Total Quality Management - II
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Lecture – 11
The Analysis of Variance (ANOVA) – IV

A very good morning, good afternoon, good evening to my dear friends who are taking this TQM II lecture, and I am Raghunandhan Sengupta from IME department IIT Kanpur. So, welcome to this 12th lecture which would basically be in the third week, which means you have completed already 5 lectures in the first week, another 5 in the second week; 10 completed.

So, this is the second in the third week. So, you are discussing about different tests for the etching problem, and how they can be framed, considering that main assumptions which I have, may be going back and foot, but do understand that the main assumptions being that the variance for the errors are independent on each other and they are not changing. So, they are not time dependent, because; so if that happens; obviously, things go out of control and we also consider the balanced problem, the unbalanced problem in which the case, the sample size is same for each and every treatment.

It is different for different treatments and we have also proposed the problems that how we can consider and the null hypothesis, where all the mean values are same, alternative hypothesis at least 2 of the mean values are different. We can consider that the variability are the same in the alternative case. We can consider that the variabilities are different and based on that we consider that how we can use the T distribution. If we remember T distribution is only to be used for the case. When we are using the mean values, either the mean values or the difference of the mean values, whatever it is.

And in the underlying distribution we also, always considered to be normal, and the concept of chi square and f distribution also came up depending on whether it is less than time greater than type, not equal to; based on the fact, we are only interested to study something to do with the standard deviation. So, f would be for the case, when you are trying to find out the ratios of two standard deviation from two population, and the chi square would be based on the fact that you are trying to understand something to the standard deviation of that population, given some information from the population itself.

And we also saw the degrees of freedom would change depending on what is the efficiency loss or the samples observations you take, utilize it once, twice, thrice so on and so forth, depending on what you want to find out and then the tests, what was done.

And we also saw that how the q q plots could be used to utilize, how the box plots could be utilized in order to understand whether there is normality, whether there is tone normality and all the subsequent topics. Again, I am mentioning the slides are fine, but best would be if you refer to the book, maybe a little bit on the higher end the book is, but trust me it is one of the classic books which can be utilized to study in the concept or design of experiments. You can go slowly, you do not have to understand each and every topics for the, actually the derivation, the formulas, but try to appreciate the end result of the of the of the formula which is being utilized, and how the results for the problems which are solved, how they can be utilized using the derived formulas or theorems.

So, considering the orthogonal line and the, the combined orthogonal contrast; so if you have treatments, which is under control to which is another level 1 and 3 is under level 2. So, coefficient the orthogonal contrasts are given in the table.

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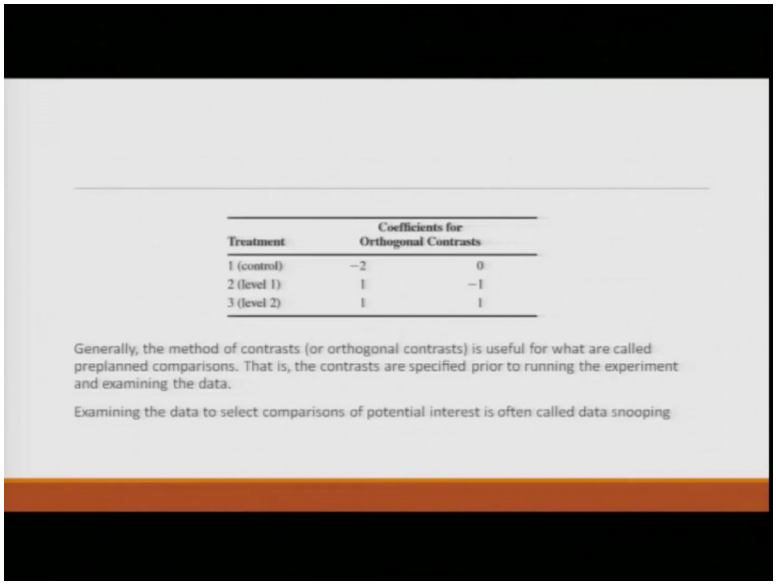

So, these values are consequently and respectively 2. I am going through the column 2 1 1 and the other values are 0 minus 1 1. So, generally the method of contrast or orthogonal contrast is useful for what, what are called the pre-planned comparison. So, you are basically going to compare them according to some set procedures; that is the

contrast as specified prior to running the experiment and examining the data. So; obviously, you have said the goals accordingly and done the experiment. Examining the data to select comparison of potential interest, is often called data snooping and you, basically you check the data.

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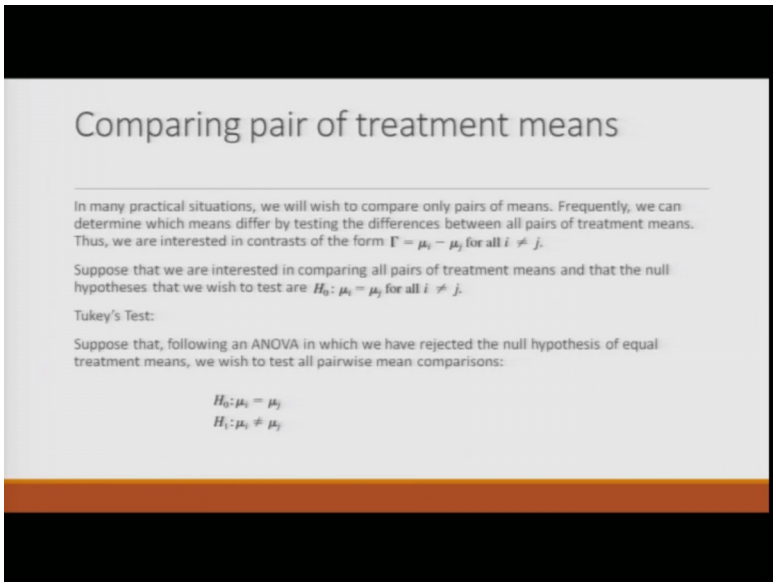

And what information can get from this data company pair of treatment, means any many, in many practical situation we wish to compare only pair of the means frequently, we can determine which we can determine which means differ by trusting the difference between all pairs or treatment means. Thus we are interested in in contrast to form my, the difference on the means as mu i minus mu j, where i is not equal to j.

