Total Quality Management-II
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Lecture – 09
The Analysis of Variance (ANOVA) – II

A very good morning, good afternoon, good evening my dear friends; welcome to this TQM 2 under NPTEL MOOC series and this is the ninth lecture or class we are taking. So, which is the penultimate or the second last class for the second week and; obviously, after the tenth class you will have the second set of assignments. So, the first set of assignments because we I did hear that what type of assignments would be given for TQM 1 also similarly for quantitative finance also, but it will be based on whatever is being covered.

So, we will slowly expand our (Refer Time: 00:49) and cover more interesting topics. So, coming back to the discussion of what the lecture we are have we were basically discussing in the eighth lecture we basically discussed about the ANOVA and how; how the effect of any factor on an experiment and none with respect to the numbers we are taking, how the average values are there, how the errors are there, what is the assumptions or the errors and how any fluctuation in the mean of the so, called means because if you remember mu i is equal to mu plus Tau i.

So, that mean value is basically the mean of the means for all the processes. So, that will become clear as we proceed.

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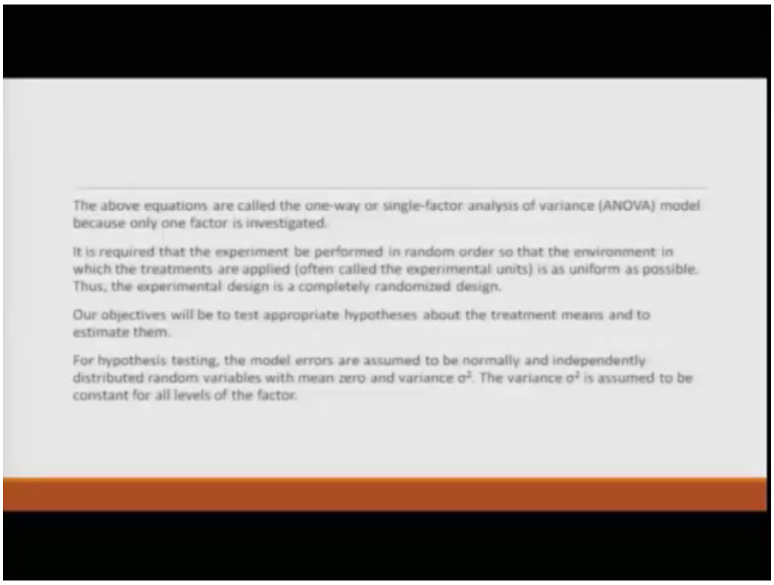

The above equations are called the one-way or single-factor analysis of variance (ANOVA) model because only one factor is investigated.

It is required that the experiment be performed in random order so that the environment in which the treatments are applied (often called the experimental units) is as uniform as possible. Thus, the experimental design is a completely randomized design.

Our objectives will be to test appropriate hypotheses about the treatment means and to estimate them.

For hypothesis testing, the model errors are assumed to be normally and independently distributed random variables with mean zero and variance σ^2. The variance σ^2 is assumed to be constant for all levels of the factor.

So, let me continue reading it further the above equations are called the one way or one or single factor analysis of variance model so; obviously, if there are more like i j k the variables are there they can be two factor models and so, on and so, forth. Because there is only one factor is being investigated it is required that the experiment to be performed in random order.

So, that the environment in which the treatment had applied often called the experimental units is as uniform as possible. Thus the experimental design is completely randomized in design; our objective would be to a test appropriate hypothesis what are the hypothesis we will come to that later about the treatment means and to estimate them accordingly.

For hypothesis testing the model errors are assumed to be normal and independent which I did mention about epsilon the mean value was 0; obviously. So, so let me read it further are assumed to be normally an independent distributed random variables with mean 0 as I did mention in the last class and with a variance of sigma square which means I am not poking.

Putting any suffixes for sigma square which I am assuming that the errors are not increasing and decreasing; we have a fixed value of variances which is sigma square. So, the errors are independent of each other this is very important; the variance sigma square is assumed to be constant for all levels of factors as I mentioned and there is no in dependency between the errors because if that happens; obviously, a whole concept and will change.

