressed the main effects may be estimated directly on the model level of 2 to the power k minus p on the suffix and the 4 design one to the resolution being on the on the on the subscript. So, resolution 4 designs are used extinguishing extensively as screening experiments based on which we analyze what are the next combinations will consider. So, any 2 to the power k minus p design

must cont cons contain at least 2 to the power k, 2 into k runs so obviously, it will mean that we are taking those combinations in this effects such that we have the minimum set of informations available for us. So, resolutions 4 design that contain exactly 2 k runs are called minimal design. So, this is the minimum number of such runs we can have in order to get the effects described in the least number of such the combinations, so obviously, if we go for higher levels of combinations; obviously, we will get better results, but anything less would not be tenable.

Resolutions of 4 designs may be obtained from resolution 3 designs by the process of fold over. So, if you remember we are going from the second to the third, third to the forth we basically fold over and take their maximum amount of information based on which we proceed.

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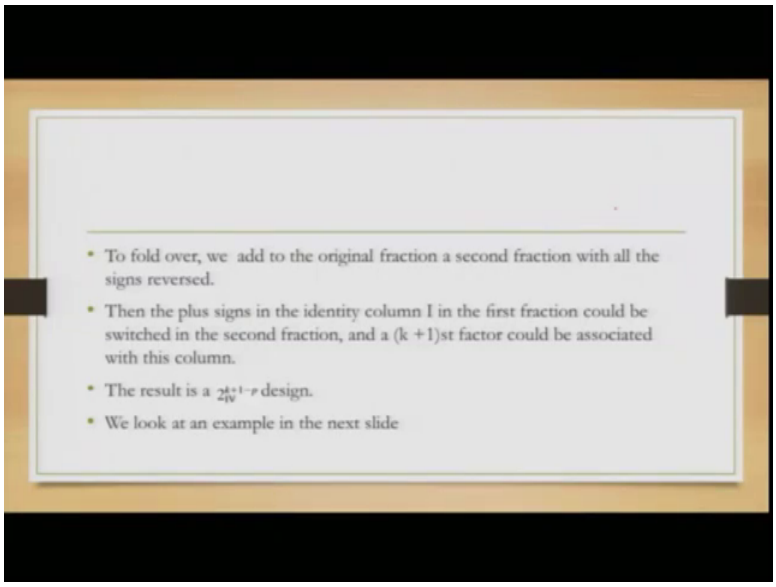

To fold over we add to the original fraction or the second fraction with all the signs reversed. So, if we remember as you are doing the folds will basically do the combinations on the plus and minus sign and continue doing that that such that we get the maximum set of information. Then the plus sign in the iden and in the identic column for the generators is we would basically the first fraction that could be switched in the second fraction and in this way we basically find out the different combinations of the plus and minus which can be done.

So, this would result in an design of 2 to the power k mi k plus 1 minus p in the superscript and in the subscript you will basically have 4 which is with the level of design which you are basically considering. Sorry that I have been using the word prefix and suffix is basically superscript way to the power, 2 to the power which basically gives you the different design variables which are there and the level of such resolutions and the design level to which the resolutions would be basically mapped is basically the subscripts which you are basically considering.

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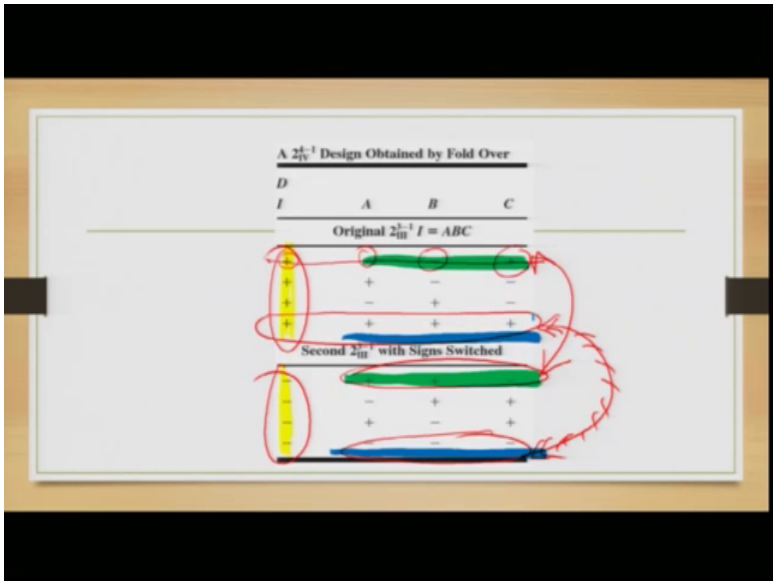

So, let us look at an example in the next slide. So, we have design obtained by fold over which is given by 2 to the power 4 minus 1 and design level of 4. And here the variables are A B C. So, basically A B C are the basic design variables based on which you are trying to proceed. So, here the original one was basically 2 to the power 3 minus 1 because 3 was basically the number of variables which you had which is k. So, which are resolution of 3 and the generator was A B C. So, the combinations were given with the generator was which gave us the best factor.

