Total Quality Management – II

Prof. Raghunandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 23
Two level Fractional Factorial Design – I

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you, who are all of you are taking this TQM – II course under the NPTEL MOOC series of lectures and I am Raghunandan Sengupta from the IME department IIT, Kanpur. So, if you remember that we had just started the concept of factorials and factors based on it is the factors are not based on which your analysis would be done not the ANOVA analysis is basically further on where the factors are in such a way that is some of them are quantitative variables, some are qualitative variables based.

On this we will try to analyze how the combinations could be done to find out the best effect from those and now obviously, we go for to find out the total square, sum of the squares, then the degrees of freedom then mean squared errors and finally, test the hypothesis depending on the level of significance which has been already been stated in the problem whatever it is.

Now, in this for an example, if you consider say for example, the battery problems. So, in the battery problems we had the type of material. So, material would not be the type of material can be divided on a qualitative framework, like say for example, the quantum of magnesium is more, silicon is more or say for example, carbon is more whatever the factors be or it may be the num amount of impurities which may be needed say for example, I am doing the doping experiment in physics and I need to basically add some impurities in order to basically change the conductivity of the materials.

So, amount of impurity may be there or say for example, you are doing the some kind osmosis experiment and then the density would change, those are called in a sense qualitative in nature and if you consider the temperatures so, they were basically temperature that the two end one was 70 125 and another was basically 15. So, when we are trying to do that you will also try to under lesson the, understand these are quantitative in nature. So, try to combine the quantitative factor qualitative factors would basically done in such a way that we are able to do a one to one correspondence between them such that overall picture and overall analysis can be done in the best possible manner. So, to continue that in this 23rd lecture, we will basically dwell into this problem in more details.

So, if you remember the consider the general model of the same a simple regression model and what are the simple regression model consider you had basically three factors I am talking about those factors ABC. So, your three factors capital A capital B and capital C. So, y ijk. So, i would be for A factors starting from 1 to small a, j would be factor B starting from j is equal to 1 to small b, k would be the factor corresponding to C starting from 1 to small c. And, the last suffix under y would basically the l, ijkl the fourth one would basically dis simplify or denote the number of samples which you have basically starting from 1 to small n.

So, if that y ij I want to find out the effect that would basically be equal to mu, mu is basically the averages of the averages of the average. So, we would basically take the averages for A averages of B average of C and average of n and then find out mu which is the called the population expected value and if it was not available you will try to replace that with mu hat which is the best estimate then the other terms which would be very logical come out from this those analysis would be tau suffix i, I will repeat and come to that again I know it I, I am basically repeating the same example time and again, but it will make sense why this changes are being done in the factorial design models.

Then we will basically have tau suffix i then you had beta suffix j and you have basically gamma suffix k corresponding to the dispersion of the variations which you would have for the for the A factor for the B factor and the C factor respectively. Then when you are going to the next stage next stage means continuing the equations trying to find out the combination of two it will be tau into beta for factors A and B; obviously, they would be suffix tau beta suffix i j.

So, I am not mentioning very expressively the suffixes then when you are going to find out the effects or of for combined effects of B and C it would be beta in to gamma; obviously, with the suffix j and k and when I am going to of the combination of A and C it would basically be tau into gamma suffix i into k and when you are going for the combination of them it will be tau beta gamma suffix ijk.

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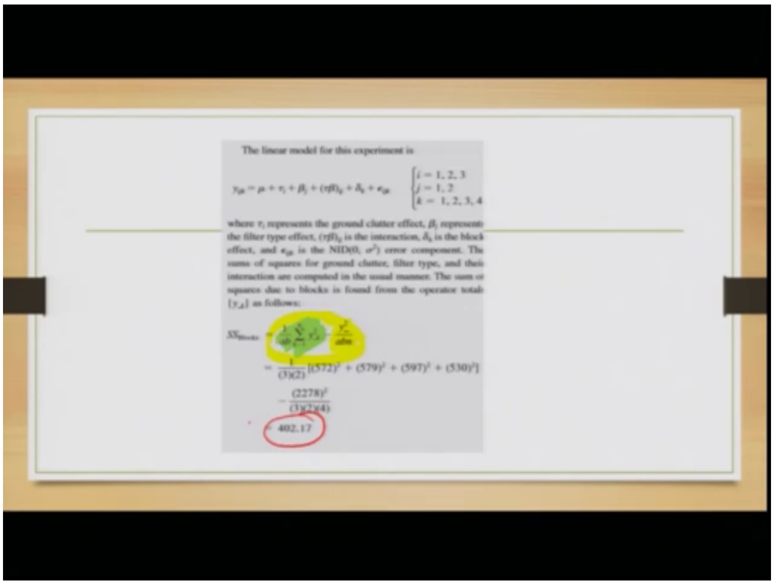