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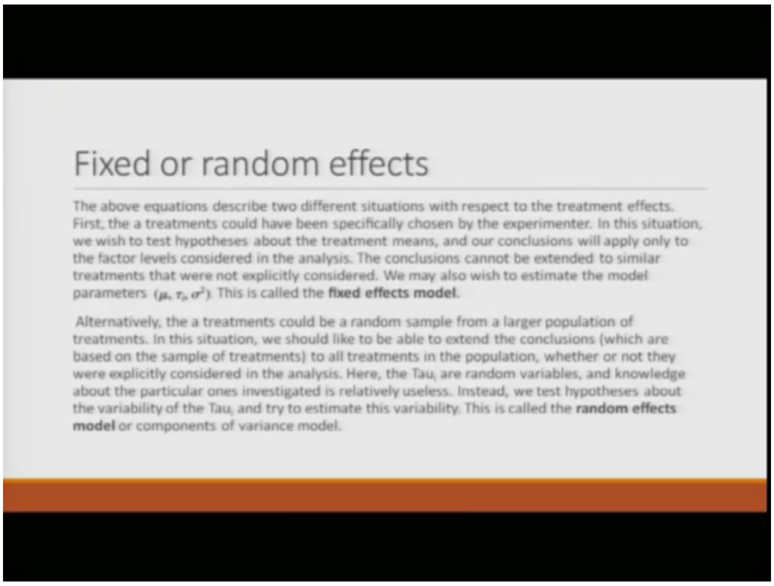

Fixed or random effects

The above equations describe two different situations with respect to the treatment effects. First, the a treatments could have been specifically chosen by the experimenter. In this situation, we wish to test hypotheses about the treatment means, and our conclusions will apply only to the factor levels considered in the analysis. The conclusions cannot be extended to similar treatments that were not explicitly considered. We may also wish to estimate the model parameters (μ, τ_i, σ^2). This is called the **fixed effects model**.

Alternatively, the a treatments could be a random sample from a larger population of treatments. In this situation, we should like to be able to extend the conclusions (which are based on the sample of treatments) to all treatments in the population, whether or not they were explicitly considered in the analysis. Here, the Tau_i are random variables, and knowledge about the particular ones investigated is relatively useless. Instead, we test hypotheses about the variability of the Tau_i and try to estimate this variability. This is called the **random effects model** or components of variance model.

The above equations describe two different situation with respect to the treatment effects; first the treatments or the a treatments could be a our treatment could be specifically chosen by the experimenter this can be the whichever treatment we are doing it is 1 2 3 4 till eighth one in this situation we wish to test the hypothesis about the treatment means. And our conclusions would apply only to the factor levels consider then analysis; the conclusions cannot be extended to similar treatments that were not explicitly considered.

So, if they are not be considered; obviously, we cannot draw conclusions in those lines. We may also wish to estimate the model parameters which are now mean mu only without the suffix which is the average of the average the word I am using in a very lame a sense, but it will become clear to you later on. One would basically be the Tau i that is the fluctuations with respect to the mean of the means are that we can find out the means for each and every experiment or the treatment which is mu suffix i and; obviously, the errors as I said we have a mean value of 0.

So, that would not be considered and we are also considering the errors and independent they have a variance of sigma square; so, this is also the parameter on the model. So, this is called the fixed effect model because the effects are fixed kept due to a fixed level considering the sigma square are not changing.

Alternatively the treat the treatment can be random samples from a large population of treatment. In this situation we should like to be able to extend the conclusion which are based on the sample and the sample of treatments to all treatments in the populations whether or not they are explicitly considered analysis. So, hence here the Tau r Tau i’s are random variables and knowledge about the particular ones investigated is relatively not of that importance for us; instead we test the hypothesis about the variability Tau and what f x is has.

And we try to estimate this variability this is called the random effects model or components of variance model which we will consider later on.

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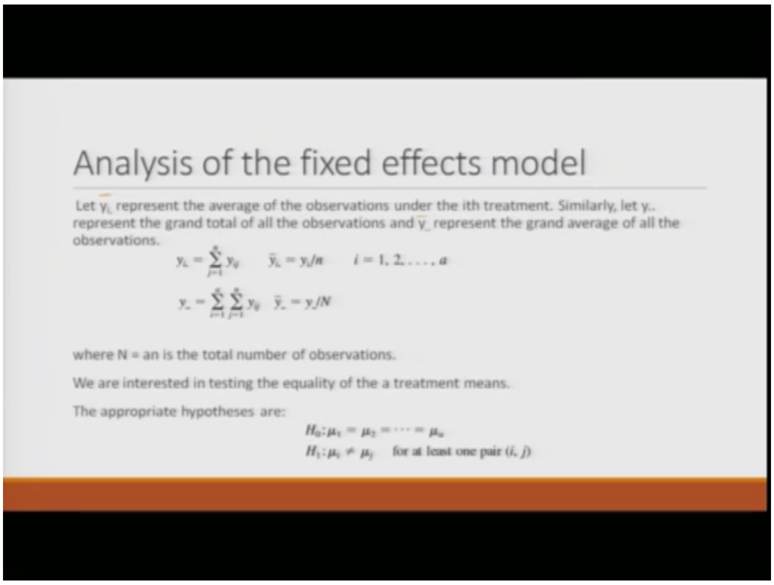