So, let us consider the first row. So, this was plus, so indicator of the generator was plus effect this was A was minus B 1 minus C was plus, similarly if I go to the last row. So, they were all positive; that means, they were at the highest level of interaction in the their effect on the on the decision making process.

Now, with signs switched services signs which would means that each combinations of the of the signs which we had earlier would now we just we reversed; that means, if they were positive if they were would be negative and vice versa; that means, the overall effects it was high, it would be now low and considering that. So, in this effects the signs of all the all the generator or indicator I would basically now B minus. So, as you see here.

And the combinations of the signs for A B C would just be reverse. So, if you had initially minus and minus and plus for C. So, that they are now plus and minus similarly for the last column last row if you had plus. So, they are basically minus minus minus. So, they are basically a complementary of each other. So, I will just make a note and this is what you had in this we had for indicator, this is for the indicator then this we had for A B C combined with A B C in the switched over this we had this is for the switched over this is for the original one. So, this just a simple concept which you are trying to utilize.

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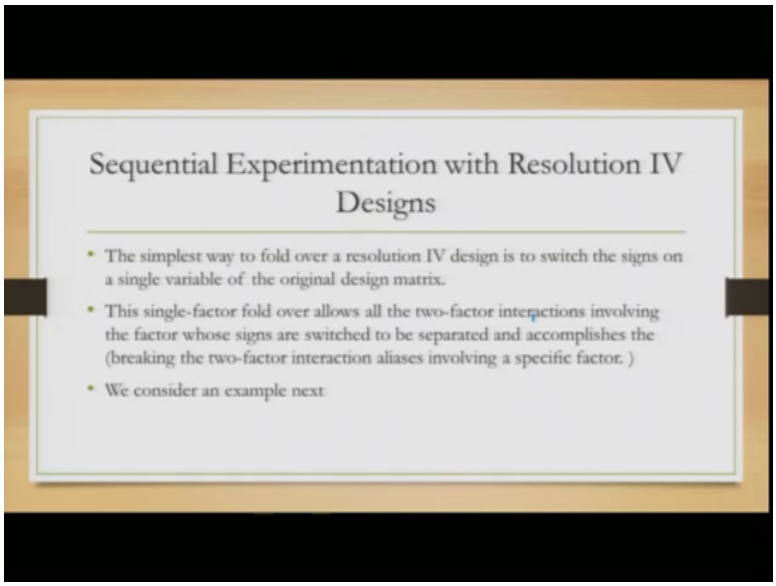

Now, sequential experimentation with resolution 4 designs. So, the simplest way to fold over resolution 4 design is to switch the signs on a single variable or the original design matrix and do it for all the variables. The single factor fold over allows all the two factor interaction involving factors of whose signs are switched to be separated and accomplished an and basically accomplishes the the overall breaking of the two factors

into their and there so called positive and negative effect we will consider that accordingly.

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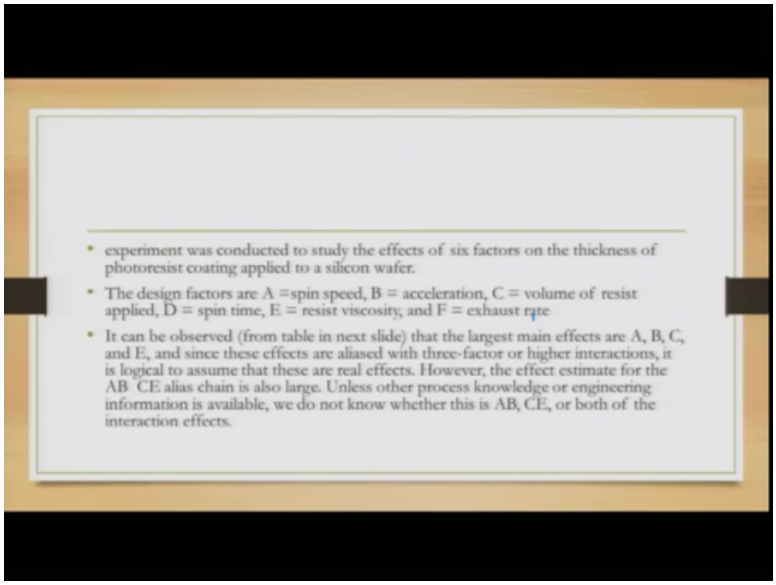

