Total Quality Management - II
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Lecture – 20
Factorial Designs – VI

Welcome back my dear friends, a very good morning good afternoon and good evening to all the students and the participants for this NPTEL MOOC course. And this is the TQM II lecture series and this is lecture number 20. As you can see in the slide 20 means that you have we will be halfway through after this lecture and complete about 4 weeks. Each week you have 5 sessions, each session or lecture whatever you mentioned is for half an hour.

Now, if you remember we were discussing after in the fag end of the 19th lecture and also I did mention as we were proceeding that given the factors, which will affect your like the interrelationship of the factors and their effects. So, we had considered in detail two problems, one was basically etching problem and depending on the power and one was basically the material used for the temperature for the battery.

Now in the initial example for the etching problem and though in between these two examples I did mention though briefly the smelting process of aluminium, so it can be thought about different above examples in the same light. We did consider that there were only two factors point 1. Point number 2, when we consider the factors the factors effects were coming by from a factor a, which has numbered the suffix was I from 1 to small a factor b which were number J is equal to 1 to small b and k basically denoted the number of observations which was basically from 1 to small n and similarly you find found out the degrees of freedom, we found out the total sum of squares. So, they are basically coming from a a b a b b then from the errors or some of them was the total errors.

Then we considered the concept of later on in all these examples, the concept of the mean squared error; based on the mean square error as we found out that was found not by dividing the sum of the squares divided by degrees of freedom; we found the mean square the ratios on the mean square in different formats give us the f value and we basically either rejected the null hypothesis or accepted the null hypothesis based on what the problem formulation was.

Another important thing which we should remember is that, whenever we are considering the factors, we did consider that there was a mean value mu which was basically average of the average or the average of the average of the average depending on how many such factors were there if you remember I did mention that time and again.

And we also had the concepts that when you are considering the average of the average of the average of the average of the average, we took the best estimate which was mu hat which was basically either the average value of y can sing one time that is y bar or y bar bar y enough triple bar and the suffixes, which were ijk if those are dot which assumed I know I am repeating, but please bear with me if it was a not in the suffixes it means that we are summing them up for those rows or columns or the number of observation which you have.

So, this factor was along the row, so obviously a was along the row. So, the suffix would be the dot would be the first dot. If the summation was along the columns for factor b the dot would be the second dot and if you are basically summing up for all the observations i and j keep varying like i is equal to 1 j is equal to 1 then you sum it up then i is equal to 2, j is equal to 1 sum it up and all these combinations, we had a dot in the third place in the suffixes.

Similarly, we went to the effect of trying to utilize the capital T statistic which is tuckeys test and then the Bartlett tests were there, then we consider the concept of tau i's beta j's tau i's are the variations happening for the factories beta j were because, the suffix would make it very clear i being 4 factors 1 to small a which is for capital factor a, then beta j was basically for all the factors along the columns from j is equal to 1 to small b. Based on that we calculated and we then if there was a combination to extend it we consider it is tau into b suffix ij. Then in the later part which I am coming and we will be discussing more in details, we consider there were three factors abc and then we consider tau beta gamma and tau i beta j gamma k and k was from 1 to small c for the factor c, and l was basically the last suffix was basically for the number of observations 1 to small n.

Then we in the last few pay attention to the last slide in the 19 lecture that was a detailed 1, but was very easy to understand. The first column was the factors their effects and the total sums, then come the degrees of freedom the column wise, then we had the mean squared errors, then you have the f statistic and then basically whether it was greater or less than we commented whether we accept or reject h naught.

