Total Quality Management - II
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Lecture - 24
Two level Fractional Factorial Design – II

Good morning, good afternoon, good evening my dear friends and participants for this course, I am Raghunandan Sengupta from the IME Department IIT Kanpur. And you are all taking this TQM - II course under the NPTEL MOOC lecture series. And this is the 24th lecture and if you remember that at the last end fag end of the 23rd lecture I was discussing that for the fractional factors how you can reduce the overall number of significance of calculations and then based on that you proceed.

So, we consider A separately where fact effects of B and C are negative and then we considered B separately where the effects of A and C are negative, then we want to combination, so A B and then and so on and so forth.

And I also mentioned that how you can analyze these problems on A very simple diagram of the size of either A square or A cube only for the level of 3, and then 3 dimension because for higher dimension it will be difficult for us to understand.

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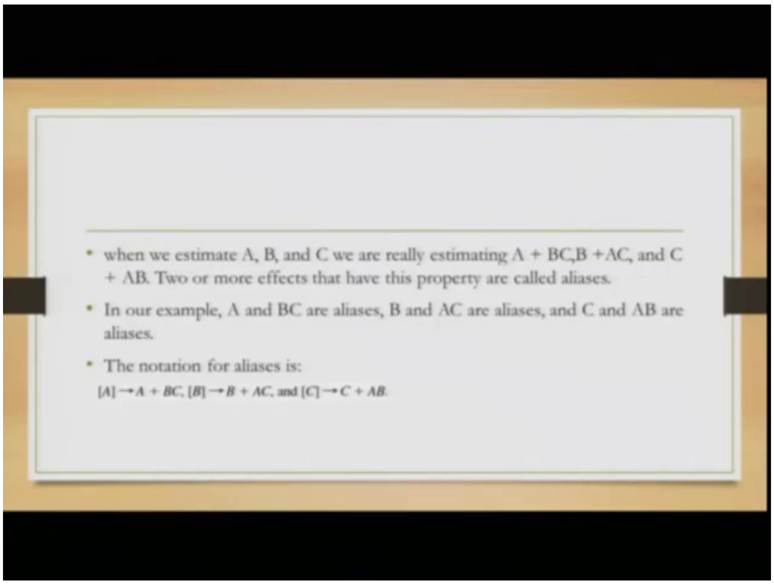

So, let us continue the discussions further. So, when we estimate ABC we are really estimating the combinations like this. We are want to find out A plus the combinations effects of B and C, when we are trying to find out the B it is B effect plus AC and when it is only C it is C effect plus A B.

So, 2 or more effects that have this property are called the aliases and based on the aliases, we can find out the level of significance which is there in our example A B C are combined together; aliases B and ac are 1 set and finally, C and A B are another set.

The notions of these combinations are given as A is related or find found out by the combinations of A plus BC, B is found out the combination of B plus AC and C is found out by the combinations of C plus AB.

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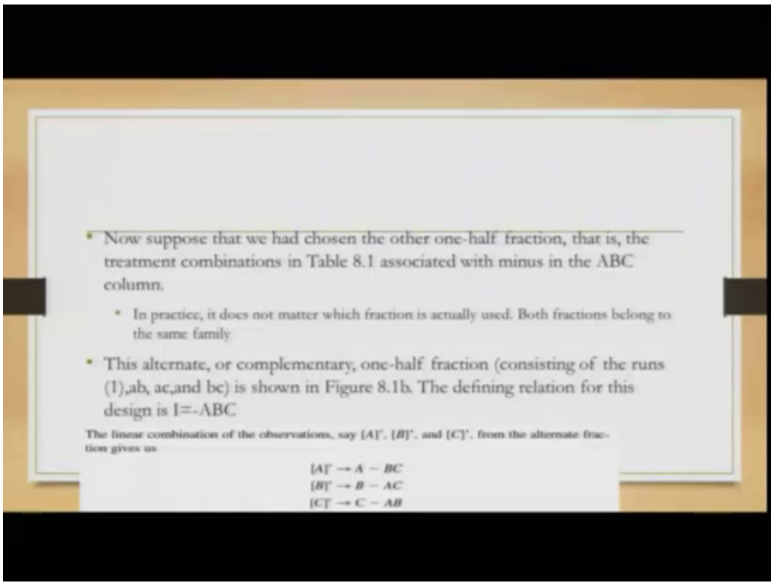

Now, suppose that we have chosen the other 1 half of the fraction, that is the treatment is combined in such A way that we have the table 8.1 associated with minus in the ABC column. So, in practice it does not matter which fraction is usually used. So, the both the fractions belong to the same family so, based on that we can do the calculations.

This alternative or complementary 1 half fraction consisting of the runs of I or AB or AC and BC are shown and this defines the combination such that you can find on the overall effect of the I or the identity values for the combination of ABC if there are 3 factors the linear combination on the absorptions are given as A prime, B prime, and C prime. So, A prime is basically combinations, which gives you the effects of A and the effects not coming from B and C because B and C does not have the effects of A as such. For B prime it will be and without A C and for C prime it will be C without A B.