The experiment was conducted to study the effect of 6 factors on the thickness of photoresist coating applied to a silicon wafer the design factors are basically A B C is what we consider spin speed, acceleration, volume resist applied, spin time resist resistance viscosity and the exhausted. It can be observed from the table that the large largest main and main effects being basically A B C E and since these effects are analyzed with 3 factor or higher interactions it is logical to assume that these are real effects.

However, the effect estimate for the combinations of A B C analysis chain is also large. So, it will basically continue depending on the different combinations which we have. Unless other process knowledge or engineering information is available we do not know whether this which is A B C E or both of the interactions can be done accordingly.

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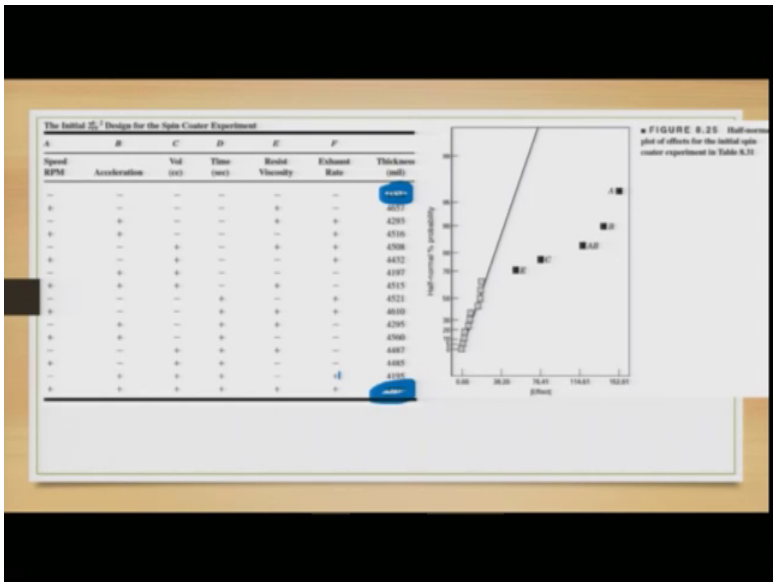

So, the initial two to the power because there were 6 independent factors, which was A B C D E F and so hence it will be 2 to the power k minus 2, so k is 6. And the level of design was 4, so 2 to the power 6 minus 2 and the subscript was 4 superscript 2 to the power k k minus p which is 6 minus 2, so design on the spin coating experiment is given, we have the speed. So, A was the speed, so different combination sum of high effect low effect are combined together.

Similarly if you go to B, if you go to C, if you go to D, if you go to E, if you go to F. So, which on acceleration volume time volume in C C, time in seconds the viscosity of resistance. So, arises is viscosity consider like that and the exhausted. And thickness in millimeters are given starting from for say for example, for the combination when they at the lowest level minus the value is 4 5 2 4 and where their highest level the values 4 5 1 0. So, I am only reading the first value and the last value.

So, combinations are given. So, half normal plots of the effects of initial spin coating experiment is given which means that we have basically trying to find out and fit in normal distribution and check whether the overall distribution effects combined effects are normally distributed with a certain mean and the certain variance.

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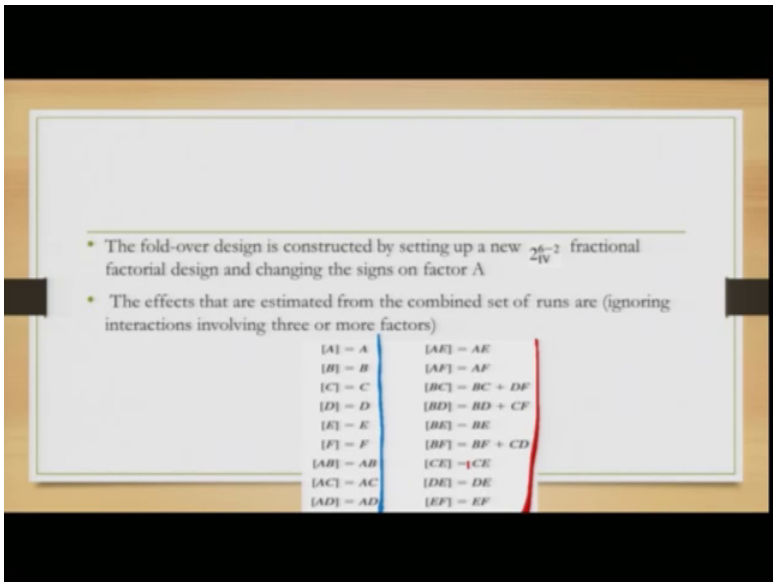

