Total Quality Management - II
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Lecture - 12
The Analysis of Variance (ANOVA) – V

A very good morning, good afternoon, good evening to my dear friends, who are taking this TQM II lecture and I am Raghunandan Sengupta from IME department IIT Kanpur. So, welcome to this twelfth lecture, which would basically be in in the in the third week. Which means you have completed already 5 lecture in the first week among the 5 in the second just 10 completed. So, this is the second in the third week.

So, here discussing among more different tests for etching problem, and how they can be framed considering that main assumptions which I may be going back and front, but do understand, that the main assumption being that the variance for the errors are independent on each other and they are not changing. So, they are not time dependent because on that happens so obviously, thinks go out of control. And we also consider the balanced problem, the unbalanced problem in which the case the sample size is same for each and every treatment, it is different for different treatments. And we also propose the problems that how we can consider the null habit is were all the mean values are same, alternative hypothesis is at least 2 of the mean values are different.

We can consider that the variability as are the same in the alternative case we can consider the variabilities are different, and based on that we consider that how we can use the T distribution. If remember T distribution is only to be used for the case, when we are using the mean values. Either the mean values or the difference on the mean values whatever it is. And the underlined distribution we also always considered to be normal. And the concept of chi square, and f distribution also came up depending on when is less than time below than time not equal to based on the fact that we are only interested to study something to do in the standard deviation.

So, f would be for the case when you are try to find out the ratios of 2 standard deviation from 2 population. And the chi square would be based on the fact that you are trying to understand something to the standard deviation of that population giving some information from the population itself. And we also saw the degrees of freedom would change depending on what is the efficiency laws on the samples, observations you take you utilize once twice thrice so on and so forth depending on what do you want to find out, and then that the test what were done.

And we also saw that how the cube plots could be used utilized, how the box plots could be utilized in order to understand. Whether these normality, whether these non-normality, and on the subsequent of it is. Again, I am mentioning, the slides are fine, but best would be if you refer to the book. May be little bit on the higher in the book is, but trust me it is one of the classic book which can be utilized to study the concept or design of experiments. You can go slowly, you do not have to understand each and every topics for the actually the derivation of the formulas. But try to appreciate the n result of the formula, which is being utilized and how the results for the problems which have solved how they can be utilized using the derived formulas or the theorems; so considering the orthogonal and the combined orthogonal contrast. So, if you treatments which is under control 2 which is under the level 1, 3 is under level 2. So, coefficient the orthogonal contrast are given in the table.

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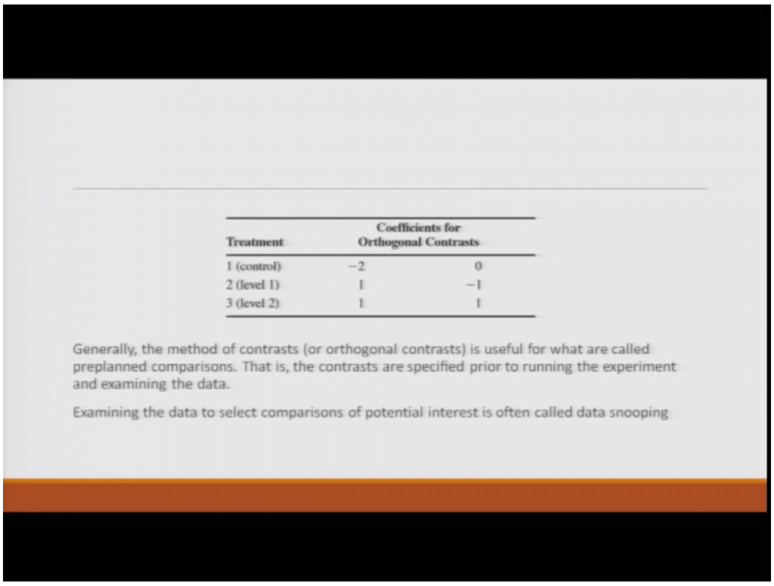

So, these values are consequently and respectively minus 2. I am going through the column, minus 2 1 1. And the other values are 0 minus 1 1. So, generally the method of contrast or orthogonal contrast is useful for what? What are called the pre-planned comparison?

So, you basically going to compare them according to some set procedures; that is, the contrasts are specified prior to running the experiment and examining the data. So obviously, you have set the goals accordingly and done the experiment. Examining the data to select comparisons of potential interest is often called data snooping. And basically, you check the data and what information can get from this data.

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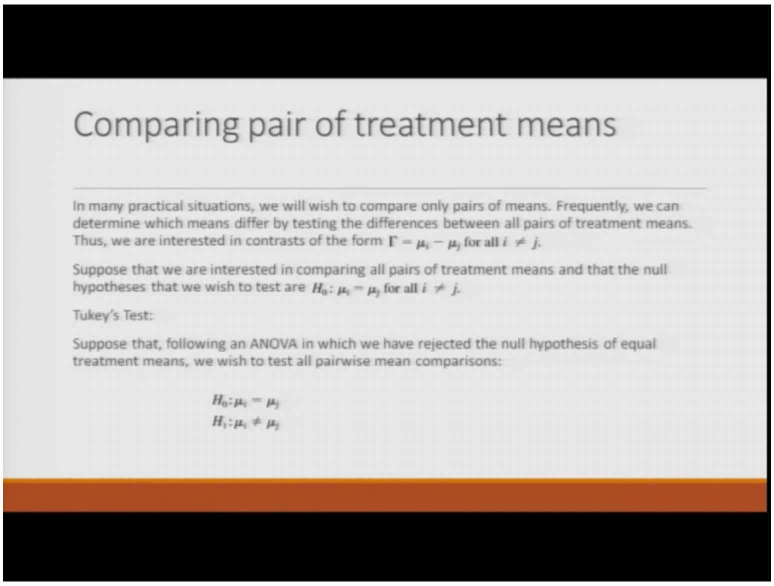

Comparing pair of treatment means, in many practical situation we wish to compare only pair of the means. Frequently, we can determine which we can determine which means differ by testing the differences between all pairs of treatment means. Thus, we are interested to in in contrast to form the difference on the means as mu i minus mu j; where i is not equal to j.