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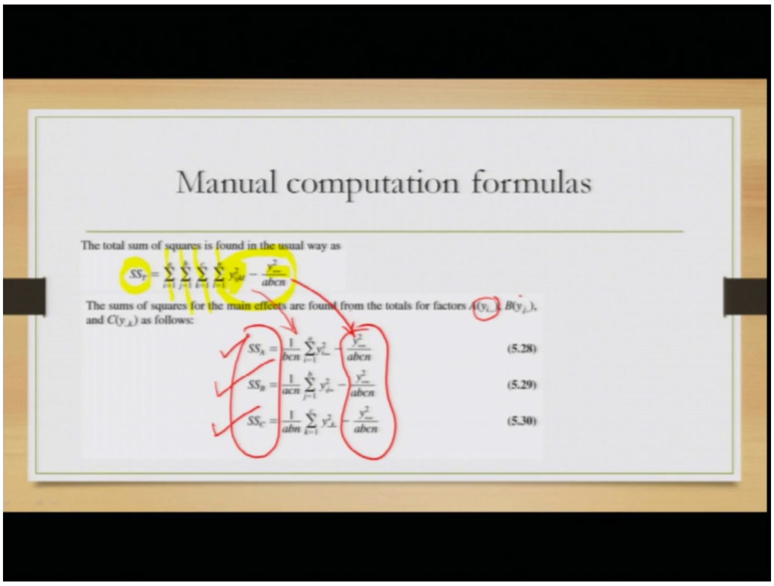

So, based on that we again continue the sum of the squares is found out like this, so here I will try to highlight and use the highlighter so you are summing up for all the fours i j k l. So, here you will note down there are 4 dots and the summations are i is equal to 1 to 2 over i is here sorry i is equal to 1 to small a, j is equal to 1 to small b which is for factor b. Similarly i was for factory a, for factor c it is k is equal to 1 to small c and for the observations l is equal to 1 to small n you and add up, so these are the sums of the squares you want to find out. So, this is the sum of the total squares you use this formula find it out and then put your calculations accordingly.

So, these are the squares of each observations, these are the square of the average of the average of the average of the average you are taking average 4 times and then divide by a b c and which are the total observation. So, in the total set of observation b into b into c into n. So, the sum of the squares for the main effect are found from the total of the factors from a from b and c, and why we are naming giving the suffixes I make it clear to you.

So, if you note down here a is for factor 1 with the first one so obviously to be summed up for j k l. So, if you find out the first one is i, the second third fourth are dot dot dot dot because you are summing up. If I go to the b one so obviously b is basically for the column or the second factor, so obviously dot should come in the second place dot dot should come in the first third and fourth place sorry my mistake. So, j would basically change and the summations would be done for I which is a for k which is c and for l which is n.

Similarly, when I take the third factor c, the summation would be done for the first second and the fourth and k would be fixed and then the summations would be done which is for a factor, I is equal to 1 to a, b factor j is equal to 1 to small b. So, a and b are small b's a's and b's and third places is changing mu z j and then the summation being done for l is equal to 1 to small n. So, you find out some of the squares for a b c, which is the first factor second factor third factor, then when you summing them up when you used utilize this, so corresponding values would become very simple to understand.

So, for a it means that you are keeping if at ith level and summing up for the second factor third factor and n observations, this is what it is dot dot dot in the second third fourth place and given for the sum of the squares it will be divided by b into c into n. For SS B which is sum of squares for b here the second suffix would keep changing, but if we sum of summed up as I am repeating it for the first third and the fourth places as rightly done here and it will be divided by a into c into n. And for SS C the summation would be done for each value of k it will be done for the first second and the fourth as it is shown here and the division would be a b into n.

Similarly you have these values coming out here y square and the total set of observation is obviously a into b into c into n. So, you find out sa sb sc. So, you can find out combinations of ab ac bc and then summing up all these things and subtracting the whole sum from SS t which is the total will give you the sum of the square of the errors.

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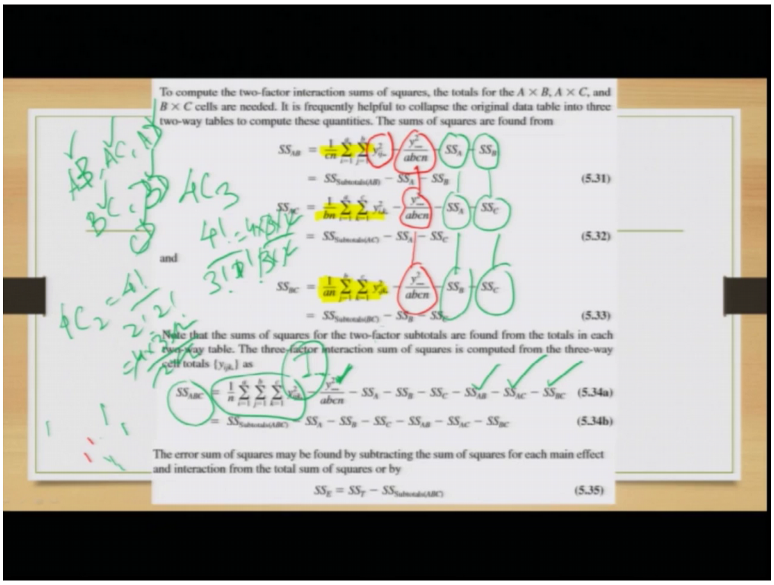