The fold over design is constructed by setting up a new model when initially we had considered 2 to 2 to the power 6 minus 2 and the design was of third level, but now you are considering the fourth level. So, the fractional factorial designs is considered 2 to the power 6 minus 2 and for 4 design and changing the sign of the factor of a will basically come achieve that.

The effects that are estimated for the combined set of runs are, so we are in ignoring international involving 3 or more factors. So, basically they would be combined effect would be A and A E then B E and A F. So, we have basically what I am talking about is the two sets at would which level you will consider, one set or one side or one bucket of affects would be A B C D E F, A B, A C and A D I am reading these values and in the other set effects will consider these values. So, these values are A E, A F, B C plus D F. So, you have basically taking the combination B D, C F, C F again the combinations then B E, B F plus C D, C E, D E and E F. So, they are considering them as combine factors which will give us the overall effect.

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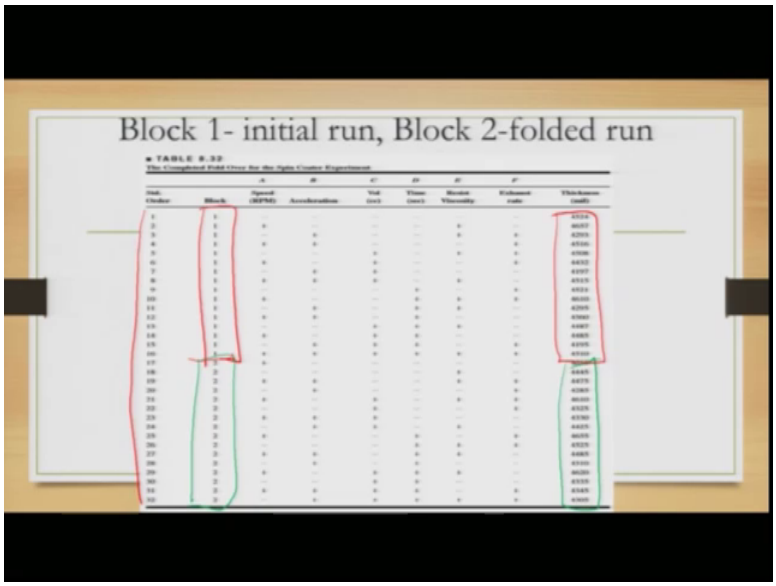

So, for the block one initial run. So, so (Refer Time: 22:04) are the effects which you mentioned was basically one was block 1, one was block 2. So, the block 1 and block 2 ah this initial runs on the block 2 folded runs are given. So, sorry for this is resolution because is very intake very I will not use the what cluttered, but a lot of information set is here. So, here the standard orders of the runs was basically given from 1 to 32 which is these values then you have the block. So, the blocks would basically the first set would be one as mentioned here, and the second one let me change the colour would be 2.

So, if I have the effects, so effects are basically the factors are the same initially the, so call if I considered the basic factors or basic variable based on which you are proceeding. So, there as A B C D E F if you read the top most row as I take my cursor which is again I am repeating A which is first speed B is for acceleration C is for the volume in C C, D is in basically time in seconds, E is the resistance of the viscosity on for the resistance and F is exhausted and the thickness are given. So, if we remember are the thickness are for given initi and the here there given for one set which is basically for block. So, say for example, for the block 2, 2 means the second combination which you are taking, one means for the first combination we are taking.

So, if you consider this is the first block and if you partition them. So, all the different combinations are plus and minus are taken for block 1 and block 2; that means, from

block 1 for the initial run, block 2 we are trying to basically fold and considered their combined effect.