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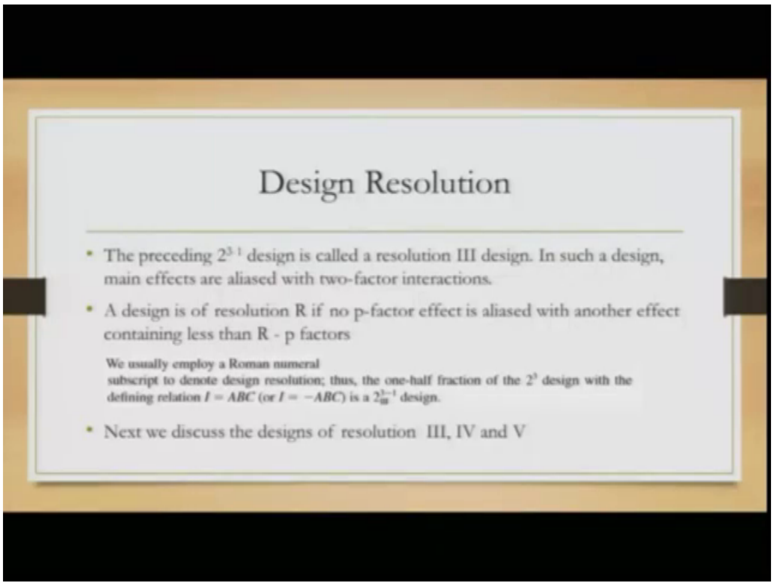

So, design and resolution the preceding 2 to the power 3 minus 1 design is called the resolution of the third degree or third design. In such a design, main effects are analyzed with 2 factors interactions. And combinations can be done accordingly for A higher factor for 4 factors ABC we can do it for A combination of 3 and lower levels for A fifth factor we can do it for combinations of fourth third 2 and, but obviously, what would be significant would depend on what is the level of affects you want to have. A design is of resolution R if no p factor effects is aliased with another effect continuing less than R minus p factor. So, these are the levels of the values which you have.

We usually employee A roman numeral for the third design, fourth design, fifth design, whatever it is, subscript to denote the design resolution thus 1 half fraction or 2 to the power 3 design with would be defined in such a way it will be basically 2 to the power 3 minus 1, on the superscript and the subscript would be roman 3 considering the reservation of 3 design which we have for our example.

Next we design the design of the resolution of the fourth design factor, fifth design factor, and the sixth design factor we can do it accordingly.

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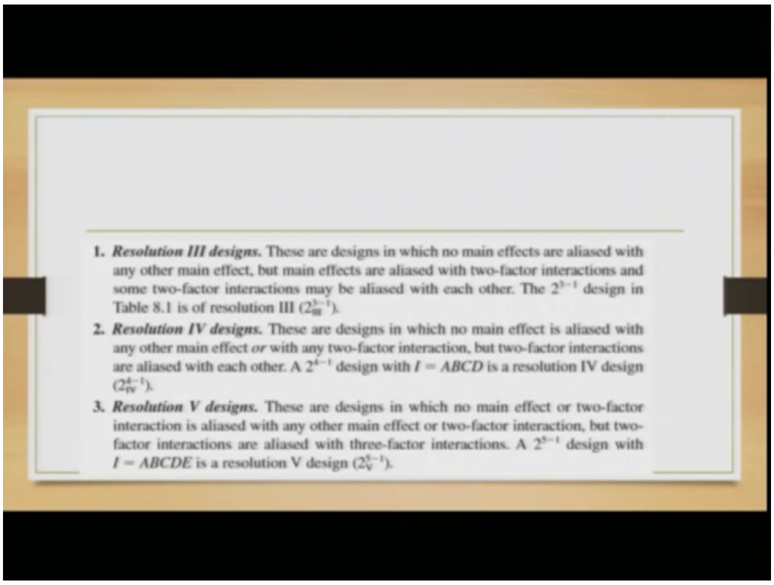

Resolutions of the 3 design so, these are designs in which no effects are aliased with any other main effect, but main effects are aliased with 2 factor interaction and some 2 factor interactions which are there, which may be aliased with each other. So, so the effect of 2 to the power 3 minus 1 design is given in table 8.1 if which is of the resolution 3. So, hence you will have basically 2 to 3 minus 1 and sub subscript roman 3.

So, resolutions of 4 design. So, they are design which no main effect is aliased with any other main effect or with any 2 factor interactions are not combined. So, what you are trying to do is that for A level of 3, where you have ABC you will break down into small subsets and find out the fx combined together in such a way that the overall effect is basically mirrored such that the combinations give you the best resolution of the effect on A 3 design factor. If you have 4 design factor such that you basically want to go to a high level of significance.

So, that can be done in 2 ways number 1 where you have the effects of ABCD. So, based on the ABCD you can basically have the effects such that the combination of A B C D taken 3 at a time, would give you the first level of significance, then ABCD taken 2 at a time would give the second level of significance and then you can basically find all the single effects. And combine them for the single effect, the double effect, and the triple effect in such a way that the overall significance comes out with the maximum possible extent.