So, second example if there are such treatments, you would like to compare the difference of any 2 combinations in the means taken 2 at a time from this a set. Suppose that we are interested in comparing all pairs of treatment means, that the null hypothesis then can be basically formed it as H suffix not which is the null hypothesis means, mu suffix i is equal to mu suffix j; where i is not equal to j. And obviously, the alternate hypothesis is with the complimentary part. So, remember that the word have complimentary and mu (Refer Time: 05:02) for the first time, but obviously, you would have understood in the sense. That whatever the H naught is the total experiment, or whatever achievement, you can do you are trying to divide into and apportion that into 2 2 sets.

One would be H naught, and now is the; if you want to test the alternative hypothesis it will be the complimentary part. So, this is the Tukey test. So, suppose that follow following in in an Anova in which we have rejected the null hypothesis of equal treatment, we wish to test all pair comparisons; which means in H naught it will be each of them are unequal and H a; that means, H mu i is not equal to mu j for i is equal to 1 to j depending on whatever combinations you have. Minimum one; obviously, it will fail; that means, you support H a. More than that; obviously, you will for support H a. So, Tukey's procedure makes use of the distribution of the students range test. So, in the students range test what you find out? Want to find out is the in the ratio to maximum the minimum.

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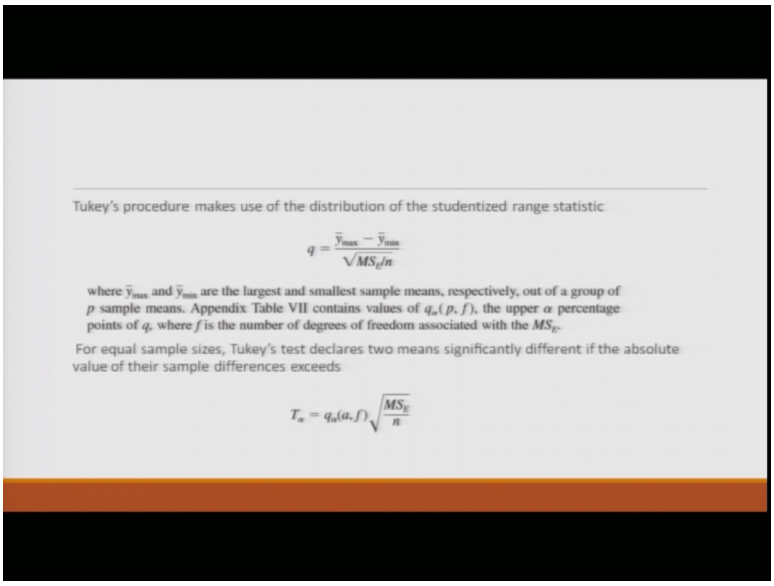

So, you want to find out the difference between the max values. And the min value and in the denominator, you will basically divide it; if you remember, if you go step back watch when you trying to basically find out the standard deviation of x bar n; which is the sample mean. So, it is basically mean value was mu, and variance was sigma square by n. So, if you find out the transformation into z distribution, it will be x bar and minus mu divided by sigma by square root of n. So, if you basically go into the denominator in this problem, it is basically square root of means squared of the errors divided by n.

So, it makes one to one significance, based on what facts which we have already learnt. And also, the fact what is there in front of us. So, where y bar max, and y bar min on the largest in the smallest sample means respectively out of a group of p sample mean. So, this this appendix, which you find in the book is basically would have all the values of this this keys of q suffix a alpha. Alpha is basically the degrees of freedom concept which you are trying to understand. P is the sample mean and while f is the number of degrees of freedom.

So obviously, you can find out the different values of q corresponding to these 3 values, the parameters alpha p and f and if the tables which is there table 7 in in in (Refer Time: 07:32) will give you all the values corresponding to this Tukey's test. For equal samples size is Tukey's test declares to means significantly different if the absolute value of the sample differences exceed. The value of T value, which is given as here, where q suffix alpha in the bracket p comma f would basically coming from the table, then you basically you will basically have the square root of means squared error divide by n. Which is basically something to do with that T table would be the capital T.

Tukey test not the small T, which is the T distribution. You will use these values and test whether less than greater than or not equal to based on that will be the support H naught, or basically reject H naught. So, these are the values which are utilized.

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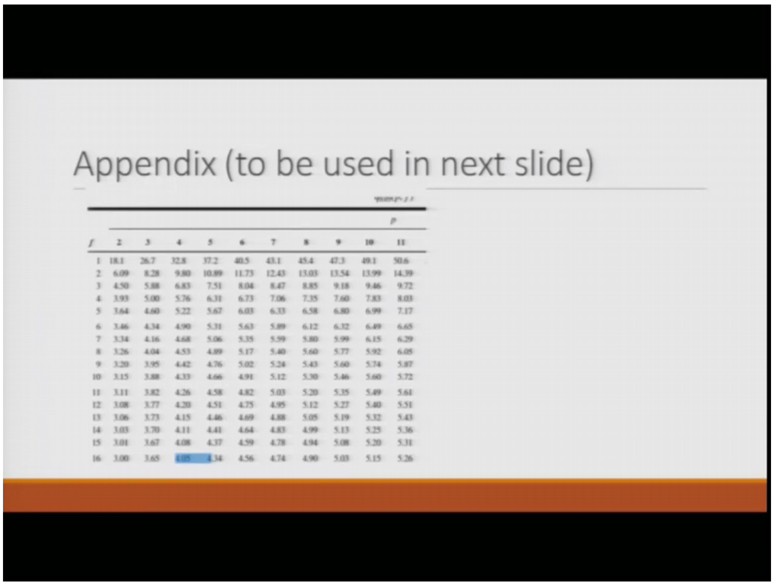