So, say for example, if there are such treatments we would like to compare the difference of any two combinations on the means taken 2 at a time from this a set. Suppose that we are interested in comparing all pairs of treatment between means, then the null hypothesis, then can be basically formulated as H suffix naught which is the null hypothesis, means mu suffix i is equal to mu suffix j, where i is not equal to j and; obviously, on the alternative hypothesis would be the complementary part.

So, remember that the word I am complimentary, I am utilizing for the first time, but; obviously, you would have understood in the sense that a whatever the h naught is the total experiment or whatever achievement you can do, you are trying to divide into and apportion that into 2 two sets one would be the H naught and; obviously, if you want to

test the alternative hypothesis it will be the complementary part. So, this is the Tukey's test. So, suppose that follow, following in an ANOVA in which we have rejected the null hypothesis of equal treatment. We wish to test all pair comparisons which means in h naught, it will be each of them are unequal and a h H a; that means, a h H nu mu i is not equal to mu j for i is going to, want to depending on whatever combinations, you write minimum one; obviously, it will fail; that means, you will support H a more than that; obviously, will for support H a.

So, Tukey's procedure makes use of the distribution on the students range test. So, in the students range test what you find out, want to find out is the, in the ratio, it will be in the maximum and the minimum. So, you want to find out the difference between the max value in the min value, and in the the in the denominator you will basically divide it. If you remember, if we go step back watch when you are trying to basically find out the standard deviation of x bar n which is the sample mean. So, it is basically mean value was mu and variance was sigma square by n. So, if you if you find out the transformation into z distribution, it will be x bar n minus mu divided by a sigma by square root of n.

So, if you basically go into the denominator in this problem, it basically square root of mean squared of the errors divided by n. So, it makes a one to one significance based on what facts which you have already learned and also the fact what is there in front of us. So, where y bar max and y bar mean are the largest in the smallest sample means respectively, out of a group of p sample means.

So, this, this appendix which you find in the book, which basically would have all the values of this, this case of q suffix a alpha, alpha is basically the degrees of freedom concept which you are trying to understand, p is the sample means and while f is the number of degrees of freedom.

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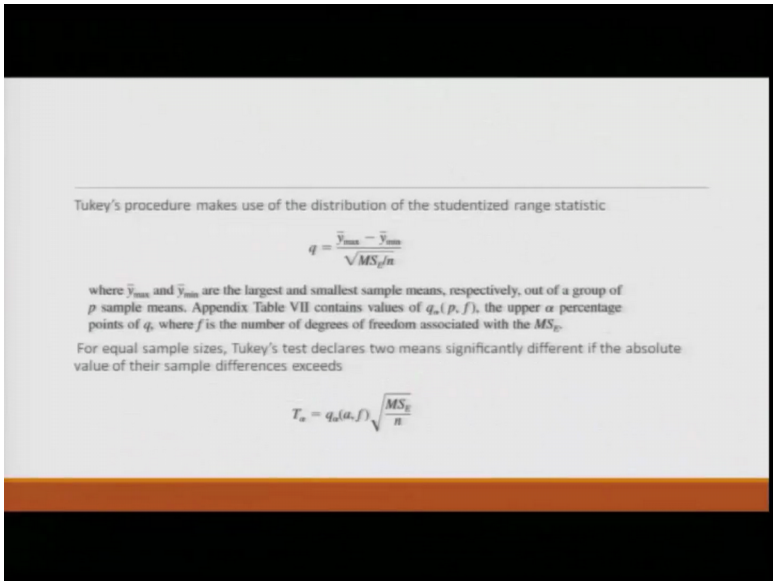

So obviously, you can find out the different values of q corresponding to these three values; the parameters alpha p and f and in their tables which is there, table 7. In Montgomery will give you all the values corresponding to this Tukey's test for equal sample sizes. Tukey's tests declares two means significantly different, if the absolute value of the sample diff differences exceed the value of t value, which is given as here, where q suffix alpha in the bracket, p comma f would basically be coming from a table, and then you will basically have the square root of mean squared error divided by n, which means basically something to do with the t table would be the capital T Tukey's test, not the small t which is the T T distribution. You will use these values and test where this less than greater than or not equal to, based on that you need the support H naught or basically reject H naught.