So; obviously, in that case the resolution or the subscript notation would be given as 2 to the power 4 minus 1 suffix 4. So, the 4 minus 1 would give you the number of combinations you are doing and the design factor would be given by the subscript, which is basically that is superscript will be given by 4 minus 1 as I mentioned. And the subscript would be given by the roman number 4, such that it will mimic to what level of significance you want the experimenter to do the experiment or find out the resolution.

For resolutions of 5 designs so, these are the design factors in which you will have 2 to the power subscript being it will be roman 5, which because it is of the level of 5 degrees; 5 degrees is not the degrees of significance it is basically the 5 design experiment which you are going to do? And the superscript would be 5 minus 1. So, it will give you that 2 to the power 4 or the total number combination based on which you can find out the effects to the maximum possible level.

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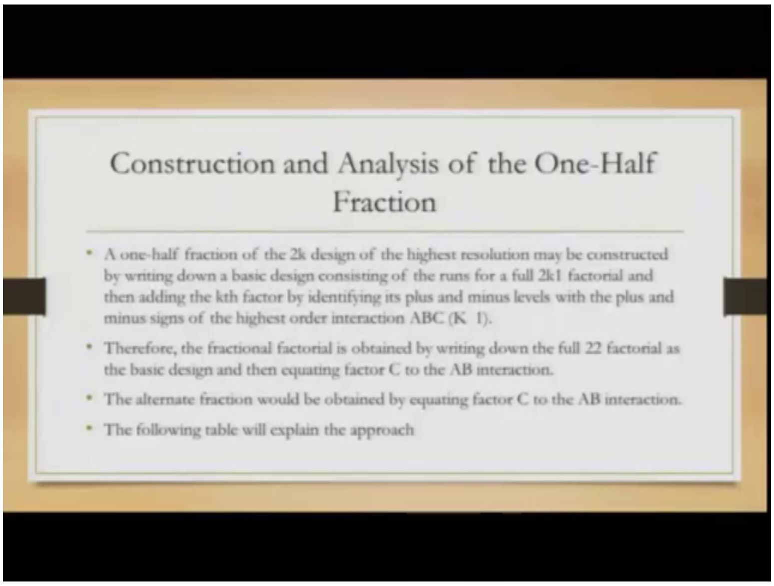

Construction and analysis of the one-half fractions so, a one a fraction of the to the bar or 2 to the power K design of the highest resolution may be constructed by writing down a basic design consisting of the runs of those factors of 2 to the power K minus 1, which is the fractional fractions and then adding the k th factor by identifying it is plus and minus levels and then adding the pluses and minus in such a way that you find out the overall effect and what is the level of significance.

So, if you remember in the table or the chart, on the leftmost column you will have all the factors and the topmost row you will have all the factors written separately. So, what you do is that you find out the combination of A S effects on A on B on C on D, if there are 4 factors. Then try to find out the effects of A on A B effect of A on BC effect of A on CD and continue in such a way such that you will give the plus and minus sign to basically denote the level of significance. One you have the level of significance this is not numeric value this is the plus and minus sign one you once you are sure about this level of significance you add them up in your overall factorial design such that you get the effect of the maximum possible way which is possible.

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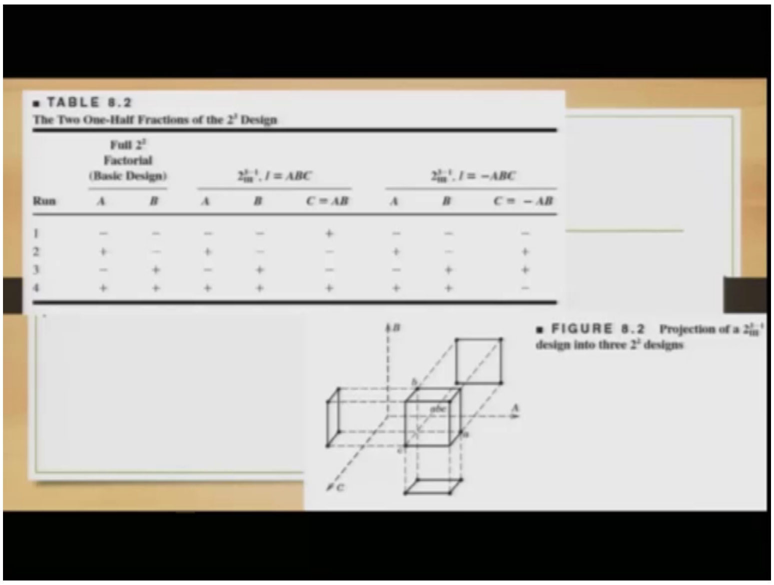

■ TABLE 8.2
The Two One-Half Fractions of the 2^3 Design

	Full 2^2 Factorial (Basic Design)		2^{3-1}_{III}, $I = ABC$			2^{3-1}_{III}, $I = -ABC$		
Run	A	B	A	B	$C = AB$	A	B	$C = -AB$
1	−	−	−	−	+	−	−	−
2	+	−	+	−	−	+	−	+
3	−	+	−	+	−	−	+	+
4	+	+	+	+	+	+	+	−

■ FIGURE 8.2 Projection of a 2^{3-1}_{III} design into three 2^2 designs

So, the 2 half fraction of the to the power 3 design problems are given like this. For runs which is given on our leftmost column which is 1 2 3 4 and the full to the power square factorial basic designs are given which you have basically A B combined together. We have basically to the power 3 minus 1 suffix numeric 3, which is given in the second block and to the power 3, which is given in my mind to the power 3 minus 1 and by 3 which is given in the third block basically, signifies the levels accordingly.