Now, coming that is what I did mention. So, let us go slowly step by step. To compute the two factor interaction, sum the squares in the total for a into b would be one combination, a into c would be the second combination, b into c would be the third combination. So, now we need to find out the sums for these three sets of combinations.

It is frequently helpful to collapse the original data table into 3 way 2 way tables, so it consider this. When there were two factors a b c and n, it was a matrix and it was a cube sort of thing cube or a cube or whatever and you took the slides and slices. So, if you are taking the slides along the vertical as I am showing, then obviously it will mean that if I am measuring a along this direction. And so, which means keep a fixed and summing up for all the slides would basically mean, I am summing up for b and also which is the second factor for m. If I am looking at the orthogonal direction from this side from a left hand side, which means I am taking slices like they are happening and they are slowly moving away from me, but they are parallel which means that I am basically taking keeping b fixed which is j fixed summing out for a factor and n.

Now, if I and for the case when I am basically trying to find out for a and b at i and j value for n, it means that I try to find out for any one cell all the sums. If we go to higher dimension now it is not a cuboid or a cube, but basically they would be 3 axes for one for a, one for b, one for c and obviously n would basically also come into the picture, because it is the total number of observations which you have. So, if you are able to visually the first case when there are two factors and observations is n, it will be very easy for you understand, how the formulations are done for the higher factors like in this case we are considering now two at a time and then finally consider 3 at a time also.

So, let me continue. So, the sum of the squares can be found out, as I mentioned is that to collapse the original data into 3 2 way tables, rather than combining them as the matrix. So, here the sum of squares would be a b, so this combined effect of a and b, it will be. Now if you like if I extend it will be y square now I would pause and think that if it is a b, which means that I am trying to take the factors of a and b considering c which is the third one is being summed up for the whole set, similarly the number of observations for n is basically being summed up for the whole set and rightly pointed out is where I am going to circle is this.

If you see i and j are corresponding to a and b and the third and the fourth dot correspond to the fact that c is not being considered in the picture, because you are summing up for all the fact the factor values for c, and for all the values of n. And obviously the second point second term obviously remains minus with a minus sign y square in suffix dot dot dot dot, that means you are summed up for all the 4 in IJKL divided by a into b into c into n.

So, and in the total sums obviously would mean that the you have to subtract the individual sum of the errors also coming from a individually and b individually, so which are what I am going to mark now with a different color, so it will easy for us to note it yes. So, this would be SS A which you have already found out this is SS B, so they would be subtracted in order to find out SS AB.

When I move to the sum of the square a c, the concept exactly remains the same you only replace b with c. So, if I am basically replacing b with c so; obviously it will become this concept this part which are now used black red again, this remains same as this part when you use a different color green. So, this is a remains same it says be replaced by SS c because, it is now a combination of a and b and in the extreme left the term onto the right hand side of the equality would be let me use an another color well let me continue the highlighted one.

So, if you notice this and this they are exactly the same only the concept of b has been replaced by c. So, it was b so obviously sum would have been for all the c’s and ends so now it is c, so all the sums would be for b and n. If I go to sum of square of bc obviously now the a factor would be there. So, this is 1 by e n divided by n and the summations are for j is equal to 1 and k is equal to 1 to respect values of b and c and y squared would mean that if you are taking b and c so obviously b x and c is the second and the third would be gnk the first and the fourth would be summed up. So, hence they would be dot the term case, so the term y square dot dot dot dot remain the same and the values of this is pn SSc are just the replacement which you know.