Analysis of the fixed effects model

Let $\bar{y}_{i.}$ represent the average of the observations under the ith treatment. Similarly, let $y_{..}$ represent the grand total of all the observations and $\bar{y}_{..}$ represent the grand average of all the observations.

$$y_{i.} = \sum_{j=1}^{n} y_{ij} \qquad \bar{y}_{i.} = y_{i.}/n \qquad i = 1, 2, \ldots, a$$

$$y_{..} = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij} \quad \bar{y}_{..} = y_{..}/N$$

where N = an is the total number of observations.

We are interested in testing the equality of the a treatment means.

The appropriate hypotheses are:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_a$$
$$H_1: \mu_i \neq \mu_j \quad \text{for at least one pair } (i, j)$$

Now, analysis of the fixed effect model as I said; So, let y i represent the average of y i bar be depends the average of the of the observations under the ith treatment. Similarly let y double dot represent the grand total the average of the average of the not the average basically i in that sense; obviously, with the average of the average of the of the total number of observations and y bar double dot represents a grand average of all the observations.

So; obviously, in that case you if y i dot would basically be the summation of y i and j summed up for all the j's and y i dot bar would technically be this the average of that would definite be y i dot divided by n.

So, n here n is basically the observations which I have for each and every treatment and then I will be able to find out the averages for y 1 bar dot y 2 bar dot. So, that would 1, 2, 3, 4 would basically correspond to the treatment of the experiment change I am basically trying to do.

Similarly, y double dot would basically be the sum of all the values corresponding to i n j and the average of that would basically be the sum would be basically y double dot divided by total number of observations which you considered as capital N. So, it will be n plus n plus n and how many such small n are there that a small n so; obviously, the grand sum which is capital N.

So, here it is given where capital N is equal to a into n is the total number of observations we are interested to test the have equality of this treatment. So, hence the appropriate hypothesis is under H naught which is the null hypothesis which we technically as I mentioned we would like to disprove is that all the means are equal which is mu i's are all equal; hence it can be written as mu 1 is equal to mu 2 is equal to mu 3 till mu suffix it with respect to the alternate hypothesis that at least one pair are there where they are not equal.

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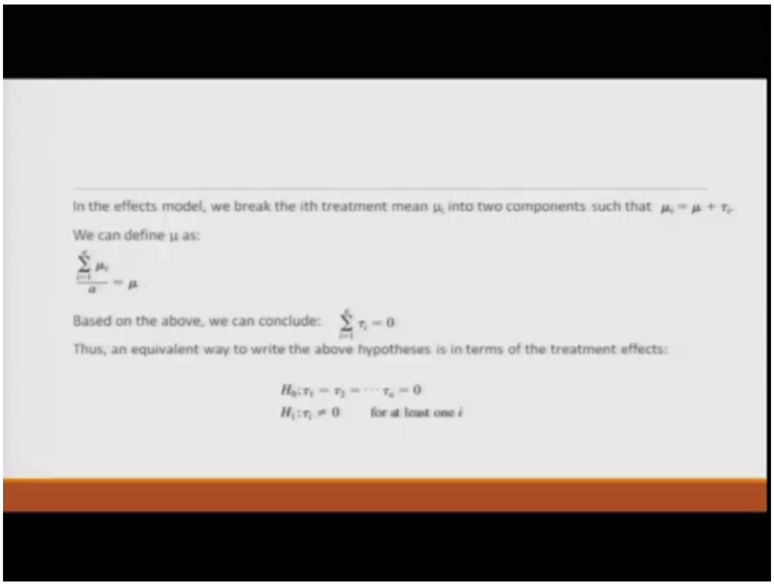

In the effects model, we break the ith treatment mean μ_i into two components such that $\mu_i = \mu + \tau_i$.

We can define μ as:

$$\frac{\sum_{i=1}^{a} \mu_i}{a} = \mu$$

Based on the above, we can conclude: $\sum_{i=1}^{a} \tau_i = 0$

Thus, an equivalent way to write the above hypotheses is in terms of the treatment effects:

$$H_0: \tau_1 = \tau_2 = \cdots \tau_a = 0$$
$$H_1: \tau_i \neq 0 \quad \text{for at least one } i$$

In this effect model we break the ith treatment mean mu i into two components which I have mentioned as mu i is equal to mu plus Tau y we can define this value of mu as the summation of y s which with the summation of all the means divided by the number of treatments which is sum of mu 1, mu 2, mu 3 to mu 4 till dot dot till mu a divided by a based on the above, we can conclude that basically that averages of the fluctuations which we have from the mean of the mean of the means is basically 0.