f	p = 2	3	4	5	6	7	8	9	10	11
1	18.1	26.7	32.8	37.2	40.5	43.1	45.4	47.3	49.1	50.6
2	6.09	8.28	9.80	10.89	11.73	12.43	13.03	13.54	13.99	14.39
3	4.50	5.88	6.83	7.51	8.04	8.47	8.85	9.18	9.46	9.72
4	3.93	5.00	5.76	6.31	6.73	7.06	7.35	7.60	7.83	8.03
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65
7	3.34	4.16	4.68	5.06	5.35	5.59	5.80	5.99	6.15	6.29
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87
10	3.15	3.88	4.33	4.66	4.91	5.12	5.30	5.46	5.60	5.72
11	3.11	3.82	4.26	4.58	4.82	5.03	5.20	5.35	5.49	5.61
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51
13	3.06	3.73	4.15	4.46	4.69	4.88	5.05	5.19	5.32	5.43
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31
16	3.00	3.65	4.05	4.34	4.56	4.74	4.90	5.03	5.15	5.26

So obviously, you will have f, f which is the degrees of freedom along the first column, and the values of p's are given on the top most row. And in a values basically given as the Tukey's value capital T, with the corresponding alpha value. So, alpha values can change you will have different type of tables.

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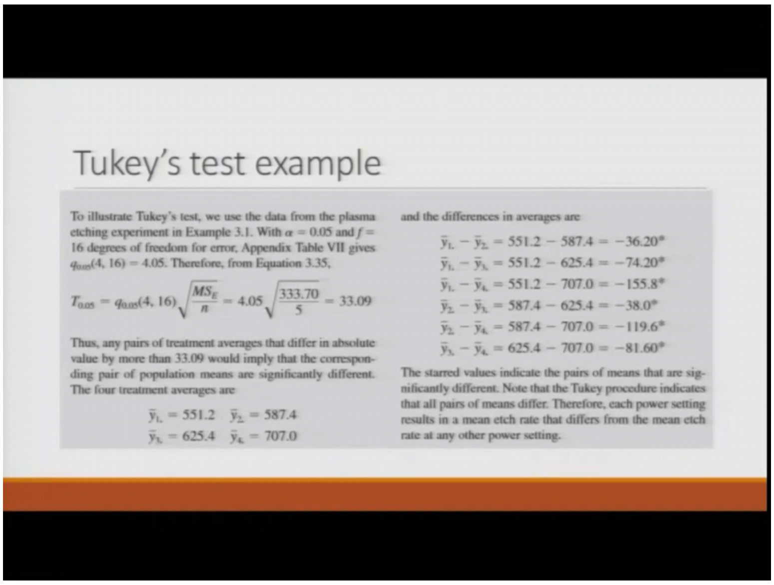

So, let us consider the Tukey's test as an example. So, to illustrate Tukey's test, you use the data from the plasma etching example; where alpha considered is 0.05.

So, 1 minus alpha would be 1 minus 0.05. F is given as 16 degrees of freedom. So, if you remember there would 20 observation; that means, 5 into 4 and for each case you have to find out the mean. So, so the how many such as were there? As was 1 2 3 4 which was corresponding to 160, 180, 200 and 220 based on that you find out the total degrees of freedom would be 20 minus 4; which is 16. So, appendix 7 in the table which we just consider if you give a q value with the corresponding alpha is 0.05, value of p as 4, and p is basically the number of samples which you have; which was a and 16 in the degrees of freedom, the value come of q comes out to be 4.05. Therefore, we can find out the capital T, which is the Tukey's value as 33.09.

Thus, any pairs of treatment average that differ in an absolute value by more than 33.09 or 0 9 whatever it is, would imply then the corresponding pair of population means a significantly difference. So, what we have is that 4 treatment values are given. So, when you find out the differences. These values are given. And the difference in the averages are given as the 36.27. I am not talking about the negative or the positive value. I am just giving the absolute value. 74.2 155.8 38 119 81. So, that the starred value which is you see all of them are starred. So, that means, they are greater than 33.09, whatever the values we find out using the tables in the Tukey's capital T value.

The starred value indicates the pairs, and means that are significant different. Note that the Tukey's procedure indicates that all pairs and means are different. Therefore, each power setting results as a mean rate etch rate is changing. So, which will means that you have to basically understand the mean values for each etching depending upon the treatment levels 1 8 160 180 200, 220, are different.

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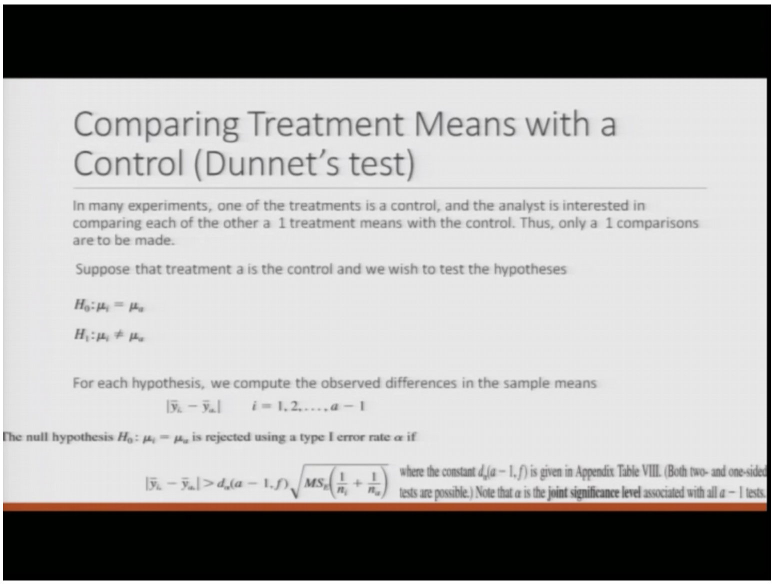