So, note that the sum of the squares for the two factors of totals are found from the totals in each two way table. So, you are basically into two way tables, the 3 factor interaction sum of the squares is compounded in the similar way. So, what you do is that you find out the sum for all ijk dot, so obviously it will divided by n and i would be summed up for 1 to small a, j would be summed up from 1 to small b c would be some c would be summed up from j is equal to k is equal to 1 to small c, minus the value, which you found out y squared dot dot dot divided by abc and remain the same and now you have to basically minus the values which is SS A, SS B, SS C individual then combination of SS AB, SS B c, SS ac which are coming like this SS AB, SS BC and SS ac and finally now if I go to 4 concepts so obviously in the sum here I will just repeat it slowly you had ABCD. So, if it is ABCD so obviously you would have the sum of the squares of ABCD and what would be coming out here.

Let me go one by one so let me mark it as 1, so 1 would change to 4 summations. So, i 1 i 2 i 3 i 4 where i 1 i 2 i 3 i 4 change from 1 to small a, 1 to small b, 1 to small c, 1 to small d considering the factors are capital A, capital B, capital C, capital D and obviously 1 by n remains outside. So, inside what you have when I am marking my this pointer would be y square now rather than 4 dots there would be 5 dots, the first dot for capital A, second dot for capital B, third dot for capital C fourth dot for capital D and the last dot force n; which is the observations. Deviation would be a into b into c into d into n small n. Now what factors you will negate from there would be first singular one SS A suffix value, SS B suffix value suffix I am mentioning for a and b's SS C suffix value.

Now, you will consider the combinations of two taken at a time it will be AB. So, you will have AB, then AC and AD. So, combination of A changing B CD will come then combination will be BC, BD then would be CD, so you will basically have the combinations as shown. So, it will be taking 2 at a time for 4, so basically it will be 4 C 2 which is factorial 4 factor 2, so 4 into 3 into 2 by 2 into 2 so 3 to 6 1 2 3 4 5 6 so 6 are done.

Now, we will come to the combinations of 3 taken at a time, so obviously that would be 4 C 3 factorial 4 by factorial 3 by factorial 1. So, it will be 4 into 3 into 2 and this 3 into 2 which is 4. So, what are the 4 combinations? It will be ABC, ACD, BCD and the combination, so you will have the last one and combine them accordingly. And the last term would basically be the combination of ABCD. So, they would all come up and you can do your calculations accordingly.

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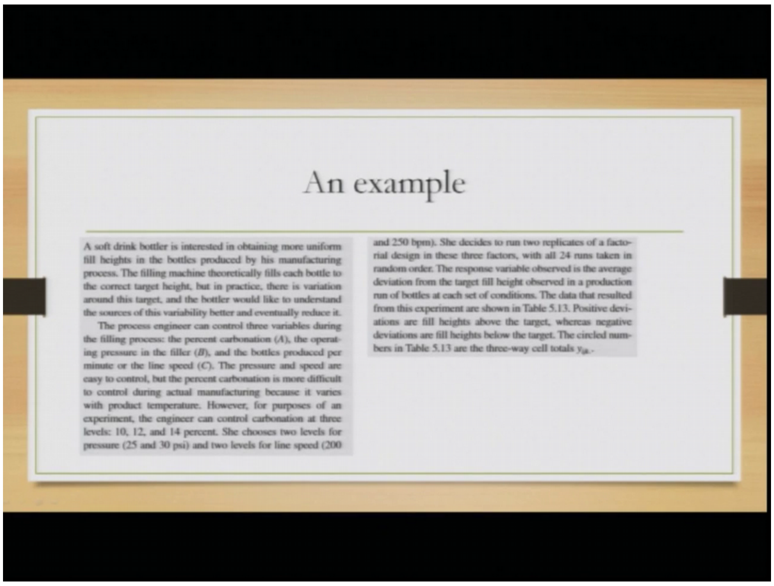

So, a soft drink bottler is interested in obtaining more uniform fill heights in bottles produced by his manufacturing process, the filling machine theoretically fills each bottle to correct target height, but in fact if there is a variation not on this target and the bottle would like to understand, the sources of this variability better and eventually reduce it. The processing engineer can control 3 variables during the filling process the percentage carbonation a operating pressure or for pressure for filler b and bottles produce per minute which is in the line speed c. The pressure and speed are easy to control, but percentage carbonation is more difficult to control during actual manufacturing process. However, for process of an experimental experiment, the engine a can control combination at 3 levels 10, 11 and 14 percent and she chooses two levels as pressures 25 and 30 psi pound square inch and 2 levels of speed which is 200 and 250.