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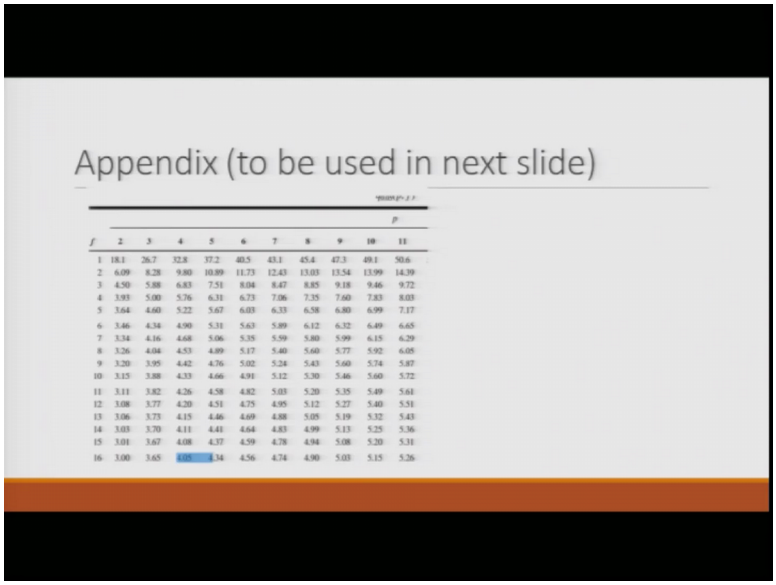

So, these are the values which are you place. So; obviously, you will have f, f which is the degrees of freedom, along the first column and the values of p s are given on the topmost row, and the inner values are basically given as the Tukey's values capital T with the corresponding alpha value. So, alpha values can change, you will have different type of tables.

So, let us consider the Tukey's test as an example. So, to illustrate Tukey's test we use the data from the plasma etching example, where alpha considered is 0.05. So, 1 minus alpha would be 1 minus 0.05, f is given as 16 degrees of freedom, because if you remember they were 20 observation; that means, 5 into 4. And for each case you have to find out the mean. So, so the, how many such case were there, a was 1 2 3 4, which was corresponding to 160 180 200 and 220. Based on that you find out the total degrees of freedom would be 20 minus 4 which is 16. So, appendix 7 in the table, which we just considered, it will give a q value with the corresponding alpha is 0.0 value of p as 4.

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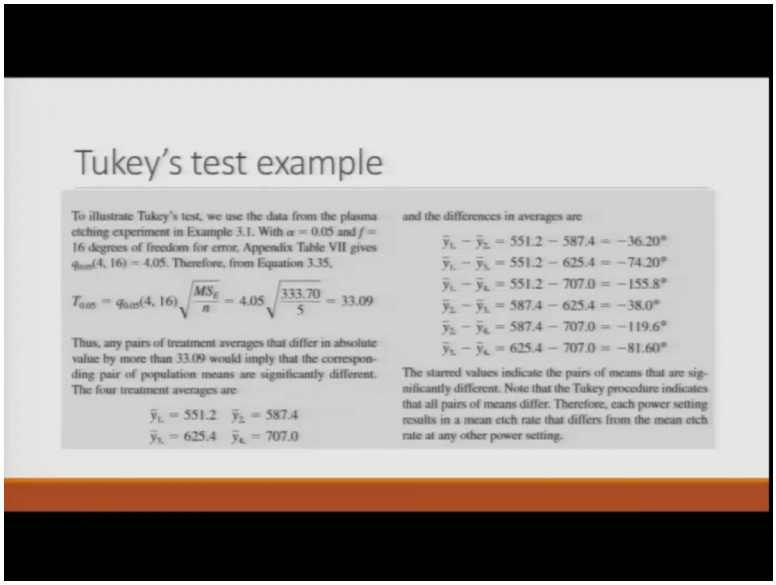

And p is basically the number of samples which you have, which was a and 16 be the degrees of freedom, the value come of, q comes out to be 4.05. Therefore, we can find out the capital T which is the Tukey's value as 333.09. Thus any pairs of treatment average that differ in absolute value by more than 33.1 or 09. Whatever it is, would imply that the corresponding pair of population means a signatory difference. So, what we have is that for treatment values are given.

So, when we find out the differences, these values are given and the difference in the averages are given as 36.27 70. I am not talking about the negative, the positive value I am just giving the absolute value 74.21 55.8 3800 1981. So, the start value which you see. So, all of them are starts. So; that means, they are greater than 33.09. Whatever the values, we find out using the tables in the Tukey's capital T value. The start values indicate the pairs of means that are significantly different. Note that the Tukey's procedure indicates that all pairs of means are different. Therefore, each power setting results as a mean rate, etch rate is changing. So, which would mean that you have to basically understand the mean values for each etching, depending on the treatment levels 18 161 to 82 200 and 220 or different.

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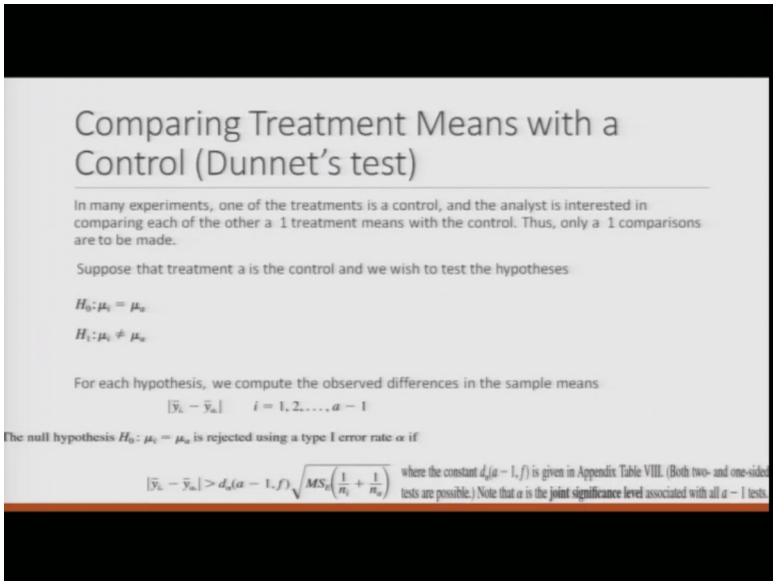