So, comparing treatment means with a controlled, and which is the Dunnet's test. In many experiments one of the treatment is a control, and the analyst is interested in comparing each of the other a other one treat with mean a control value. Thus, only a one comparison are to be made. So, what are those? Suppose the treatment is in control, and we wish to test the hypothesis which is that. So, they are, want to a, we keep the a of x. So, the null hypothesis would be mu i.

So, i is equal to 2 3 4 till the second, the second last one which will be the a minus 1. So, say for example, you are considering 10. And the fifth you want to fix so; obviously, you will try to compare the first to the fifth, second to the fifth, third to the fifth, 4th to the fifth, and 6th to the fifth, 7 to the fifth, 8 to the fifth, 9 to the fifth, and 10 to the fifth. So, that is what given a value can change. So, here H naught is mu i is equal to mu a. And on the alternative hypothesis would be mu i is not equal to mu a. For each hypothesis we find out the compute the observe difference in the sample means, and those values are given as. So, what they would be? So, you have to find out the averages. Average would be along the row; which means, for sample 1, sample 2, sample 3, sample 4 and sample 5 for that etching example. So, that you will need to find out y bar i dot. So, that difference has to be found out with respect to y bar a dot which is the fixed.

So, called average is which we have. So, here if we remember, and if you see it here it means i is changing from 1 2 3 4 till a minus 1. So, the null hypothesis would be mu i is equal to mu a with respect to H naught. So, H H naught was given like by mu i is equal to mu suffix a. And would be the complimentary part. So, you will so, the null hypothesis is rejected using the type one error rate of alpha. If you remember I did mention that there are 2 type of errors alpha and beta, in the case of hypothesis testing will keep, beta fixed at certain level, then basically do our experiments concept of alpha. It can be done the other way round also there is no problem.

So, you find out the differences. Node of that is greater than, the value which is given which is d suffix alpha a minus 1 comma f. F is basically the total degrees of freedom. In a minus 1 would be the so-called sample size. With the constant d suffix alpha a minus 1 comma f is given in the table 8. So, both that one side and 2 side values are given. Note that alpha is the joint, significance level as we did a minus 1 test. Because a was the total number of etching. So, minus 1 because you are basically keeping one of them fixed.

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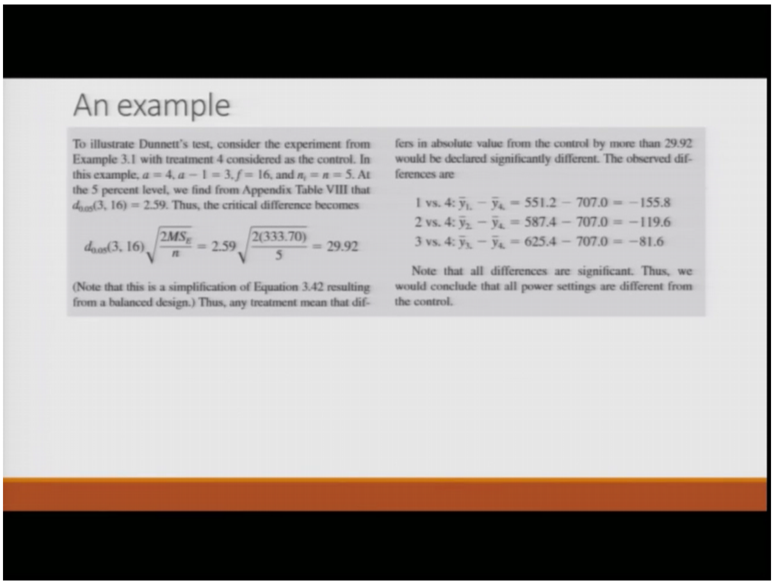

To illustrate this and direct test, consider the experiments from example 2.1, which is the etching example. Treatments are 4 in number considered.

So, a is 4, a minus 1 is 3, f is 16. So, that is 20 minus 4. And n i is because all of them are equal, then they are 5. So, n 1 n 2 n 3 n 4 are all 5. So, at the 5 percent level we find out using table 8, the value of d with corresponding alphas and a minus 1 and f coming out to be 29.92. So, for basic form that fact. So, we try to find out the differences. So, the differences as the observed difference are given where I am basically pointing my finger. So, these are they are without the negative and the positive sign, I am just giving the values. It is 155.8 119 and 0.6 and 81.6; so the node that all the differences are significant. Thus, we would conclude that all part settings are different from the controlled one which is given.

So, you can basically change the control one go to the first one control, and basically compare the second third 4th with the first one. Then you basically you can fix second compare with first third and forth. You can do that experiments. So, it consider that if there is reason, reason workmen working on some trying to manufacture some tie rod, or trying to basically manufacture some gear. Or you have some special jigs and fixtures; which have been put on the sophisticated seen it is a machine. Anyone to compare the output of that machine keeping that let us fix with the other new machine which have been just purchased. So, you can do this type of experiments on standard test or compare the variability on the difference in the means and compare how those other production processes are going on with respect to the fixed one, where you have basically much control and where you aware of the quality levels.

So, the 2 sided comparison values are given which is the table 8 which were talking.