She decides to run two replicas of a factor design in these factors with all 24 runs taken in deviations in random order, the response really will observe is the average, deviation from the target filled height observed in a production, run of bottles in each set of conditions are considered. The data thus this result that results from this experiment are shown in the table 5.1133, positive deviations are fill heights above the target, whereas negative are fill heights below the target. The circled numbers in this table is the three way set together which we just mentioned, so IJKL would be the nomenclature.

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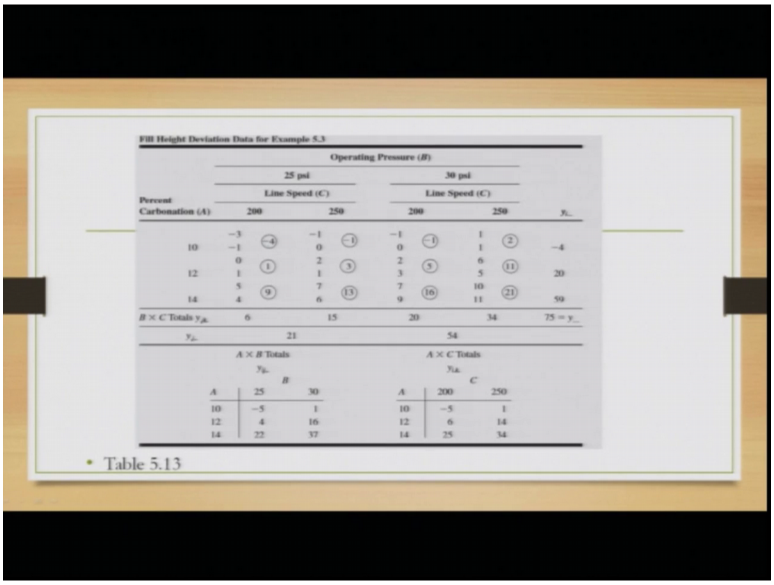
Fill Height Deviation Data for Example 5.3

Percent Carbonation (A)	Operating Pressure (B) 25 psi, Line Speed (C) 200		25 psi, 250		30 psi, Line Speed (C) 200		30 psi, 250		$y_{i..}$
10	−3, −1	−4	−1, 0	−1	−1, 0	−1	1, 1	2	−4
12	0, 1	1	2, 1	3	2, 3	5	6, 5	11	20
14	5, 4	9	7, 6	13	7, 9	16	10, 11	21	59
$B \times C$ Totals $y_{.jk.}$	6		15		20		34		$75 = y_{....}$
$y_{.j..}$	21				54				

$A \times B$ Totals $y_{ij..}$

A	B: 25	30
10	−5	1
12	4	16
14	22	37

$A \times C$ Totals $y_{i.k.}$

A	C: 200	250
10	−5	1
12	6	14
14	25	34

Table 5.13

So, operating pressures b is there, so you have 25 psi 30 psi and the percentage carbonation which you see in the first column, which is 10 12 and 14 and the line speed being 200 you had the values given as minus 3, minus 1, 0, 1, 5, 4 and for 9 speed corresponding to 250. You will have the values of minus 1 0 2 1 7 6 similarly you have for 30 psi 9 speed of 200 and 250, the values are minus 1 0 2 3 7 9 and I am not reading the circled one, which I did mention what the circuit want was in the last slide.

If those underline speed for 250 or 1 1 6 5 10 11, the values are 2, 11 and 21 and the why I dot dot dot that mean is summing up for a j for k for l or coming out to be minus 4, 20 and 59. Similarly when I go into the combinations of b and c total values, b and c would obviously, mean that we are keeping g and k fixed and you are summing out for the first factor A and the number of observations. So, those values are given and the combinations values are found not to be 6 15. So, I am reading these values 6, 15, 20 and 34 and the y value comes out to me the y summed up value comes up to be 75.

Similarly for combinations of A into B and A into C the values are given. So, the A and B value is 10 12 and 14, B values or 20 30, then the AC combinations are given a values are 10 12 14 and c values are 200 and 250.