So, if you note down here I just highlighted. So, this was y ijk. So, there is only 2 factors here. So, if they were three it would be l here. So, mu would basically with the have with the overall population average tau corresponded to A betas corresponding to B there would be gamma also which is not there because you are considering two factors only. So, when I combine them tau and beta which is tau beta suffix ij would be combination of A and B.

Now, this delta if you remember and obviously, I will come to that later on this epsilon would basically be the error terms which is epsilon ijkl if they are three factors and if you note down there are i is changing from 1 to small a, where small a is 3, j is equal to changing from 1 to small b, where b is equal to 2, a is changing from 1 to small n, where n n is basically 4. So, this delta would be the corresponding the se effect which will be coming out when you are basically try to com combine them and trying to find out the factors as such.

So, I will repeat it where tau suffix i represents ground clutter which is for a beta j a factor A beta j represents the filter type effect which is factor B tau beta suffix ij is the interaction and delta k is the block block effects which you are trying to find out con considering the blocks are done in such a way if you want to divide the effects. So, you basically consider one block and consider that to be non deterministic or stochastic such that all external factors are out of your control. So, whatever effects are coming out are in the white noises.

So, delta k is the block effects epsilon ijk is the normally distributed error terms with 0 expected value and sigma square the variance. Now, remember another thing I know I am repeating so, but please pay attention to this sigma square which I am taking are the error terms variance which do not effect each other. So, that means, we are considering that the effects of error term from one time to the other are not there. So, hence it is basically the error term is in not dependent on time.

The sum the squares for the ground cluster which is factor a filter type which is factor B and interactions are computed in the usual manner the sum of the squares due to the blocks is found out. So, you will find out the block block effect considering y dot dot k or so, because k you are keeping fixed and you are trying to find the sums for effect of factor A sums for effect of factor B.

And, then the sum of the square of the blocks would be very use very simply given I will just highlight the two formulas which you I know that you know that, but still in order to if I repeat it there is no harm in trying to basically learn it in more depth and trying to find out any queries which I am may have been left and you if you think you can write it to the forum obviously, we will answer that, but better if you basically understand it once again the repetition would definitely help.

So, we have the effects. So, technically this in later point which I am now highlighting which is y square and dot dot dot means you have basically sum of all i's, all j’s and all n that is factor A, B and the sample size and divided by total number of observations v is equal to a to b into n and obviously, in the initial term which is blocked. So, obviously, blocks means you will keep k fixed and you will sum them up for a and sum them up for b which is as done.

So, I will try to highlight using a lighter color if possible, yes. So, this is the effect which I am com talking about considering that I find out the squares of dot dot k, that means, the blocks them and block them and for each block I find out for all the i's and all the j’s summed up and then divide by the total number of observations, what is the total number of observations for any block the total number observations would be summed down for all a summed up for all b. So, it will be a into b.

So, once I find out this value I will just. So, the value comes out about 402.17. So, you just put it make a table put plug in the values and then solve it accordingly.

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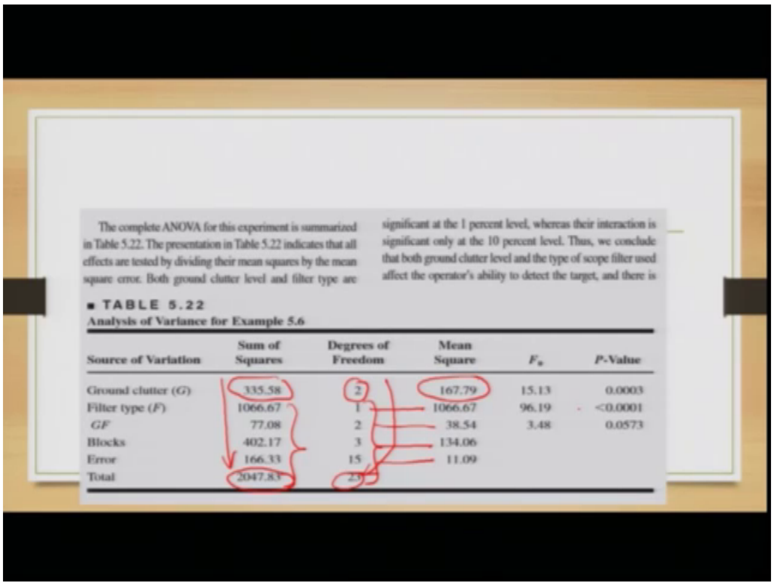