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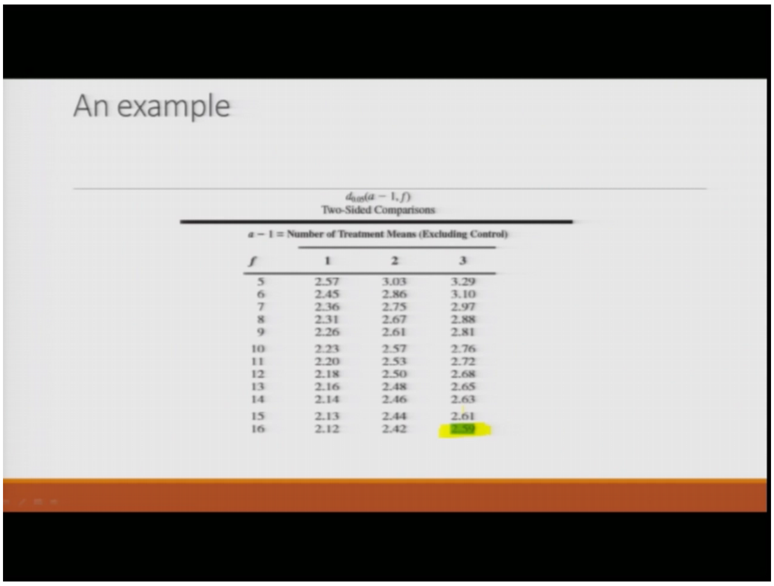

$d_{0.05}(a-1, f)$
Two-Sided Comparisons

	$a-1$ = Number of Treatment Means (Excluding Control)		
f	1	2	3
5	2.57	3.03	3.29
6	2.45	2.86	3.10
7	2.36	2.75	2.97
8	2.31	2.67	2.88
9	2.26	2.61	2.81
10	2.23	2.57	2.76
11	2.20	2.53	2.72
12	2.18	2.50	2.68
13	2.16	2.48	2.65
14	2.14	2.46	2.63
15	2.13	2.44	2.61
16	2.12	2.42	2.59

So, the a minus 1 which is the number of treatment means exclude in the control ones are given. So, here basically 1 2 3, so which is the second column, third column, forth column, and the first column is f which basically the degrees of freedom. So, if you remember a degrees of freedom here it is 16. And if there a minus 1 value is 3. So, if you if you noted. Even though I didn't mention in in table 7, but I try to basically highlight this. So, 2.59 would be utilized as the d value and the calculations done correspondingly.

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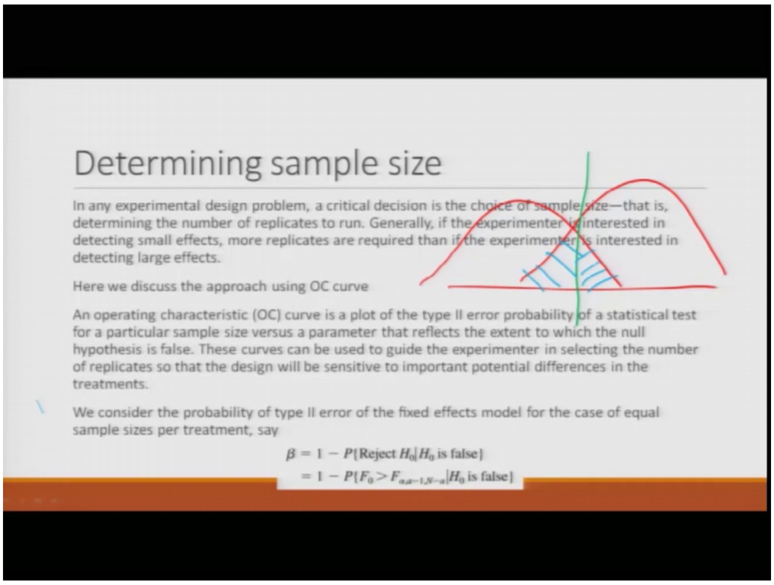

So now you want to determine that the sample size. So, the next discussion would be determining the sample sizes. In any experiment design, problems are critical decision is the choice of the sample size; that is, determining the number of replicates to run.

So, generally the experiments is interested in detecting small effects more replicates are required, then if they interested to determine large affects, you basically use this. Here we discuss an approach this problem using the OC curve, the operating characteristic curve which we did people have done this TQM one course, we have consider that. So, an operating characteristic of is a plot of the type 2 error probability of a statistical test. For particular sample size versus a parameter that reflects the extent to which the null hypothesis is false.

So, these curves can be used to guide the experiments to selecting the number of replicates so that the design will be sensitive to important potential differences in the treatment and you can basically take decisions accordingly. We consider the probability of type 2 error. So, type 2 if you remember type 1 and type 2. Type 1 was error was given by alpha, and type 2 basically given the by the value of beta. So, type 2 error of the fixed effect model for the case of the equal sample size per treatment. So, here beta would basically be. Because is going in a complimentary part.

And if you remember the example which I gave of like, you are a banker and a manager, and you want basically disburse a loans of you have basically cutoff points based on which you decide whether you want to give the loan, and whether you want to not give the loan. So, any a set of persons who have 60 points credit points and numb up, you will give the loan. And any set of persons who have scores less than 60, you would deny the loan.

So, obviously, beta would be if you consider the complimentary part, and let me draw the diagram so again for the benefit. So, this was the line you had 2 distributions. And I used so technically those were the H naught and H as. So, let me change the color to. So, this was the straight value. So now, you have the values just give me 1 minute. So, this was the so called alpha and beta value we have considered. Based on that we did r or l this is r. Try to basically give the rules accordingly ah. So, beta would be 1 minus probability of rejecting H naught when H naught is in is false ah. And obviously, that would be counter part of alpha. So, in this case you will basically have for the f test it will be 1 minus probability of F naught being greater than that f under H naught would be greater than F alpha or 1 minus alpha wherever less or greater depending on higher framed hypothesis. And a minus 1 would basically be the sample size. Because you once one year keeping fixed, and the others you are trying to compare with a fixed one.