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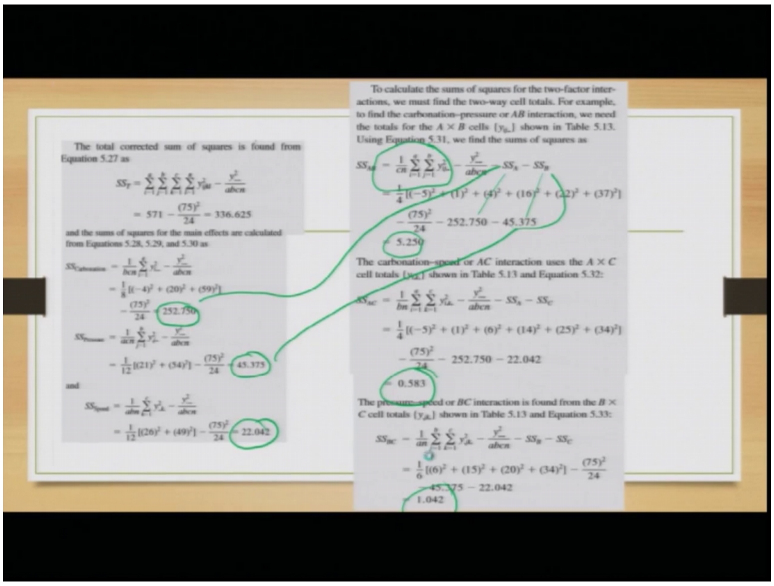

The total corrected sum of squares is found from Equation 5.27 as

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn} = 571 - \frac{(75)^2}{24} = 336.625$$

and the sums of squares for the main effects are calculated from Equations 5.28, 5.29, and 5.30 as

$$SS_{Carbonation} = \frac{1}{bcn}\sum_{i=1}^{a} y_{i...}^2 - \frac{y_{....}^2}{abcn} = \frac{1}{8}[(-4)^2 + (20)^2 + (59)^2] - \frac{(75)^2}{24} = 252.750$$

$$SS_{Pressure} = \frac{1}{acn}\sum_{j=1}^{b} y_{.j..}^2 - \frac{y_{....}^2}{abcn} = \frac{1}{12}[(21)^2 + (54)^2] - \frac{(75)^2}{24} = 45.375$$

and

$$SS_{Speed} = \frac{1}{abn}\sum_{k=1}^{c} y_{..k.}^2 - \frac{y_{....}^2}{abcn} = \frac{1}{12}[(26)^2 + (49)^2] - \frac{(75)^2}{24} = 22.042$$

To calculate the sums of squares for the two-factor interactions, we must find the two-way cell totals. For example, to find the carbonation–pressure or AB interaction, we need the totals for the $A \times B$ cells $\{y_{ij..}\}$ shown in Table 5.13. Using Equation 5.31, we find the sums of squares as

$$SS_{AB} = \frac{1}{cn}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij..}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B = \frac{1}{4}[(-5)^2 + (1)^2 + (4)^2 + (16)^2 + (22)^2 + (37)^2] - \frac{(75)^2}{24} - 252.750 - 45.375 = 5.250$$

The carbonation–speed or AC interaction uses the $A \times C$ cell totals $\{y_{i.k.}\}$ shown in Table 5.13 and Equation 5.32:

$$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} y_{i.k.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_C = \frac{1}{4}[(-5)^2 + (1)^2 + (6)^2 + (14)^2 + (25)^2 + (34)^2] - \frac{(75)^2}{24} - 252.750 - 22.042 = 0.583$$

The pressure–speed or BC interaction is found from the $B \times C$ cell totals $\{y_{.jk.}\}$ shown in Table 5.13 and Equation 5.33:

$$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{.jk.}^2 - \frac{y_{....}^2}{abcn} - SS_B - SS_C = \frac{1}{6}[(6)^2 + (15)^2 + (20)^2 + (34)^2] - \frac{(75)^2}{24} - 45.375 - 22.042 = 1.042$$

So, the total corrected some on the sequoias found out which is 336. 625 and the sum of the squares of the main fix are calculated from equations, as we already discussed the values are like this. So, for the sum of squares of the carbonation is 252 I am just reading without the different name, the decimal for sum of the square of the pressure it is 45 and the sum of the square of the speed is 22.