So, comparing treatments means with a controlled and which is the Dunnet's test in many experiments. One of the treatment is a control and the analysis is interested in comparing each of the, other a one treatment mean with the control value. Thus only a 1 comparisons are to be made. So, what are those; suppose the treatment a is in control and we wish to test the hypothesis which is that. So, they are a one to a, we keep the a fix. So, the null hypothesis would be mu i. So, I is equal to 1 2 3 4 till the second and the second last one, which will be the, a minus 1. So, say for example, you are considering 10 and the fifth you want to fix. So; obviously, you will try to compare the first to the fifth, second to the fifth, third to the fifth, four to the fifth, then six to the fifth, seven to the fifth, eight to the fifth, nine to the fifth and ten to the fifth.

So, that is what is given. So, a value can change. So, here H naught is mu i is equal to mu a, and on the alternative hypothesis would be mu i is not equal to mu a. For each hypothesis we find out the, compute the observed difference in the sample means, and those values are given as. So, what they would be. So, you have to find out the averages. Average would be along the row which means for sample; 1 sample, 2 sample, 3 sample for example, 5 for the etching example. So, that you will need to find out y bar i dot. So, that difference has to be found out with respect to y bar a dot, which is the fixed, so called averages which you have.

So, here if you remember and if you see it here; here it means i is changing from 1 2 3 4 till a minus 1. So, the null hypothesis would be mu i is equal to mu a with respect to H naught; so H naught. So, H naught H naught was given like y mu i is equal to me, mu

suffix a and H a would be in the complementary part. So, you will; so the null hypothesis is rejected using the type 1 error rate of alpha. If you remember I did mention that there are two type of errors; alpha and beta. In the case of H hypothesis testing, we keep beta fixed at a certain level and then basically do our experiment with the concept of alpha, it can be done the other way around not round also.

There is no problem. So, you find out the differences mod of that, is greater than the values which is given, which is d suffix alpha a minus 1 comma f. f is basically the total degrees of freedom and a minus 1 would be the so called sample size with the constant d suffix alpha e minus 1 comma f is in the table 8. So, both that one sided and there are two sided values are given. Note that alpha is the joint significance level, associated with a minus 1 test, because e a a was the total number of etching. So, a minus 1, because you are basically keeping one of them as fixed.

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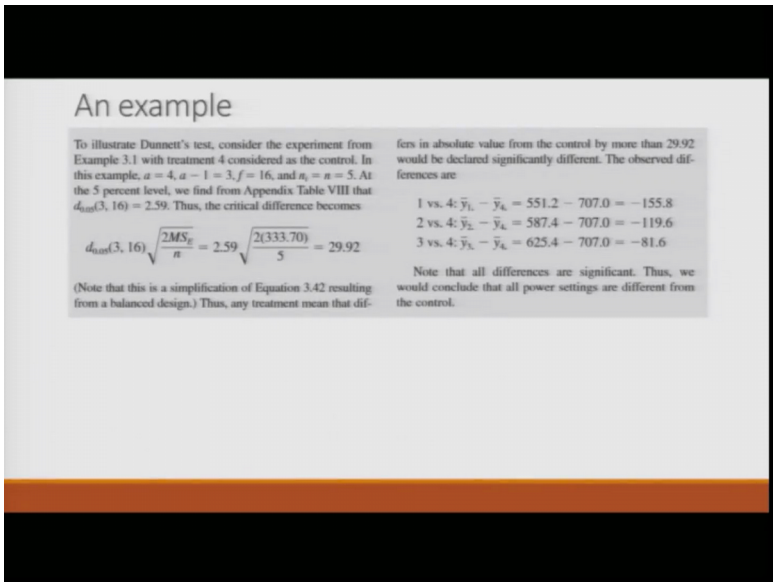

To illustrate this standard test, consider the experiment from the example 3.1 which is the etch, etching example treatments are 4 in number considered. So, a is 4 a minus 1 is 3 f is 16, because that is 20 minus 4, and a n is, because all of them are equal, they are 5. So, n 1 n 2 and 3 and 4 are all 5. So, at this 5 percent level we find out using table 8, the value of d with corresponding alphas, and a minus 1 and f coming out to be 29.92. So, based on that fact we try to find out the differences; so the differences, as observed differences are given when I am basically pointing my finger. So, these are the, they are without the

negative of the positive sign. I am just giving the values, there is 155.8 119 and 0.6 and 8 to 1.6.

So, note that all the differences are significant, thus we would conclude that all power settings are different from the controlled one, which is given. So, you can basically change the controller one, go to the first one control and basically compare the second, third, fourth with the first one. Then you basically can fix second, compare with first, third and fourth. You can do that experiment. So, you consider that if there is this and there is an workmen working on some, trying to manufacture some tied or trying to basically manufacture some gear or you have some special jigs and fixtures, which have been put on the sophisticated CNC machine, and you want to compare the output of that machine, keeping that that has fixed with the other new machines which have been just purchased.