So, in this case treatment one was the last one which is fixed which is the 221. And the total degrees of freedom as we already know would be 4, I am repeating it please excuse me, it will be basically 4 into 5 minus 4 which is 16. And based on the fact that H naught is false. Because, under H naught you will consider H naught to be true. Under the alternative one it will be false.

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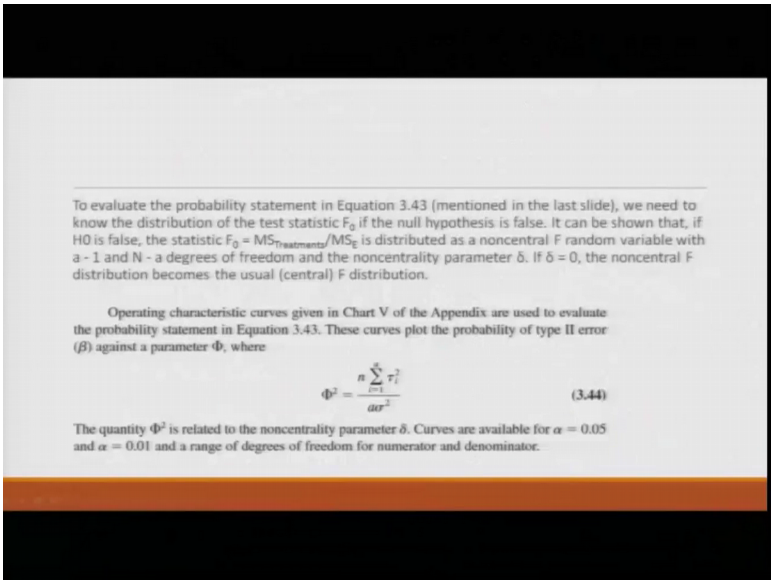

So, to evaluate probability statement of equation 3.43 as per the book; so mentioned earlier, we need to know the distribution of the test as statistics F naught if the null hypothesis is false. It can make sure that if H is false, then F naught which basically the ratio of means square of the treatment by means square of error is distributed and non-central f distribution, with a minus 1 and n minus 1 degrees of freedom and non-centrality parameters respectively. So, a minus 1 would basically be the degrees of freedom. And if and the parameter value is basically delta.

So, if delta is 0, the non-centrality values the overall skewness would not be there. So, operating characteristics of given in in chart 5 in the appendix are used to evaluate the probability statements of equity related to the equation we are just discussed. And will consider that the curved plots of the probability of type 2 errors are given against the parameter value of capital 5. So, when the capital value is given by this equation 3.44 which is basically of the ratio of; so, if you remember tau was basically the differences between the average of the average of means minus the corresponding average to each and every treatment; which is basically mu i minus mu.

So, obviously, they would be an error. And one of the habit is we did frame it as tau is equal to 0, which is H naught with respect to the null hypothesis means tau were not 0. Or any one of the tau values were not 0. So, in the ratio we have in the numerator the sum of the tau value square, because you are considering basically intrinsically even though I did not mention we are considering the squared error loss to be true ah. Because that gives us some implication of the concept of the variance if you trying to be basically minimize squared error loss it gives us some implication that we are trying to basically minimize the variance.

Divided by; so, that would be the numerator will be multiplied by n the sample size for each and every treatment divided by a, a is basically the one which is the total number of such treatment which you have into sigma square, if you remember we are taking sigma square as basically the standard deviations ah, square of that will be sigma square. And we also consider the errors are independent of each other.

So, the quantity capital 5 square is related to the non-centrality parameter delta. So, it has nothing to do with so called the capital 5 and the small 5 is saw or some many of you know in the case of normal distribution. So, curves are available for alpha is equal to 0.05 equals to 0.01 and so, and these values. And at a range of degrees of freedom for the numerator and denominator are given. And you can utilize those values accordingly.

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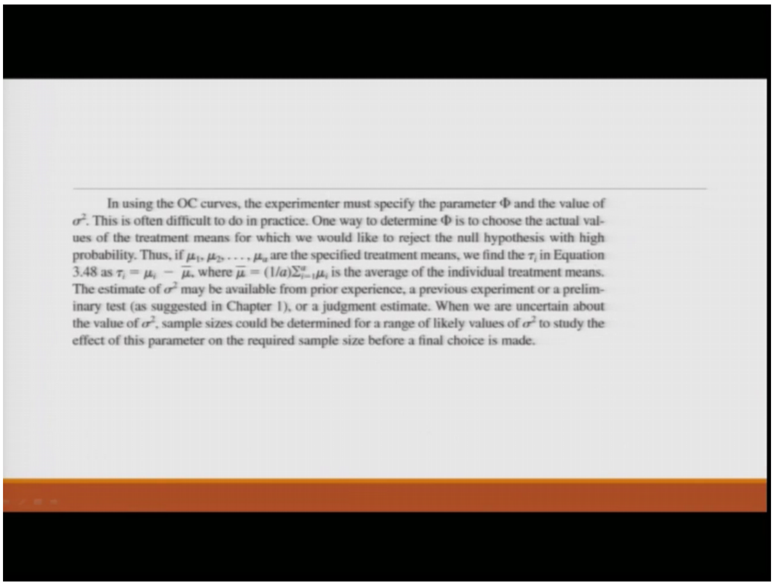

So, in using the OC curve the experimenter must specify the parameter capital 5. And the value of sigma square which is basically given by the experimenter, and depending on the data which were utilizing. This is of 10 difficult to do in practice. So, one way to determine capital 5 is to choose the actual value of the treatment means for which you would try to basically like to reject null hypothesis with high probability. Thus, if mu 1 mu 2 mu suffix one mu suffix 2 and so on. And so, for mu suffix a; this specified treatment means, we find tau I is as per the equation 3.48. So, these values are given. So, mu tau I would be mu i minus mu bar. Mu bar basically be being utilized as the best estimate for the average of the average from the population. We are using the sample to basically estimate the population average mean. And this mu bar is basically given by the sum of all the means divide by a.