The complete ANOVA for this experiment is summarized in Table 5.22. The presentation in Table 5.22 indicates that all effects are tested by dividing their mean squares by the mean square error. Both ground clutter level and filter type are significant at the 1 percent level, whereas their interaction is significant only at the 10 percent level. Thus, we conclude that both ground clutter level and the type of scope filter used affect the operator's ability to detect the target, and there is

■ TABLE 5.22
Analysis of Variance for Example 5.6

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Ground clutter (G)	335.58	2	167.79	15.13	0.0003
Filter type (F)	1066.67	1	1066.67	96.19	<0.0001
GF	77.08	2	38.54	3.48	0.0573
Blocks	402.17	3	134.06		
Error	166.33	15	11.09		
Total	2047.83	23			

So, the complete ANOVA for the ex experiment summarized in table 5.22. So, the in the in table what you have is again repetition, but I am sure you will understand the first column consists of all the factors and they corresponding to effects taken two at a time, three at a time as a case will be the second column the sum of the squares corresponding to this factor, third column the degrees of freedom forth column is basically finding out the mean square error and fifth column is basically F, F value depending on what your hypothesis is and that is ratios of the mean square at different levels and the last column basically gives you the concept of what is the value of in the level of significance which you have said for yourself for the experiment of the ANOVA model whatever you have.

So, the table 5.22 indicates that all the effects are tested by dividing the mean square by the mean square errors. Both ground cluster effect and filter types are significant at one person level. So, it will definitely the level of significance will change depending on the overall importance you want to give for those effects. Whereas, their interaction is significant only at a 10 percent level thus we conclude that both ground cluster level and as well as the clutter level and the type of scope filter used effect the operators ability to detect the target and hence this values I will just repeat are the ground effect which is a is

335.58 the degrees of freedom is 2, so, 335.58 divided by 5 2 gives you the mean square or 167.79.

Similarly, the filter type effects the combination of a and b which is G and F the block effects error terms on the total values come up to be as shown here the degrees of freedoms are as shown here. Now, remember one thing the sum of all the errors basically should add up to these value to 204 7.83 that is 2047.83. Sum of all the degrees of freedom should add up to 23, so, this 2 plus 1, 3; 4 5, 5 plus 15 is 20, 20 plus 3 is 23. So, this values basically is there similar here. Then you basically divide the sum of the squares by the degrees of freedom you get the mean squares and finding out the ratios of mean square gives the F values and then you can basically comment intelligently whether they are significant or non significant depending on level of significance.

Now, I will consider the two-level factorial design problems the factors which have been saying that whether quantitative qualitative and basically con consider them in more details within the twenty fourth, twenty fifth, twenty sixth lecture and as you proceed.

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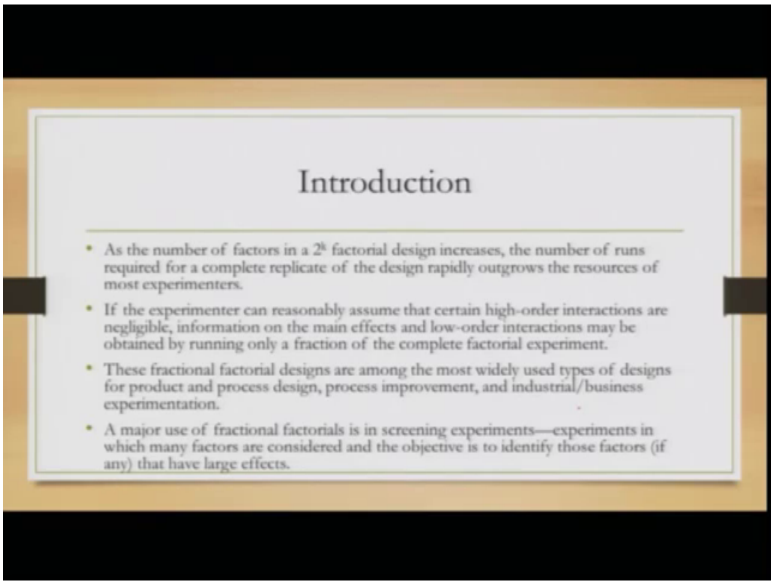