So, hence the average of Tau i is 0 this is important to note; thus an equivalent way to basically write the above hypothesis would be that all the tau's are 0 that is Tau 1, Tau 2, Tau 3 is still Tau a 0 with respect to the case then for at least one of the treatment Tau suffix I is not 0. So, this is something to do with the mean nothing to with to do with the variability of Tau. So, it may be possible the mean values of Tau are is 0, but still there is high variability.

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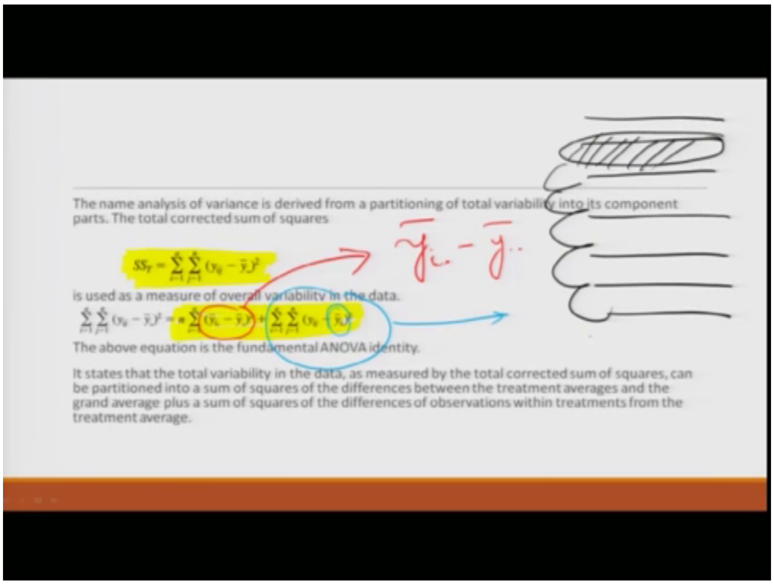

The name analysis of variance is derived from a partitioning of total variability into its component parts. The total corrected sum of squares

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$$

is used as a measure of overall variability in the data.

$$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2 = n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2$$

The above equation is the fundamental ANOVA identity.

It states that the total variability in the data, as measured by the total corrected sum of squares, can be partitioned into a sum of squares of the differences between the treatment averages and the grand average plus a sum of squares of the differences of observations within treatments from the treatment average.

$\bar{y}_{i.} - \bar{y}_{..}$

The name analysis of variance is derived from the partitioning of the total variability into its component parts. So; obviously, the component parts would be 2 1 is the sum of the squares which is basically sum of the squares. So, now, I will highlight it in particular for your convenience.

So, this is the sum of the squares which I mentioned so if you look here is basically you are trying to find out the differences of each and every observation from his grand total mean value which is y bar dot dot in the suffix which means u dot dot; the first dot means for all the experiment or the change of the of the etching which you are doing 1 2 3 4 till a and the second dot is for all the observations which are there under each and every a.

So, if you find out the sum of the totals of the squares. So, you add them up square them up add them for and sum them up for all the values changing from i is going to want to a and j is equal to 1 to n it is used as a measure of the overall variability in the data. So, in that case if you expand that this Tau is basically has two components. So, the two components would be this which I am just highlighting for the second time. So, one is with respect to the differences of the of the of the mean value. So, what are the mean values? So, now, I will basically highlight each and every observation carefully.

So, the first part which is no coloured red with respect to y bar i dot minus the y double bar a y bar dot. So, basically it means that I am basically trying to sum up for each experiment, sum them up consider the random variable and basically find out what is the difference from the average of the averages that is part 1; number 2 where I will again change the colour in order to make an understand. So, this would be the individual fluctuations which are happening with respect to the average of that particular experiment.

So, if you basically concentrate here this value. So, way which I am trying to now circle that y bar i dot is basically the averages for any experiment, but for all the examples under the experiment considered. So, the above equation when we basically find out it can be derived very easily; the above equation is basically the fundamental ANOVA identity. So, in that some of the squares have been divided into two parts one technically be can be considered as the error in the sample and one can be considered as the error throughout the sample.

So, it is basically something to do with the inter sample errors and in the in the intra sample errors based on which you are trying to basically find out the over sum of the square of the errors. It states that the total variability which is sum of the squares in the data as measured by the total corrected sum of the squares can be partitioned into two parts; one is the sum of the squares of the difference between the treatment averages and the grand average plus a sum of the squares of the difference of the observation within the treatment from the treatment average.