You have the effects of A which is basically if you go through the columns, which is minus plus minus plus then you have the effects of B, which is basically given by of A minus minus which signifies the level of the runs for 1 and 2 is negative for 3 and 4 it is positive.

Similarly when I go to the combinations of 2 to the power 3 minus 1 suffix numeric roman 3 and 2 to the power 3 minus 1 suffix numeric roman 3 with an effect where I the identity fact factor is ABC and in other case is negative, then you basically have the all the plus and minus signs given accordingly.

So, if I go back to the figure which is given here it is exactly what I said a few minutes back. So, what you have we are trying to basically combine ABC s in which direction the orthogonals will go is immaterial. So, long they are 90 degrees to each other. So, consider that you are measuring in this diagram B along the vertical, which is top and if you are considering C which is coming from your end to mine end. And if you are considering basically A because you consider the camera or you are the origin and if you are considering A, which will be going from your side to the left hand side would be A.

So, when I am basically do A mirror image of the combination of say for example, C and A it will basically the combination of C n A would be done in such A way that you will find out the effect of B is not there, when I try to find out the effect of A and B it will be on your side on through the wall.

So, as the C is not there and when in that case the initial example when I am considering the effect of C n A it will be the floor, because the B effect would not be there and when I am trying to find out the effect of B and C it will be the combination on the left wall without any effect of A.

So; obviously, you can break them into 3 orthogonal planes and find out the combinations accordingly.

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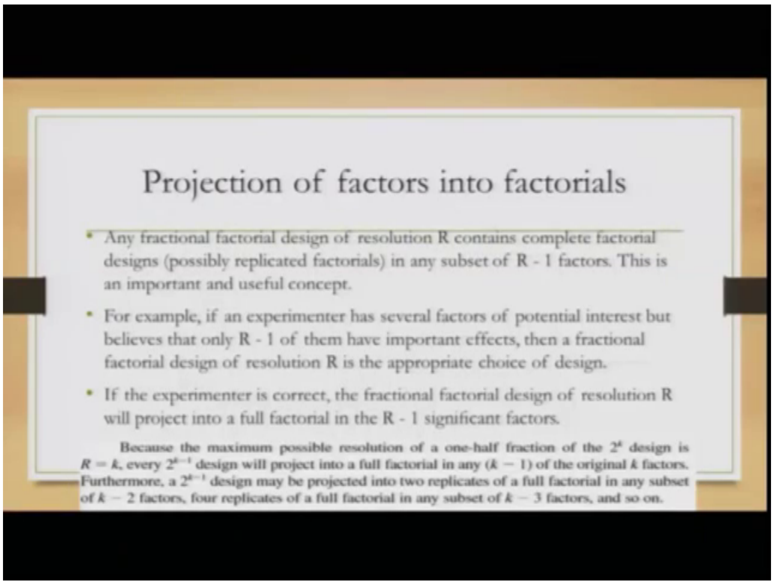

So, projections of A factors in the factorial. So, any fractional designs of resolution are contains complete factorial designs. So, possibly to replicate that in A subset of R minus 1. So, it can be done in such a combination where you basically add and subtract the combinations accordingly to find out the level of significance.

So, this is important and a very useful concept for example, if an experimenter has several factors of potential interest, but believes that only R minus 1 of them an important effects, then a fractional factorial design of resolution R it would be appropriate in order to basically bring the level of significance to that level which the experimenter wants.

So, if the experiment is correct the fractional factorial design of resolution R will project into a full factorial in R minus 1 significant factors, because the maximum possible resolution of 1 half fractions, which is 2 to the power K can be designed in such a way that the combination and can be taken together. Which means that furthermore as we take the design it may be projected onto 2 replicates, 3 replicas, or 4 replicates depending on the level of significance on the efficiency front with the experimenter wants to do.

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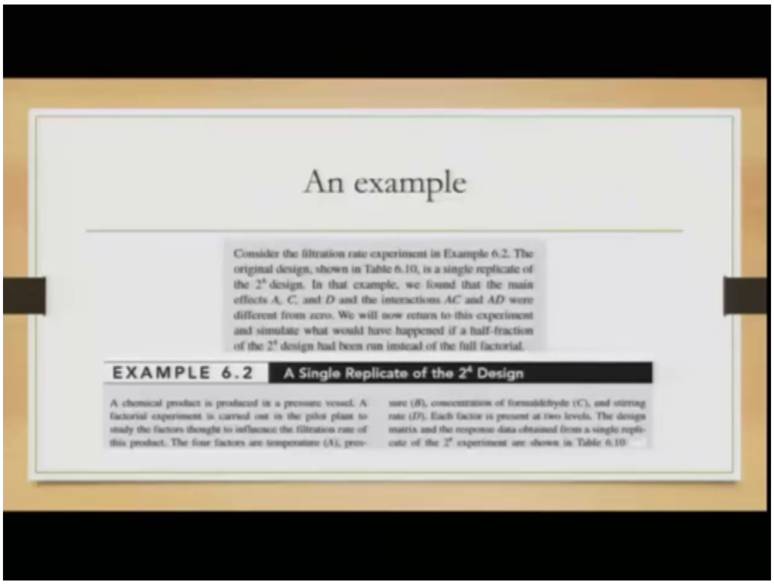