So, you can do this type of experiments on standard testing, compare the variability or the difference in the means and compare how those, other production processes are going on with respect to the fixed one, where do you have basically much control and very aware of the quality levels.

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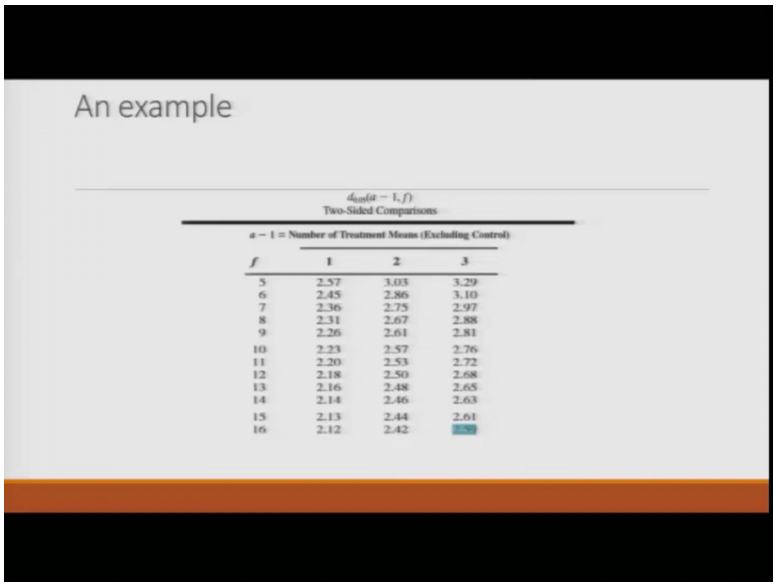

So, the two times sided comparison values are given which is the table 8, which were caught talking. So, the e minus 1 which is the number of treatment means, excluding the control ones are given. So, you have basically 1 2 3. So, which is the second column,

third column, fourth column, and the first column is the f which is basically the degrees of freedom. So, if you remember the degrees of freedom, here it is 16, and if they are a minus 1 value is 3. So, if you if you note it, even though I did not mention in tables 7, but I will try to basically highlight it. So, this 2.9 would be utilized as the d value and the calculations done correspondingly.

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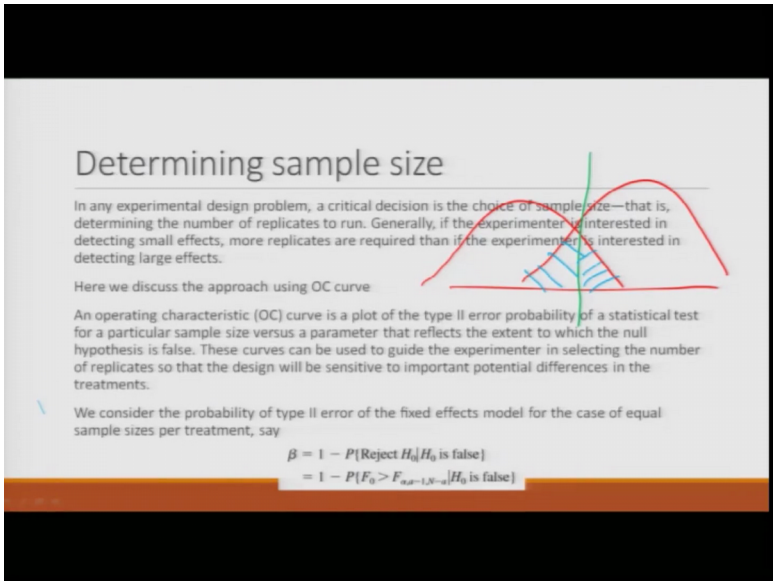

So, now you want to determine the sample size. So, the next discussion would be determining the sample sizes. In any experiment design problems a critical decision is the choice of the sample size; that is determining the number of replicates to run. So, generally the experiment is interested in determining small effects, more replications are required, then if the experimenter is interested to detecting large effects. So, you basically use this. Here we discuss an approach this problem using the OC curve, the operating characteristic curves which we did. If people have done this TQM 1 course we have considered that.

So, an operating characteristic curve is a plot of the type 2 error, probability of a statistical test for a particular sample size versus a parameter that reflects the extent to the null hypothesis is false. So, these curves can be used to guide the experimenter to selecting the number of replicates. So, that the design will be sensitive to important potential difference in the treatments, and you can basically take decisions accordingly.

We consider the probability of type 2 error. So, type, if you remember type 1 and type 2. Type 1 was, what was error was given by alpha and type 2 was basically given by the value of beta. So, type 2 error of the fixed effect model for the case of the equal sample size spot treatment. So, here beta would basically be, because it is going in a complementary part, and if you remember the example, which I gave of; like you are a banker and a manager, and you want to basically disburse loans.

Obviously, you have a cut off points based on which you will decide whether you want to give the loan and whether they we want to not give their own. So, any pay set of persons who have 60 points, credit points a number, you will give the loan and any set of persons who have scores less than 60 and you deny the loan. So; obviously, beta would be, if you consider the complementary part and let me draw the diagram once again for the benefit.

So, this was the line you had 2 distributions and I used. So, technically those, what the H naught and H s. So, let me change the color to d. So, this was the straight value. So, now, you have the red news, just give me one minute. So, this was the so called alpha and beta. Whereas, we considered based on that, we did our or analysis or tried to be basically give the rules accordingly. So, beta would be 1 minus probability of rejecting H naught, when H naught is false and; obviously, that will be counterpart of alpha.