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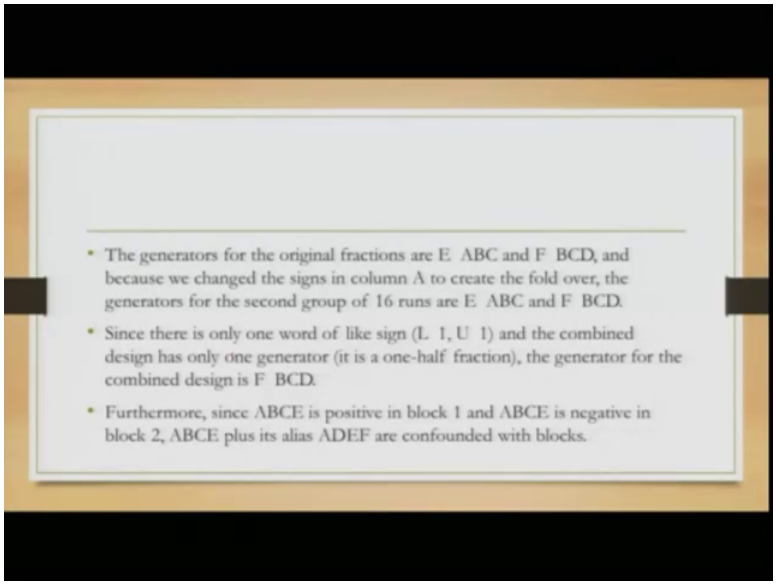

The generators for the original fractions are E A B C and F B C D and because we changed the signs in the column A to create the folds. So, column A basically was the initial set of variables which we had the generators for the second group of the 16 runs. Because if you remember the one 32, 32 runs, so we have been equally divided into 16 for block 1 and 16 for block 2 which is the for the fold one. So, the generators let me continue reading it the generators for the second group of 16 runs are E A B C and F and B C B C D since there is only one word of like signs.

So, basically L n if you remember for like signs and unlike signs and the combined design has only one generator. So, the generator for the combined is basically F and B C D. Furthermore since A B C is positive in block 1 and A B C is negative. So, if we check the selected table it will give you the signs accordingly for A B C E. So, hence A B C puts it is alias A B D A B E F, A D E F as compounded with blocks and basically find their overall effect, done in such a way that the complement each other and give us the maximum set of information based on the predictive power because your main task is to predict as high as possible.

So, here we generally we use 16 additional runs, but only gained 5 additional degrees of freedom for estimating two factors interaction. This is ah is not a terrible efficient use of experimental resources.

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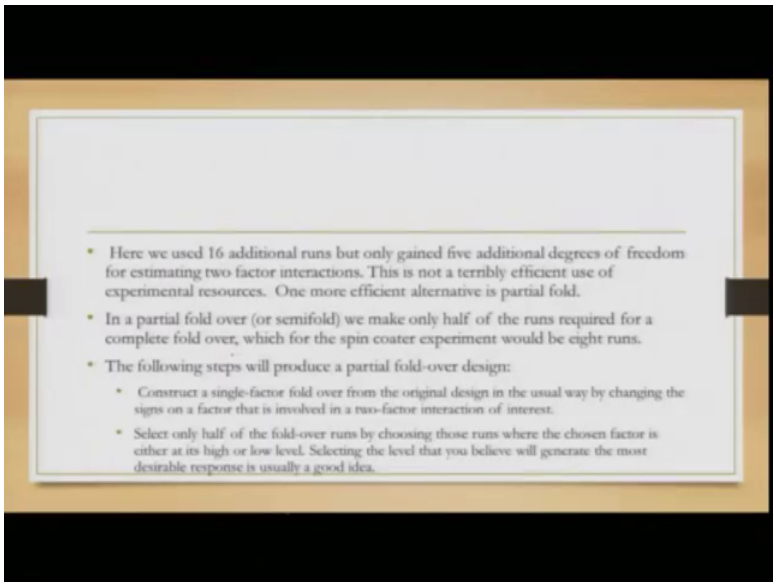

So; obviously, you can have a better or words depending on different type of combinations of variables you are taking. In a partial fold over or semi fold we make only one half of the runs required for a complete fold over which for the spin coater experiment would be 8 runs. So, depending on half of the runs if you have basically 16 half of them for block 1 or half of them for block 2 would be half of 16 which is 8.

The following steps will be will produce a partial run over design. So, construct a single fold factor fold over from the original design in the usual way by changing the signs on the factor that is involved in the two fold models. Select only half of the fold over runs by choosing those runs where the chosen factors is either at its highest level or the lowest level. Selecting the level that you believe will generate the most desirable response is usually a good idea and based on that we can basically form like the model accordingly.