So, these are the effects now I want to find out the combined effects combined effects, if you find out for A and B. This value minus y square and that and then 4 dots, 4 dots means summation in the suffix divided by A into B AC into n minus SS A and I minus SS B. So, SS A n SS B would be coming from here let me write it. So, these are basically the value of j. So, that value gives me SS AB as 5.5 point something, I am again if not mentioning the decimals, similarly I follow the concept and do the calculation SS AC the value comes out to be 0.583, I have to read the decimal because in the decimal case here and for the value of SS BC it comes out to be 1.042.

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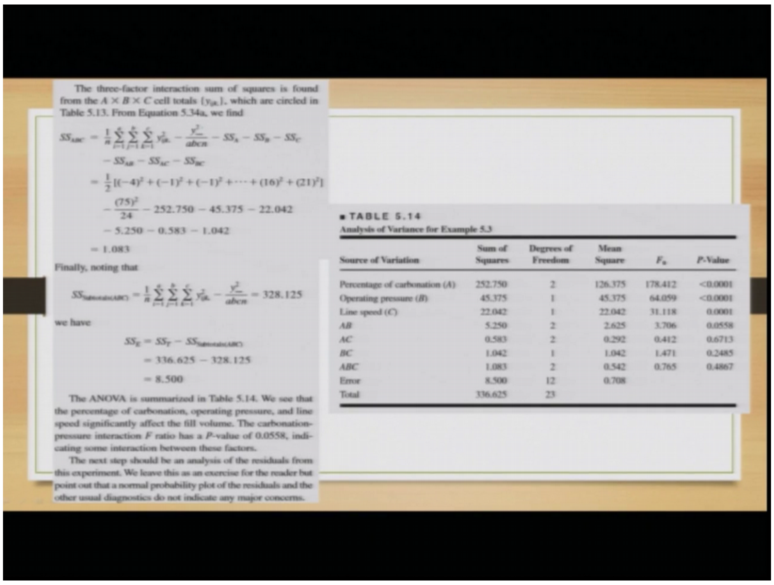

The three-factor interaction sum of squares is found from the $A \times B \times C$ cell totals $\{y_{ijk.}\}$, which are circled in Table 5.13. From Equation 5.34a, we find

$$\begin{aligned} SS_{ABC} &= \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B - SS_C \\ &\quad - SS_{AB} - SS_{AC} - SS_{BC} \\ &= \frac{1}{2}[(-4)^2 + (-1)^2 + (-1)^2 + \cdots + (16)^2 + (21)^2] \\ &\quad - \frac{(75)^2}{24} - 252.750 - 45.375 - 22.042 \\ &\quad - 5.250 - 0.583 - 1.042 \\ &= 1.083 \end{aligned}$$

Finally, noting that

$$SS_{\text{Subtotals}(ABC)} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} = 328.125$$

we have

$$\begin{aligned} SS_E &= SS_T - SS_{\text{Subtotals}(ABC)} \\ &= 336.625 - 328.125 \\ &= 8.500 \end{aligned}$$

The ANOVA is summarized in Table 5.14. We see that the percentage of carbonation, operating pressure, and line speed significantly affect the fill volume. The carbonation-pressure interaction F ratio has a P-value of 0.0558, indicating some interaction between these factors.

The next step should be an analysis of the residuals from this experiment. We leave this as an exercise for the reader but point out that a normal probability plot of the residuals and the other usual diagnostics do not indicate any major concerns.

TABLE 5.14
Analysis of Variance for Example 5.3

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Percentage of carbonation (A)	252.750	2	126.375	178.412	<0.0001
Operating pressure (B)	45.375	1	45.375	64.059	<0.0001
Line speed (C)	22.042	1	22.042	31.118	0.0001
AB	5.250	2	2.625	3.706	0.0558
AC	0.583	2	0.292	0.412	0.6713
BC	1.042	1	1.042	1.471	0.2485
ABC	1.083	2	0.542	0.765	0.4867
Error	8.500	12	0.708		
Total	336.625	23			

When I go into the third stage for a combination of ABC, I find out the summation 3 times i j k n is divided outside and then I find out y square, divided y square divide dot dot dot 4 dots dividing by A into B into C into n minus SS A minus SS B minus SS C minus of SS AC minus of SS BC minus of AC the value comes out to be 1.083.

So, with this I will close this 20th lecture and continue this example this is the first time I am not able to finish this example, I continue with this example in the 21 lecture.

Thank you and have a nice day.