Because there are such a treatments; which is the average of the individual treatment means, the estimate of sigma square may be available from prior experience or previous experiment or a preliminary test as suggested, or the judgment estimate can be made. So, when we are uncertain about the values of sigma square sample size could be determined for a range of likely values of sigma square. To study this each effect of this parameters and required sample can be chosen before a final decision is making accordingly.

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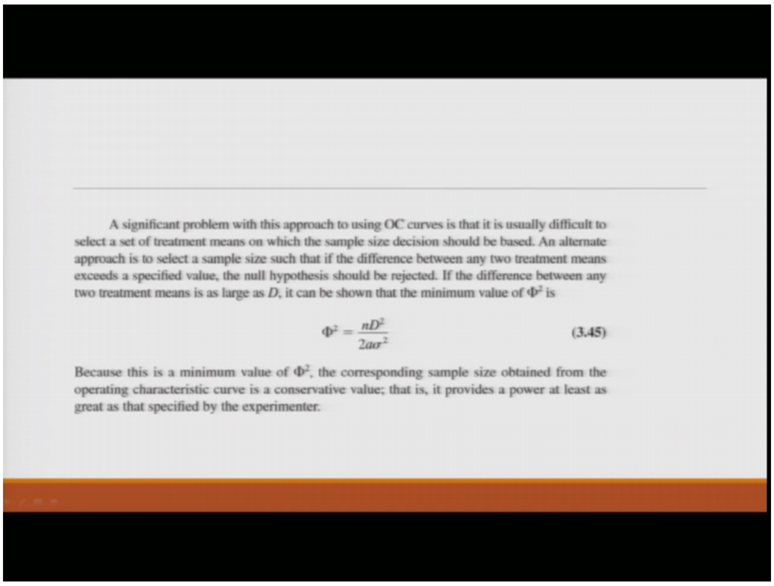

A significant problem with this approach in using OC curves is that it is usually difficult to select a set of treatment means on which the sample size decisions would be based. So, we are not certain what is could be sample size. So, it becomes difficult. And alternative approach is to select a sample size as that if a difference between any 2 treatment means exceed a specified value the null hypothesis should be rejected if the difference between any 2-treatment means is as large as the so, it can be utilized as the minimum value or sigma square. And that value is given by n into d square by 2 a sigma square.

So, this remains the same. Because this is the minimum value of sigma square, the corresponding sample size obtained; from the operating characteristics curve is a conservative value. That is, it provides a power at least as great as that specified by the experimenter.

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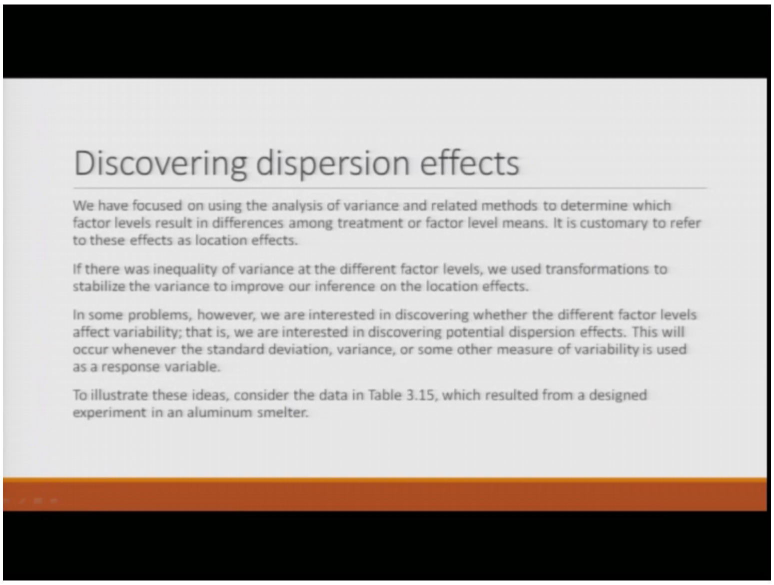

Discovering dispersion effects: so you are focused on using the analysis of variance and related methods to determine which factor values or results in difference among treatment or factor level means. So, it is customary to refer to these effects as location effects. So, we have consider the concept of difference on the means, considering something with the tau values being 0, consider something to do with sigma square as 0. So, I am talking about H naught, so obviously, ha would be the alternative part. We also consider that we are trying to keep any one of them comparison as fixed value and trying to compare the average means with rather fixed value.

So, for I example considered a which is the last one, which is fixed 220 volts, or the voltage powers sorry voltage powers. And then we saw that how different type of test, or Tukey’s test, T test, f test chi square test, Bartlett test all could be utilized depending on the framework of the problem. So, if they were inequalities of variance at different factors levels, we use transformation to stabilize them the variance in order to improve our inference on the location effects.

So, in some problems; however, we are interested in discovering whether the different factor levels affect variability; that is, we are interested in discovering potential dispersion effect which would basically be may be major parts on later on. So, you want to take some corrective actions for that. This will occur whenever the standard deviation variances or other some other measure of variability is used as a response value. To illustrate this ideas consider let us consider the data has given which results from a design experiment of an aluminum smelter.