So, as the number of factors in a 2 to the power k factorial design, so, blocks you remember. So, factorial design problem increases the number of runs required for a complete replication of the design rapidly outgrows the resources of most of the experiments. So, obviously, we have to take some correct reactions and basically make our experiments full proof based on the fact and still conduct the ANOVA test and basically come out with some conclusion. If the experiment can reasonably assume that certain high order interactions are negligible. Obviously, say for example, the effects of a and b are negligible so, obviously, you can ignore that and hence, the amount of calculation you needed to be done would reduce, but you have to take intelligent and rational decision to what level of significance you require the answer hence neglecting some factors would now make sense to you.

So, let me continue reading it that certain high order interactions are negligible the information on the main effects and the lower order interactions may be obtained by running only a fraction of the complete factorial model. So, rather than basically doing all the combinations we will run only fraction of them such the level of significance for the experiment not those values level of significance which you are said for previous for the hypothesis thing the overall efficiency is made. Hence you are able to conduct the experiment and still get good results.

These factorial factors fractional factorial designs are among the most widely used type of design of experiments for products and processes in design, process improvement and industrial and business experimentation as required. A major use of fractional factorials is in screening experiments – experiments in which many factors are considered and the objective is to identify those factors if any that have large effects and they can basically be not be ignored and the small effects can be ignored. But, obviously, keeping in mind the efficiency of the problem.

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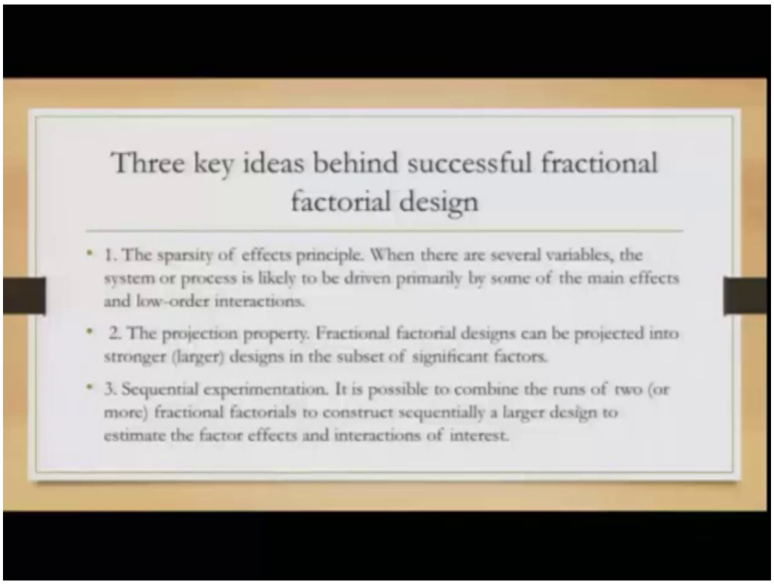

The three key basics behind, ideas behind successful fractional factorial designs are as follows. The sparsity of effects principle, this is important. When there are several variables the system or process is likely to be driven primarily by some of the main effects and low order interactions hence basically you should pay attention to that.

The second important point is the projection property, which means the factorial fractional, fractional factorial designs can be projected on to stronger which is means larger designs in the subset of the significant factors such that even if they do not make sense in the overall scheme, but when you basically project it on project it means not mathematical projection basically when you project in to find out the factorial effect on a lower scale we get much better significant answers.

Sequential experimentation means it is possible to combine the runs of two or more fractional factorials to construct sequentially larger designs to estimate the factorial effects and interaction of interest. So, that means, if you have considering that the factors A and B the level of significance or the level of significance point the form point the from the point of view of the sum of the squares is coming out to be very less, but once we combine in a and b combine or we combine a, b, c or whatever it is the effects would come out to be much more significant hence we should definitely not ignore them.

So, sequentially we have basically we trying to increase and trying to find out the overall function taking all of of them separately then you combine them two time, then three at a time and go step by step by a higher level such that we will get a much holistic and much more bigger picture for a analysis.