So, in this case, you will basically have for the f test, it will be 1 minus probability of f naught being greater than that F under H naught would be greater than F alpha or 1 minus alpha, wherever or less or greater, depending on how you are framed the hypothesis and a minus 1 would basically be the sample size, because you, once you are keeping fixed and the others you are trying to compare with the fixed one. So, in this case treatment one was the last one, which is fixed, which is the 221 and the total degrees of freedom as we all know before I am repeating it, please excuse me, it will be basically 4 into 5 minus 4 which is 16 and based on the fact that H naught is false, because under H naught if you consider H naught to be true under the alternative one, it will be false.

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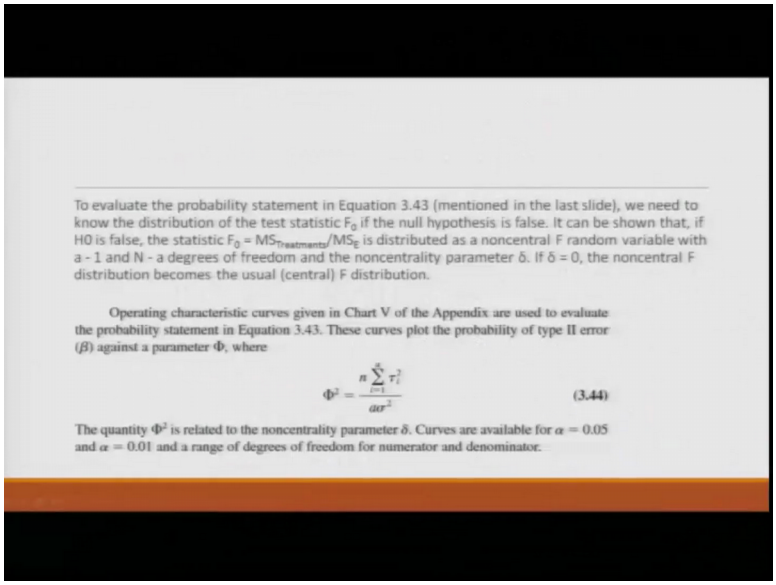

So, to evaluate probability a statement of equation 3.43 as both book. So, mentioned earlier, we need to know the distribution of the test statistics, if not, if the null hypothesis is false, it can be shown that, if h is false then f naught which basically the ratio of mean squared or the treatment by mean square of errors is distributed a non-central f distribution with the e minus 1 and n minus 1 degrees of freedom, and non-solitary parameters respectively. So, a minus 1 would basically be the degrees of freedom, and if and the parameter value is basically delta.

So, if delta is 0 the non-centrality values, the overall skewness would not be there. So, operating characteristic curves given in chart 5 in the appendix are used to evaluate the probability statements of, it related to the equation, we had just discussed and we will consider that the curve plots of the probability of type 2 errors are given against the parameter value a capital Phi. So, when the capital Phi is given by this equation of 3.44 which is basically the ratio of. Now if you remember tau was basically the difference between the average of, the average of means minus the corresponding average to each and every tip treatment, which is basically mu i minus mu.

So; obviously, there would be an error and one of the hypotheses we did frame it as tau is equal to 0, which is H naught with respect to the null hypothesis phase tau were not 0 or any one of them tau values is 1 not 0. So, in the ratio we have in the numerator, the sum of the tau value square, because you are considering, basically intrinsically even though I did not mention we are considering the squared error loss to be true, because that gives the some implication of the concept of variance.

If you are trying to basically minimize squared error loss, it gives us some implication that we are trying to basically minimize the variance divided by, so that would be the numerator will be multiplied by n, the sample size for each and every treatment divided by a is basically the one, which is the total number of such treatments, which you have into sigma square. If you remember we are taking sigma square as basically the standard the deviations square of that will be sigma square and we also consider the errors are independent of each other.

So, the quantity capital Phi square is related to the non-centrality parameter and delta. So, it has nothing to do with. So, called at the capital Phi, this small phi we saw or some many of you know in the case of normal distribution. So, curves are available for alpha is equal to 0.05 alpha is equal to 0.01 and so, and all these values and a range of degrees of freedom for the numerator and denominator are given, and you can utilize those values accordingly.

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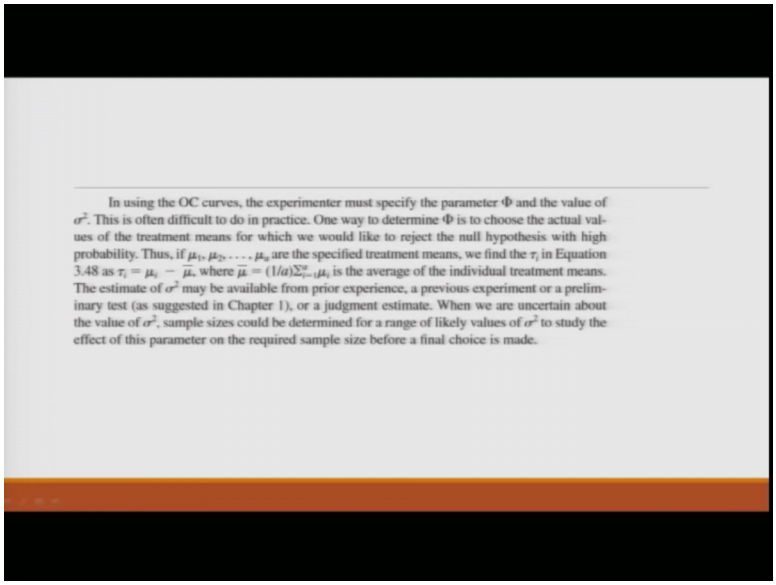

So, in using the OC curve the experimenter must specify the parameter capital Phi and the value of sigma squares, which is basically given by the experimenter and depending on the data which you are utilizing, this is often difficult to do in practice. So, one way to determine capital Phi is to choose the actual value of the treatment, means for which you would try to basically like to reject a null hypothesis with high probability. Thus if mu 1

mu 2 mu suffix 1 mu suffix 2 and so on and so on and mu suffix a or the specific treatment means we find 2 is as per the equation 3.4 a in.