So, let us consider an example considering the same filtration next latex example the original design, which basically has is a single replicate of 2 to the power 4 design, in that example we found out the main effects A C D and the interaction of AC taken 2 at a time which means you are trying to find out the effect of A and C, then you are trying to find out the effect of A and D were different from 0 levels hence they was significant. We will now return to this experiment and simulate that that what would we have would have happened if I 1 half fraction designed to of 2 to the power 4 was run instead of the full factorial model.

So, consider this example a chemical product is produced in a pressure vessel, a factorial experiment is carried out in the pilot plant to study the factors what are the effects? which is coming on for the factors? The 4 factors are A is temperature, B is pressure, C is the concentration of the form A aldehyde worth whatever we are using. And D is the stirring rate as you mix this chemicals each factor is represented at 2 levels. So, basically you will have temperature at 2 levels pressure at 2 levels level of concentration for the form a formaldehyde at 2 levels and the studying speed would be at 2 levels.

The design matrix in the response data obtained from a single replication of the 2 to the power 4 experiment are shown as given.

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So, the runs are given on the leftmost column. So, the lance runs are basically the combination based on which you are going to take the factors which we will denote are ABCD. Corresponding to pressure, corresponding to concentration and so on and so forth as well as I read. So, if I consider the runs on number 1. So, basically we are taking the run A level 1 which basically where there is no effect then and it is a static level when I consider say effect of run 2 it only gives me the level of significance.

So, 2 which is the second row corresponds to the fact that I am only taking the effect of A which is replicated by the run level which is given here. When I go to say for example, go to 3 I will use the same color to denote that when I use 3. So, 3 would not effect on basically A factor v and factor v would be there coming out in the run level, because the level label based on which the run is being run would basically be signified by the factor effect which is coming into the picture. When I take 4 it is an effect of say for example, both combined together A and B.

So, the minus signs; obviously, would mean there is no effect which is coming from there and hence the run levels would be A B combined, when I go let me take A different color it would have been best if I use different colors for different schemes, but because they become very dark. So, it is difficult for us to make the significance level. So, for the fifth 1 you have only effect of C hence the run label is given as C, suffix if I go to the I am going now down the rows, if I consider the run label number 11, it is an effect of factor B it is an effect of factors D. Hence the run label is given by A factor B D if I go to the second last, which is the fifteenth 1 I am just going arbitrarily.

So, once you understand all these things the each and every plus and minus value in each and every row would now make sense to you as you go 1 at a time. So, if I go to the fifteenth run number the effects are coming out from factor B, factor C, factor D hence the run label is given by a suffix BCD or the names BCD. And finally, the last run which is sixteenth run has an effect of A, also of B, also of C, and also of D hence the run label is given by ABCD.

And the filtration rate which you had if you find out for each such combinations are given in the last most column and D would be you class in order to basically find out the level of filtration plus the effect of all these 4 factors ABCD each at 2 different levels of significance significance means the level of the values.

So, pressure can be say for example, at 1 Pascal and another case it can be 2 Pascal's. Similarly the formaldehyde concentration may be 100 percent or say for example, 50 percent. So, based on that I have 2 level of factors and all of them being are being combined together. So, it will be 2 to the power 4, because there are 4 factors in this example.

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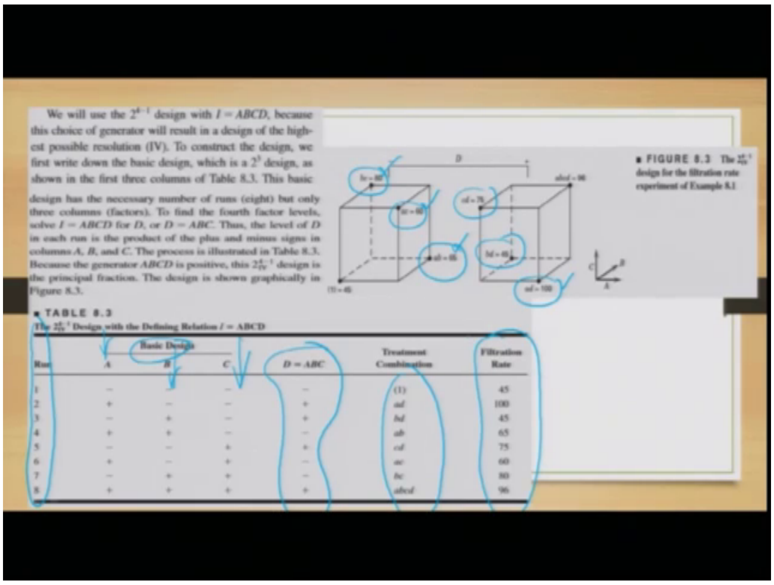

So, now we will use the design of 2 to the power 4 minus 1, because the choices the generator would result in the design of the highest possible resolution, which would basically be the resolution of roman numeric 4. To construct the design we first write down the basic deni design which is basically 2 to the power 4 minus 1, which is 2 with the power 3 design factors as shown in the first 3 columns.