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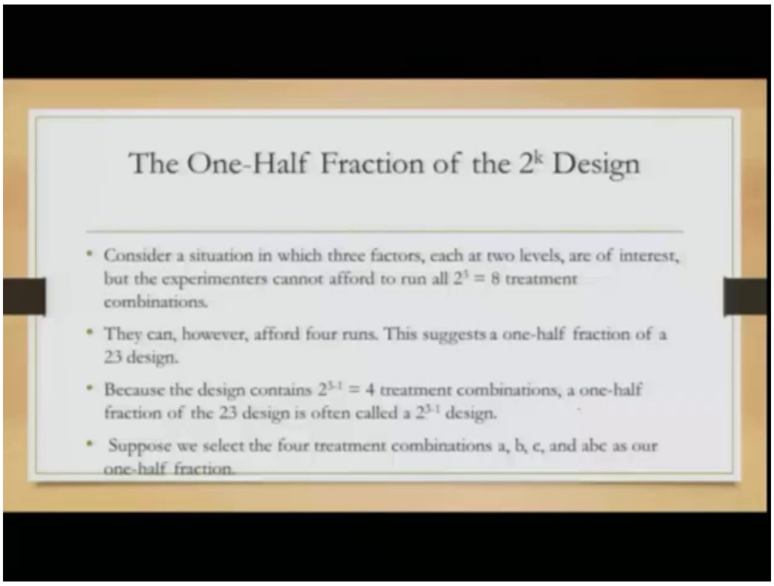

Consider a situation in three factors each at two levels are of interest, but the ex, but the experimenter if we can actually experimenter has to do the example they would be 2 to the power 3 values of treatment combinations. Because, the reason are there are three factors and each is at two level. So, 2 to the power 3 which is 8 levels of combinations we would basically find out.

They however afford four runs. They can however, afford four runs this suggests a one half fraction of a 23 design problems because the design contains technically if you find out from the overall effects it would basically 2 to the power 3, 2 to the power 3 minus 1, so that those factors to the power which you are taking is 3 minus 1, hence the total combinations of the experiment would be 4 in number and now not 8.

So, that means, you have basically the halving them from half from the original level of 8 to 4. So, which means a one half fraction of the 23 design is often called 2 to the power 3 minus 1 design. Suppose, we select 4 treatment combination a, b, c and a, b and a, b, c separately and a, b, c combined as a one half fraction problem and we can solve the prob ANOVA model accordingly.

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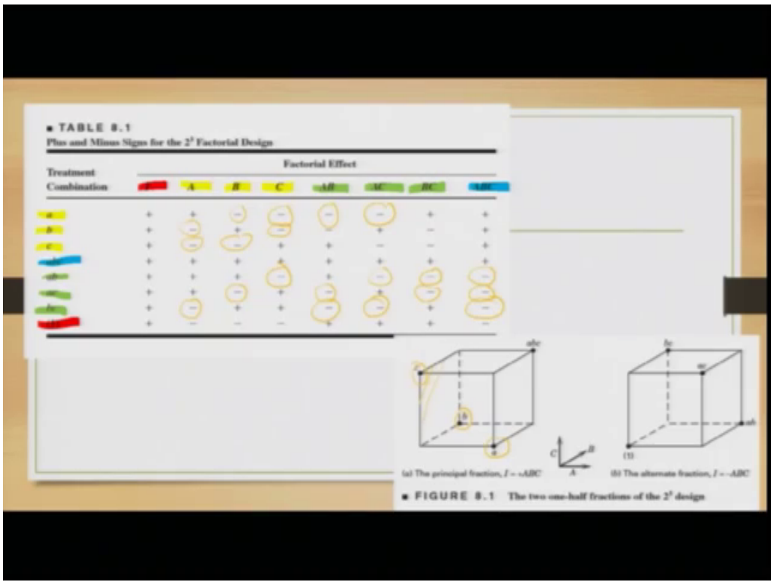

So, let us consider the effects. So, when we consider the effects factorial effects on a on a table format. So, the plus and minus sign would basically give you to whether they are positive or negative. So, con consider the treatment. So, the treatment combinations are you have basically a, b, c factors done.

So, when we are considering the fact fractional factors and one half of the model it means that we are trying to reduce the amount of computation which is required at our end, rather than considering each of them separately con or in a combination such that yes, we will get the answer to the best possible extent, but amount of calculation needed to be done increases almost exponentially because it is to the power, if you remember that.