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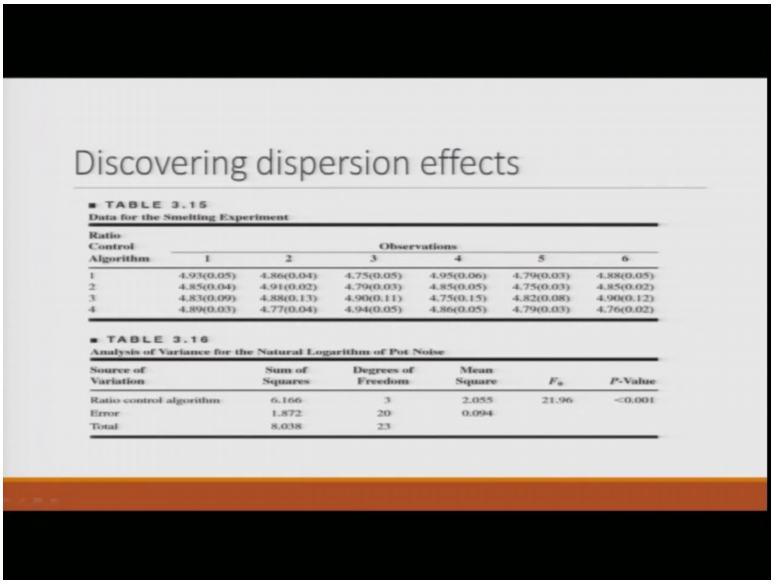

■ TABLE 3.15
Data for the Smelting Experiment

Ratio Control Algorithm	Observations 1	2	3	4	5	6
1	4.93(0.05)	4.86(0.04)	4.75(0.05)	4.95(0.06)	4.79(0.03)	4.88(0.05)
2	4.85(0.04)	4.91(0.02)	4.79(0.03)	4.85(0.05)	4.75(0.03)	4.85(0.02)
3	4.83(0.09)	4.88(0.13)	4.90(0.11)	4.75(0.15)	4.82(0.08)	4.90(0.12)
4	4.89(0.03)	4.77(0.04)	4.94(0.05)	4.86(0.05)	4.79(0.03)	4.76(0.02)

■ TABLE 3.16
Analysis of Variance for the Natural Logarithm of Pot Noise

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Ratio control algorithm	6.166	3	2.055	21.96	<0.001
Error	1.872	20	0.094		
Total	8.038	23			

So, consider this. So, we have the data for the alumining the smelting process. We have the ratio control values which are given, which are the numbers are 1 2 3 4; which is the first column in in table 3.15. And the observations are 6 in number for each and every row.

So, they start basically from 4.93, and go to 4.88. And I only keep reading first and the last value for the rows. Similarly, we have in the forth row, the values are 4.89 to in the last value which is 4.76. And the analysis the variance the test values are given. So, the ratio the control algorithm the errors in the total error total; obviously, be the sum of them are given as 6.166 for the ratio of control that the errors are given by 1.872. And the sum is basically be sum of both of these the degrees of freedom on given; obviously, degrees would freedom would also adopt to be the 7. So, ratio of the control algorithm degrees is free. For the error is 20 the total is 23. The means square values are given, which are corresponding to the first row for the value which is the ratio control algorithm 2.055 and as error being 0.094. F naught values are given as 21.96 and the p values considering to that experiment, we find out and depending on the level of significance whatever it is.

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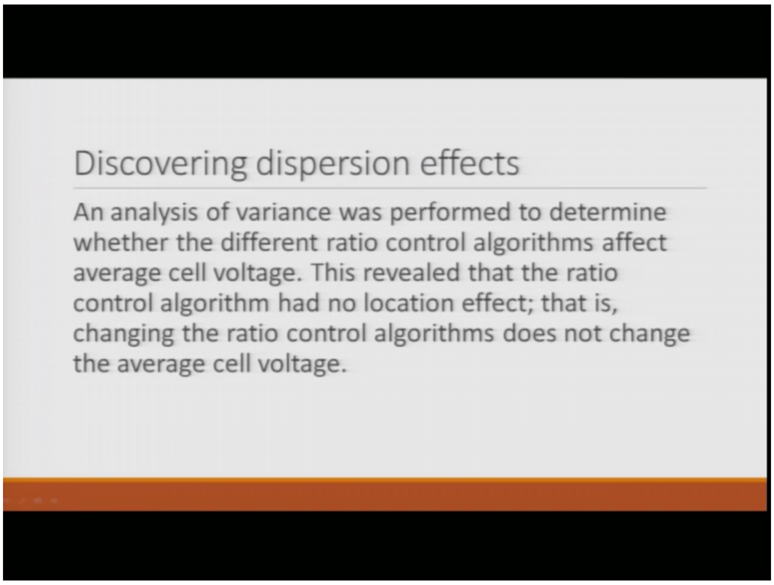

So, discovering from the dispersion effects, an analysis of variance was performed to determine whether these different ratio controls affecting the average of the cell voltages of the smelting process. This revealed that the ratio control algorithm had no location effects that is the changing the ratio control algorithm. Algorithm does not change the average cell voltages. So, and then we basically we can basically, I didn't go into the example in details. But we have done that etching problem in details in different perspective in different angles so that will be given idea. And how those examples can be informations can be utilized for solving such problems.

So, with this, I will end the twelfth lecture, and continue discussions of Anova and factorial design later on. And I wish any queries which have come up in the first 2 weeks, we would try our level best to answer them. And obviously, the assignments you have already taken care of that. And somebody of the first and second as per the norms or dates given and will continue discussions for this TQM II lecture further.

Thank you very much. Have a nice day.