This basic design has the necessary number of runs which has 8 in number, but only 3 column factors are there to find out the fourth level, which is basically the level of significance which we have to find out ABCD combined together we need to basically resolve it. Does the level of D, which is the fourth factor in each run is the product of the plus and minus signs which is given for ABC combined together and second taken separately. So, combination can be 2 at a time all as well as 3 at A time when I am doing a combination of 2 at a time the combinations are A B A C and you will have basically BC.

And, when you are basically combine them in and taking them 3 at a time it will be the combination ABC taken all 3 at the same time. The process is illustrated in the table which will just discuss which is table 8.3, because the generator for ABCD is positives hence we will have basically the design factor given has 2 to the power 4 minus 1 because there were 4 factors and we are basically decreasing the level of fx which is coming on for them.

So, it will be 4 minus 1 and the subscript would be basically be 4 in the numeric 1 which basically gives the highest number amount of resolution which is coming on for the factors. So, if you have this the combinations would be given as this. So, now, remember that drawing the 3 dimensional cuboid for 4 dimensions it would not be possible.

So, what we will do is that we will first go in this level and this may change depending on that type of experiment, which you are doing we will first draw the combination or 3 dimension for ABC and do it corresponding to the fact that the level of these are at different levels say. For example, if you remember we mentioned very category before we started the experiment or the and the example then each factor these are 2 levels. So, obviously, D would be at A level of low significance which will denote at D minus and they would be higher significance which would be D plus.

So; obviously, they would be 2 Q by 4 D 1 4 D minus and 1 4 D plus which will give me the level of significance. In case say for example, if D was at 3 levels which is say for example, D 1 D 2 D 3 so; obviously, there would be 3 cuboids 1 for D 1 for D 2 and 1 for D 3 and we can basically find out those combinations accordingly, but for this example let us concentrate on the stoop cuboids at level of D minus and D plus.

So, these are the 1. So, what you have is basically the combined factors coming out from A B C combined and they are combined in such A way that you can find out add the corner points the combination of A B, B C, B A C and the combination of A B C taken 3 at A time 3 means you are basically combining them together.

So, when you have basically the effect of BC if you remember the rates on the on the rightmost column the filtration rate. So, those values were given. So, this is 80 A combination of AC has A 60 rate filtration rate and A combination of A B has A filtration rate of 60. Similarly when you go to the higher level of D plus, the combinations come out significantly in the same way here basically CD is 75 you have the combination of BD as 45, you have the combination of A D S as 100. So, these combinations would give you the values at which you are trying to find out the level of significance.

So, again the run values are giving in the first most column I will highlight it the effect of A B C D is not there in the picture, because we are trying to eliminate D it can be elimination of A or elimination of B or elimination of C depending on how you think the importance is. So, you have the combination of basic designs A being on the second column, B being on the third column, C being on the fourth column. And the designs are given in such a way that you have basically D, which we now combines the combination of ABCD to the highest possible extent.

The treatment values are given which basically signifies very significantly in this way. So, if the combinations are A is plus B is plus and C is plus; that means, you have already taken ABC combined and if the combination of D which you are trying to find out as A combined combination of ABC if that is also plus; that means, now we have the effects of A S plus B S plus CS plus and D S plus also.

Hence the label would be denoted as ABCD; that means, you are trying to combine all of the factors effects together. If you see the second last row. So, you have basically A minus; that means, the effect of A is not there, but definitely there is an effect of B there is an effect of C, but the effect of D also which is for ABC is not there.

So, hence the effect would now be labeled by the combination of B and C. If you go to the 6 done and the run number 6 this is plus 4 A and plus of cross C. So, it will be combination of AC and so on and so forth and the filtration rates as you know down are giving the last most column, which is just a replica what we have done from the initial table.