Now, let us pay attention to slide which is in front of us, where we are trying to basically put the factors on the left most column exactly in the similar way as we do the calculation and then find out the combination of the effects to what level they are needed to be considered seriously for our calculation.

So, the treatment combinations are individually a, b, c is taken. So, which is basically the first row, second row and third row then if you go sorry I will come to that if you come to the combination of taken two at a time then we have basically that the fifth sixth and the seventh and if I consider all of them combined it is a, b, c which is the fourth row. So, and obviously, the value of i and the factors combinations are given as here, I will just highlight it.

So, we have a single single single, single single single this means only taken them into consideration. If I change the color double means two at a time ab ac abc, AB, AC, BC then you have the combined effect last level abc ABC you have the combined all of them I and I. So, now, I had used a pen color with the level of color combination as orange and try to basically find out the effects.

So, when I am considering the effects of a let us go one by one. So, let us first negate one for this there is no effect. So, if you consider a s effect it is not there for b hence it is minus it is not there for c the level and if I combine them consider the effect of significance of a on a combination of ab and com effect of a on a combination of ac are insignificant, hence they are minus here and if we consider minus means we are we are we can put the picture in both the ways. The minus sign can be utilized to highlight and higher level effect or lower level effect whatever the combination is.

Then if you consider the combination of b, so, b would be minus for A minus for C and then if we consider c it will be minus for A minus for B. When I am considering the combination of ab c this fifth fourth column is positive for all the effects because abc are there everywhere; When I come when I consider ab. So, obviously, for a b if I consider the effects combined on of them on AC is insignificant of ABC is significant and ABC insignificant go because for the k s for the last one a c effect is here for the combination bc b is there, but not a and in k in k combination ac you will find out the b is not there only the effect of a. So, that is why they are minus minus minus and this is, obviously, minus because it is only c.

Similarly, when I taken ac. So, let us consider where the minuses are. So, ac means AB minus because there is no combination of c, BC is minus because there is no combination of a and ABC is because there is the combination effect of C which is coming when I consider ac with B it is minus. Similarly, for bc this will be minus AB would be minus AC would be minus and ABC would be minus and then you can find out the combinations positive or negative.

So, when I do the to half fractional models of the 2 to the power 3 design so, you have for the combinations as this. So, you have the effects of abc drawn on 3 orthogonal planes. So, this is 3 dimensional. So, it will be easy for us to explain if it is 2 dimensional it will be on a on a flat plane, but it is higher dimension, obviously, difficult us to understand. So, we will consider the effects coming out for c orthogonal b orthogonal. So, they are orthogonal to each other any orthogonal and the combination if you do.

So, if you are considering this plane let me understand this plane which is in this room; So, if the wall which is at the back of me is the plane and if I am standing at the origin. So, on the right if you it is a vertical up is c and or horizontal straight forward towards you towards the camera it is basically b.

So, the wall which we have here would only consider the combination of c and b, the wall on which if I basically if you see my picture on the wall back it will only consider the combination of c and a because these effect would not be there and if I am considering the plane on the floor it will only consider the effects of a and b because c would not be considered there because c is 0 and effect.

And, if I am taking the united cube all the effects of a all effects of b and all effects of c can be combined together towards the hori orthogonal line or the hypothesis line which is basically from the origin to the point a, b, c and you can basically do the mapping in similar way.

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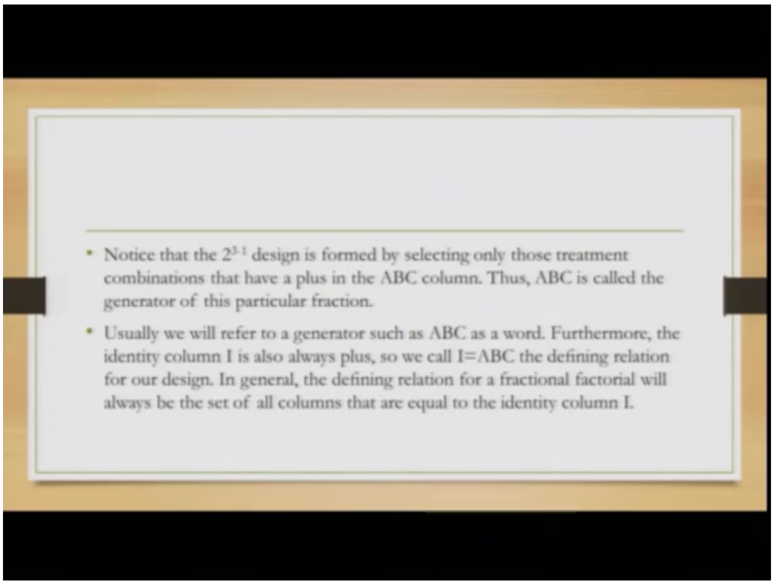