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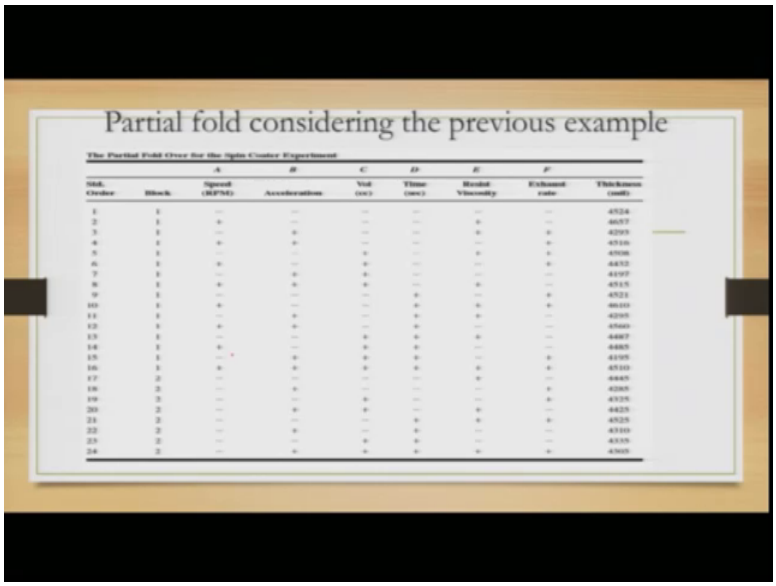

So, the partial folds considering the previous example. So, you now initial you have considered the full fold that me a basically trying to encompass the maximum set of information now you are doing half of them. So, the readings are basically again on the first column or from one to third, third 24. So, you have basically reduced them.

So, the blocks again are in the one remain the same the first block remains the same, the second one you have reduced them by half and again the A B C D E F which as speed, acceleration, volume, time and resistance viscosity and exhausted are basically combined in the plus and minus sign in order to find out the effects and the thickness which is given in millimeters is the actual value based on which we are going to do our experiment.

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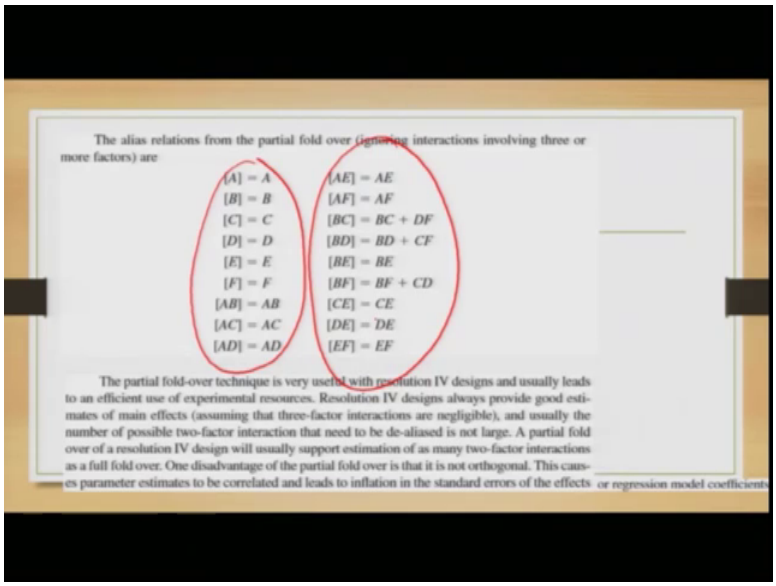

So, the alias relations from the partial folds initially (Refer Time: 27:45) done the full fold and based on that we had the block 2. The partial folds again we divide into blocks the blocks are if you do if you if you consider this for the first one starts from A to A D, for the second one basically goes from A E to E F. I am just reading, so that you understand whether the blocks are.

The partial fold over technique is very useful when resolution of 4 designs is used and usually leads to an efficient use of experimental resources because money, time, energy is important. So, you have to basically find out the maximum effect in the minimum time and minimum resources. Resolution 4 designs always provide good estimates or the main effects and usually the number of possible two factor interaction that need to be de-aliased are at the largest level. A partial fold of resolution 4 design would easily support estimation of as many two factor interaction as possible and the 4 models.

So, one advantage of the partial fold over is that it is not orthogonal; that means, we are basically not taking one to one correspondence from them. This causes a parameter estimates to be correlated and leads to inflation of the set of the data of the information which we have.

So, with this I will I can close this 29 lecture and continue discussion about this fraction factorial models in more details, and the folds and the semi folds later on in the later class. Have a nice day.

Thank you very much.