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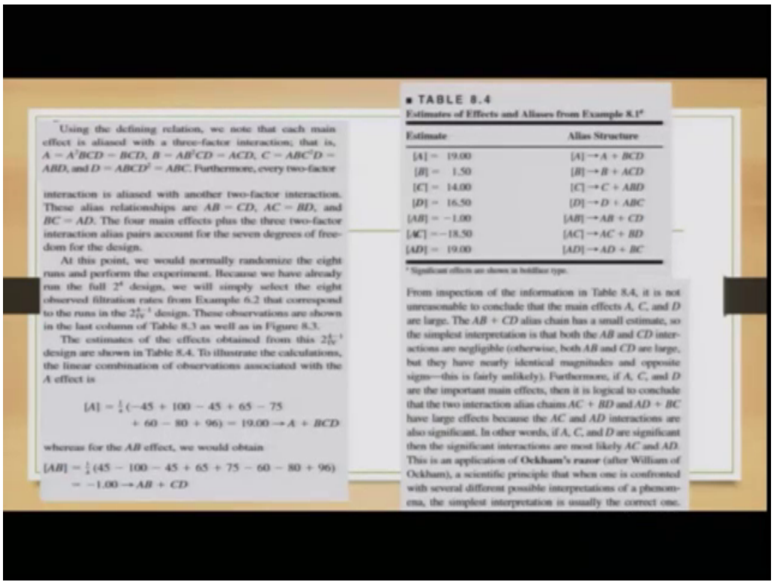
Using the defining relation, we note that each main effect is aliased with a three-factor interaction; that is, $A = A^2BCD = BCD$, $B = AB^2CD = ACD$, $C = ABC^2D = ABD$, and $D = ABCD^2 = ABC$. Furthermore, every two-factor interaction is aliased with another two-factor interaction. These alias relationships are $AB = CD$, $AC = BD$, and $BC = AD$. The four main effects plus the three two-factor interaction alias pairs account for the seven degrees of freedom for the design.

At this point, we would normally randomize the eight runs and perform the experiment. Because we have already run the full 2^4 design, we will simply select the eight observed filtration rates from Example 6.2 that correspond to the runs in the 2_{IV}^{4-1} design. These observations are shown in the last column of Table 8.3 as well as in Figure 8.3.

The estimates of the effects obtained from this 2_{IV}^{4-1} design are shown in Table 8.4. To illustrate the calculations, the linear combination of observations associated with the A effect is

$$[A] = \frac{1}{4}(-45 + 100 - 45 + 65 - 75 + 60 - 80 + 96) = 19.00 \rightarrow A + BCD$$

whereas for the AB effect, we would obtain

$$[AB] = \frac{1}{4}(45 - 100 - 45 + 65 + 75 - 60 - 80 + 96) = -1.00 \rightarrow AB + CD$$

TABLE 8.4
Estimates of Effects and Aliases from Example 8.1[a]

Estimate	Alias Structure
$[A] = 19.00$	$[A] \rightarrow A + BCD$
$[B] = 1.50$	$[B] \rightarrow B + ACD$
$[C] = 14.00$	$[C] \rightarrow C + ABD$
$[D] = 16.50$	$[D] \rightarrow D + ABC$
$[AB] = -1.00$	$[AB] \rightarrow AB + CD$
$[AC] = -18.50$	$[AC] \rightarrow AC + BD$
$[AD] = 19.00$	$[AD] \rightarrow AD + BC$

[a] Significant effects are shown in boldface type.

From inspection of the information in Table 8.4, it is not unreasonable to conclude that the main effects A, C, and D are large. The AB + CD alias chain has a small estimate, so the simplest interpretation is that both the AB and CD interactions are negligible (otherwise, both AB and CD are large, but they have nearly identical magnitudes and opposite signs—this is fairly unlikely). Furthermore, if A, C, and D are the important main effects, then it is logical to conclude that the two interaction alias chains AC + BD and AD + BC have large effects because the AC and AD interactions are also significant. In other words, if A, C, and D are significant then the significant interactions are most likely AC and AD. This is an application of **Ockham's razor** (after William of Ockham), a scientific principle that when one is confronted with several different possible interpretations of a phenomena, the simplest interpretation is usually the correct one.

So, using the definition relationship we know that each main effect is aliased with 3 factors interactions and they are as follows A is given by A square B CD, because now you have basically B C and D B S is being combined by the combination of ABC taken together.

Then you have basically the combination of B given as A B square CD, combination of C given as ABC square intended DS square means the effects, which is being coming into picture that is C square means there are 2 level of effects, which is coming from factor C.

Similarly for if it is A square it will mean the effect of factor A is coming in 2 fronts, if it is A cube it would mean the effect of A is coming on 3 fronts and similarly for D it will be given by ABCD square.

Furthermore each every factor in interaction is aliased with another 2 factors as that you can break it up accordingly and basically reduce the level of calculation, but the effects of the output would be the same; that means, you are trying to get the maximum amount of output from the information giving the number of calculations has been reduced. At this point we would normally randomized the 8 runs and perform the experiment,
because you have already run the full 2 the for design we will simply select the 8 observations in order to make our life much simple.

So, what we are trying to do is that convert 2 to the power 4 combinations into 2 to the power 4 minus 1 suffix romans 4 such that the effects is are coming out to the maximum possible extent. These observations are shown in the last column in table 8.3. So, based on that when we find out the effects the effects are given and we and we basically given them in a table. So, these effects are as follows in 8.4.

So, you have estimated values of A. So, this A does not mean A only it is A combination of ABC into whatever levels we have taken. So, the f estimate of A is 19 estimate of B is 1.5. Similarly if you go down this limit of D is 19 and the aliased structure which is given basically means A has A combination of effect of A, separately and BCD taken separately if I consider the effect of B it is basically effect of B separately and ACD separately.