So, these values are given. So, mu tau i would be mu i minus mu bar mu, mu r is basically being utilized the best estimate for the average, of the average from the population we are using the sample to basically estimate the population average mean, and this mu bar is basically given by the sum of all the means divided by a, because there are such a treatments which is the average of the individual treatment means, the estimate of sigma square and may be available from prior experience a previous experimenter or between previous test as suggested or a judgment estimate can be metal made.

So, when we are uncertain about the values of sigma squared sample size could be determined for a range of likely varies. So, sigma square, to study these each effect of these parameters and require sample can be chosen before our final decision is making accordingly.

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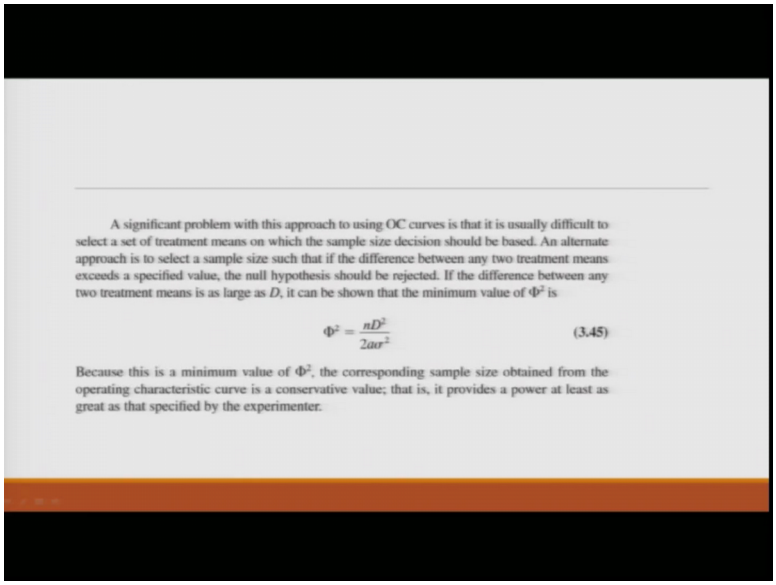

A significant problem with this approach in using OC curves is that, it is usually difficult to select a set of treatment means on which the sample size decision should be based. So, we are not certain what it should be sample size, where it becomes difficult, an alternative approach is to select a sample size as that if the difference between any true

treatment means exceed a specified value. The null hypothesis should be rejected if the difference between any two treatment means is as large as d.

So, it can be utilized as the minimum value of sigma square and that value is given by n into d square by 2 a sigma square. So, this remains the same, because this is the minimum value of sigma square. The corresponding sample size obtained from the operating characteristic curve is a conservative value that is it provides a power, at least as greater than specified by the experimenter discovering dispersion effects.

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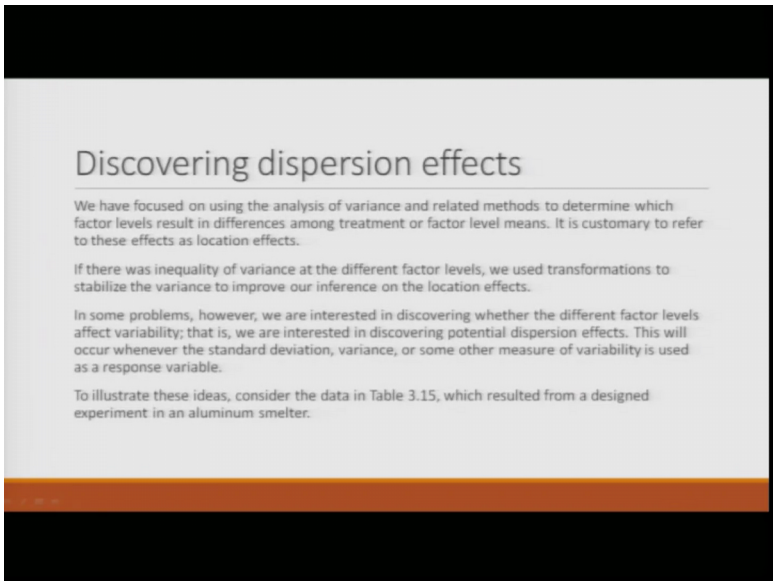

So, you have focused on using the analysis of variance on related methods to determine which factor values or results in difference among the treatment or factor level means. So, it is customary to refer to these effects as location effects. So, we have considered the concept of difference on the means, considering something to do with the tau value is being 0, consider something to do with sigma square as 0.

So, I am talking about H naughts. So; obviously, etch rate would be alternative parts, we also consider that we are trying to keep any one of them comparison as fixed value and trying to compare the average means with fixed value. So, for our example we considered a, which is the last one as fixed which is 220 volts or the wattage powers, sorry voltage powers and then we saw that how different type of tests or Tukey's stays T test, F test, chi square test Bartlett's test all could be utilized depending on the framework of the problem.