Notice that the 2 to the power 3 minus 1 design is formed by selecting only those treatments combinations that have a plus in the ABC column. Thus, ABC is called the generator of the particular fraction and you can basically find out the effects accordingly.

Usually we refer to a generator such as ABC as of as a certain word furthermore the identity column I is always is always put. So, such that we have the combination of I being equal to ABC, that means, we are trying to find out the combined effect of ABC thus defining relationship for a design. In general, defining relationships for a fractional fraction fractional factorial will always be the set of the columns that are equal to the identity column which is basically ABC. So, in case if you have ABCD it will be combination of A into B into C into D. If it is only AB AB, so, it would combination of AB.

So, that case coming back to the pictorial diagram which was explaining the effect of ac would be only ac would be on the on the wall back. So, if there is no b, so, all the effects would be found out from the origin which is where I am standing and if I find out the combined effect of ac it would basically be orthogonal the diagonal line going from origin to the point on to the top which will basically a into c. In case, if I am going to com combine the combination of b into c, so, a is not coming into the effect it would only the wall is on the left hand side.

So, here is the origin where I am standing and the diagonal would be from this origin to the point which is bc. Similarly, we can find out the combination of b into a would with a point where I am standing and it will go diagonally on along the floor in the point which is basically b into a.

Similarly, you can do the combinations accordingly, but; obviously, for a higher dimension it would be difficult for us to understand for dimensions of four and higher.

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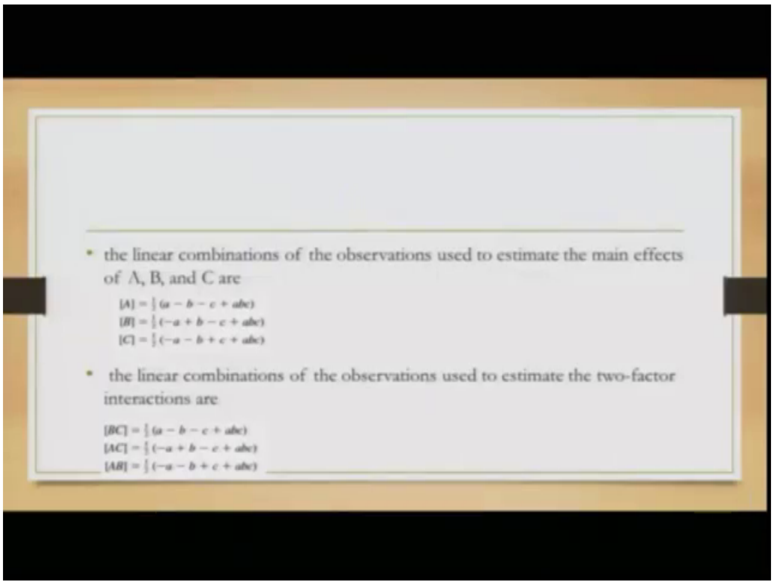

So, the linear combinations of the observations used to estimate the main results of effects of A, B, C would become taking out like this it will be combination of A for only for ab we want to find out a has an effect on itself yes, A has an effect on b, no. So, it is minus b a does not have an effect C which is minus c and once we find out the effect of ABC it would be possible because a has an effect on abc combined. When I find out the effects of B it will be minus a plus b minus c plus abc. When I try to find out the effect of C it will be minus a minus b plus c plus abc.

Now, when I go to the linear combinations the linear combination taken two at a time for BC would be because bc means a is not there which would be the effect would be for for b and c combined it will be minus minus because we are considering things considering them separately so, obviously, it would be minus effect A would be coming with the plus and ABC would be coming in a plus. So, if I go to AC it would be minus a plus b minus c plus abc, if I go to AB it will be minus a minus b plus c plus abc.

With this, I will end this 23rd lecture and continue more discussion about the effects of fractional factors and how they can be reduced to give us a better answer.

Thank you. Have a nice day.