So, what you are trying to do is they are trying to factor them in such a way that if it is only as for A it signifies 2 levels of effects; 1 is coming only from B and 1 is coming from the effect of ACB where individually all of them may have an effect on B what we are trying to do is trying to reduce the amount of calculation and find out the combined effect of A on B of C on B and of D on A B such that the combination would give us the best result.

Similarly, when we go to the combination of AD it is basically A combination of AD taken together and BC separately such that B gives effect on ad C gives an effect on AD such that the combination of BC would give us the best effect from inspection on the transformation as given in table 8.4.

It is not unreasonable to assume that the effects are done in such a way that they give us the best possible combination of result in order to find out the factorial effect of 2 to the power 4, but now the overall number of calculations have been reduced to the maximum possible extent for A 5 factor in A manufacturing process.

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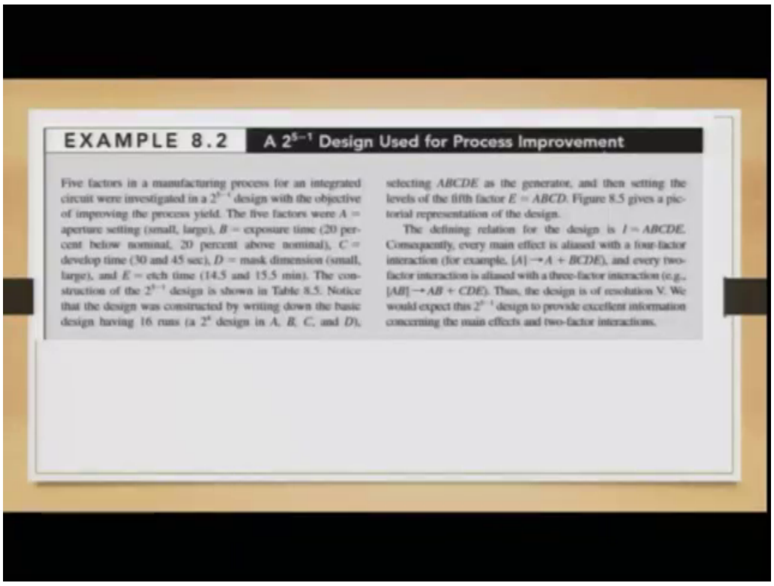

So, I am just giving you the gist and the calculations would come out very automatically. So, first you have to do is basically find on the table of the plus and minus plus and minus, basically gives the level of significance and then you basically break them into their aliases effect and find out the combinations accordingly.

5 factors in A manufacturing process for in for an integrated circuits were investigated and we want to find out the fractional factorial experiment based on the factor of 2 to the power 5 minus 1. And the suffix would basically depend on the level of significance, which would denoted by either 3 or 4 or 5 on the roman script such that it will give you the level of significance for the experiment. The 5 factors are for this manufacturing process our aperture setting which is A B which is exposure time and remember that when I am reading A affect or B affect or C effect I will also mention to what level of significance each of them has on the overall factor.

So, again I repeat it the 5 factors are A which is aperture setting which will be small and large so; that means, A has an effect on 2 levels, B which is exposure time would basically be 20 percent below nominal and 20 percent above nominal again 2 factors to effects of for those factors. C would basically be the developed time which is 30 seconds and 45 seconds again 2 levels of factors for C D is the mass dimension, which is small and large again 2 effects from coming out from D E is the H time, which 14.5 and 15.5 minutes which is again 2 levels for M D and finally, for E as and which is for 2 effects for E.

So, hence we basically we have 5 factors each at levels 2. So, it is basically 2 to the power 5, notice that the design was constructed by writing on the basic design of 16 runs which is basically 2 to the power 4 because initially you can consider only ABCD now as I can cons add another factor it becomes 2 to the power 5. Now selecting A B C D E as A generator what we are doing is that then we are setting the combination in this way either you can basically combine them so.

So, let me go back 1 step back initially you had ABCD as factors, now our fifth E factor is coming, now when you add a fifth factor it can happen in this ways 2 ways. Either start the experiment from base levels where you attach this the fifth factor is a new 1 and do the calculations afresh other way you can be you add E and combine E in such A way that it will E will give me the overall effect or the combination of ABCD taken together at a time. So, this is what I had my mean.

So, the identif so, called effect would be coming out from ABCDE, but the combination of A would be done or combination of B would be done or the combination C would be done and we have so on and so forth we will be done in such a way, the factors effects are separated in 2 fronts. Number 1 front is the combination of ABCD and on another front basically you will have the effect of E only, but combined in such a way that the overall effect would come out to the maximum possible extent thus the design is of resolution 5. So, we will basically have an experiment of a factorial design of 2 to the power 5 minus 1 with a roman suffix of 5. Such that you are able to basically bring out the maximum possible effects to the highest level such that we get the best answer.

So, with this I will close the 20 fourth lecture and continue discussing more about the fractional factorial design with this example another example. So, that will be much easier for us to appreciate how we have been able to change the overall individual factors, where the calculation becomes more cumbersome in a much nicer way such that the fractional factor models can be brought in at the picture and give us the same level of results. With this I will end the lecture and have a nice day.

Thank you very much.