So, if they were in equality of variances at different factor levels we use transformation to stabilize them, the variance in order to improve or inference on the location effects. So, in some problems; however, we are interested in discovering whether the different factor levels affect variability that is we are interested discovering a potential dispersion effects, which would basically be maybe a major concern later on. So, we have want to take some corrective actions for that, this will occur whenever the standard deviation variance or some other measure of variability is used as a response value, to illustrate these ideas consider. Let us consider the data has given which results from a design experiment of an l aluminum smelter.

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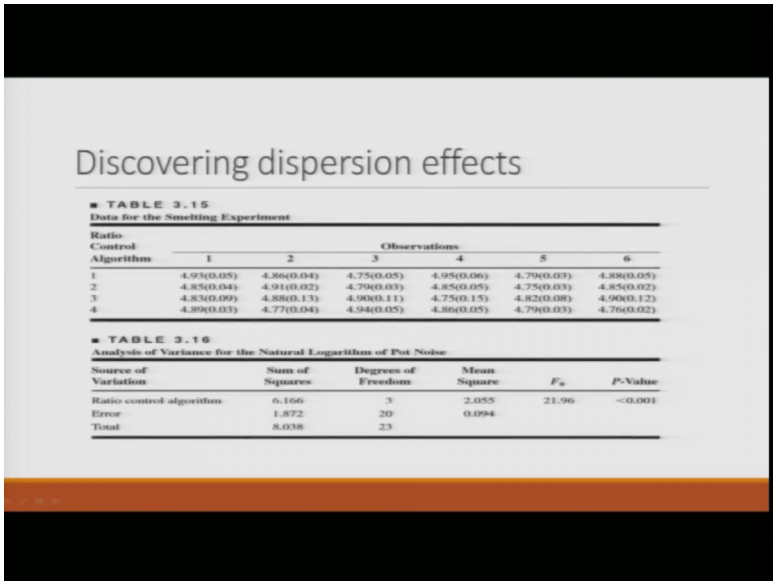

So, consider this. So, you have the data for the aluminum self-smelting process you have the ratio, the control lag values which are given, which are the numbers are 1 2 3 4, which is the first column in table 3.15 and the observations are 6 in number for each and every row. So, they start basically stuff from 4.93 and go to 4.8 it. I am only completing the first one, the last value for the rows. Similarly I mean the fourth row the values are 4.892 in the last value which is 4.76, and the analysis of the variance the test values are given.

So, the ratio of the control algorithm, the errors and the total errors a total will; obviously, be the sum of them are given as 6.166 for the ratio of the control that the errors are given by 1.872, and the sum is basically sum of both of these the degrees of

freedom are given on; obviously, the degrees would freedom would also add a, add up to be the 7. So, the ratio the control algorithm degrees is 3 for the error is 20, the total is 23, the mean square values are given which are corresponding to the first row for the value which is a ratio of the control, like with them as 2.055 and as error being 0.094 F naught values are given as 21.96, and the p value is considering to that experiment, we find out and depending on the level of significance whatever it was there.

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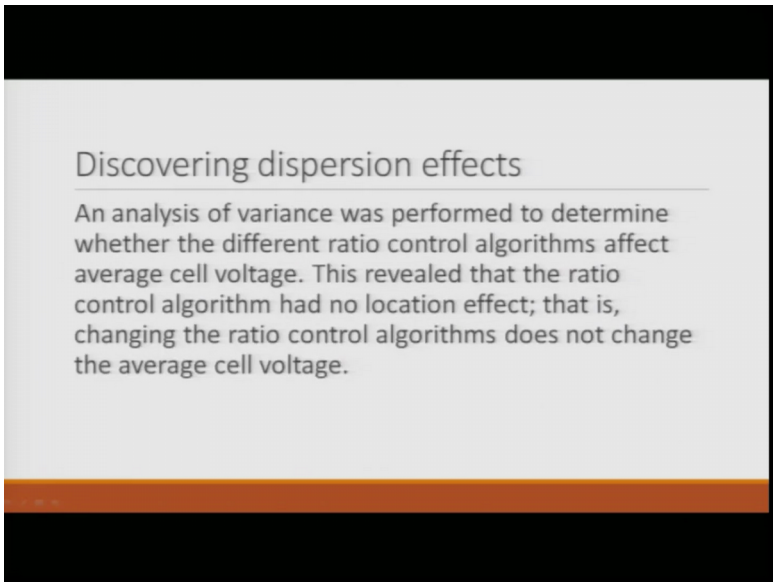

So, discovering the dispersion effects analysis of variance was performed to determine whether there is different ratio controls affecting the average of the cell voltages of the smelting process. This is revealed that the ratio control algorithm had no location effects; that is the changing the ratio. Algorithm does not change the average cell voltages. So, and then we, basically we can, basically I did not go into the example in details, but we have done that etching problem in details in different perspective in different angles. So, that will give you an idea that how those examples can be or informations can be utilized for solving such problems.

So, with this I will end the 12th lecture and continue discussions of ANOVA and factorial design later on, and I wish any queries which have come up in the first two weeks, we would try our level best to answer them and; obviously, the assignments you have all taken care of that, and somebody at the first and second as per the norms or dates given, and will continue discussions for this TQM II lecture for that.

Thank you very much. Have a nice day.