So, basically one is between sample one is basically compared with the sample. So, basically what I mean by the word samples is each treatment has a sample. So, there are a samples like this in under each you have as I mentioned by assumption n number observations. And once I find out the sum the squares of the average difference between the samples then also I add up the sum of that average of the in sample errors also. So, this total sum has been divided into two components.

So, let me if I am possible let me give a diagrammatic because the equations are very easy to understand, but in order to make things a little bit clear. So, what you have? You have the samples; so, how many such samples? a. So, one error is basically between each and another is basically when you are comparing between the samples. So, there are two components. So, this would be summed up and this will be summed up. So, in this way you very simply find out sum the squares.

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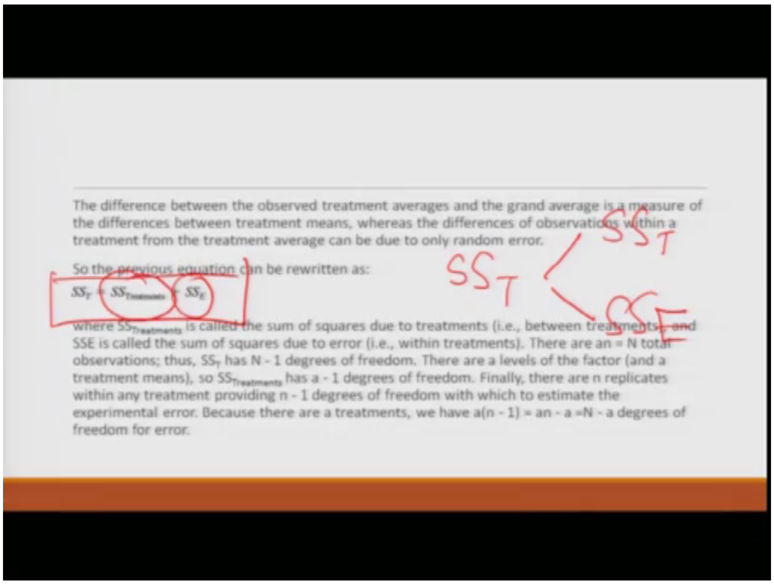

The difference between the observed treatment averages and the grand average is a measure of the differences between treatment means, whereas the differences of observations within a treatment from the treatment average can be due to only random error.

So the previous equation can be rewritten as:

$$\boxed{SS_T = SS_{Treatments} + SS_E}$$

where $SS_{Treatments}$ is called the sum of squares due to treatments (i.e., between treatments), and SSE is called the sum of squares due to error (i.e., within treatments). There are an = N total observations; thus, SS_T has N - 1 degrees of freedom. There are a levels of the factor (and a treatment means), so $SS_{Treatments}$ has a - 1 degrees of freedom. Finally, there are n replicates within any treatment providing n - 1 degrees of freedom with which to estimate the experimental error. Because there are a treatments, we have a(n - 1) = an - a =N - a degrees of freedom for error.

The difference between the observed treatment average and the grand average is a measure of the difference between it means whereas, the difference of the absorption within the treatment from the treatment average can be due to only the random error. So, there now you are slowly trying to differentiate there is into whether it is random error or the basically error inculcated out into the system.

So, the previous equation can be written as I will try to now use a colour in and the ink colour being change is basically the sum of the squares has two part the sum the squares for the treatments and the sum of the squares for the so, called the errors. So, this SS suffix treatments is called the sum of the squares due to the treatment that is between the treatment and SS suffix E is called the sum of the squares due to the treatment that is between the treatment and SS suffix E is called the sum the square due the errors.

So, the errors have some the would have an effect on the sum of the squared which is SS T. So, that is basically SS E; so, here SS T has been divided into two parts SS treatment SS E. So, this; so, this is the white noise; so folk so, called coming from the example. There are levels of factors and treatment means. So, SS suffix T has a minus 1 degrees of freedom because you are losing one day deserved freedom corresponding to the fact. Now, this would make sense based on all the seven lectures which you have done till for about the information about the variants is being not known. Finally, there are n replicates within any treatment providing N minus 1 degrees of freedom each in each case it is reducing by 1.

So, technically if I want to find out; so, N degrees of freedom with which to estimate there is, but because there are a number of treatment hence in each case reduction is 1. So, now if I try to find out the total number of degrees of freedom and reduce it will be a. So, taken in hence it will be a n total number of observations minus a. So, that is basically given by capital N minus a because we know capital N is basically sum of all the number of observations. So, in each row you have n how many such rows S you multiply them and find out capital N.

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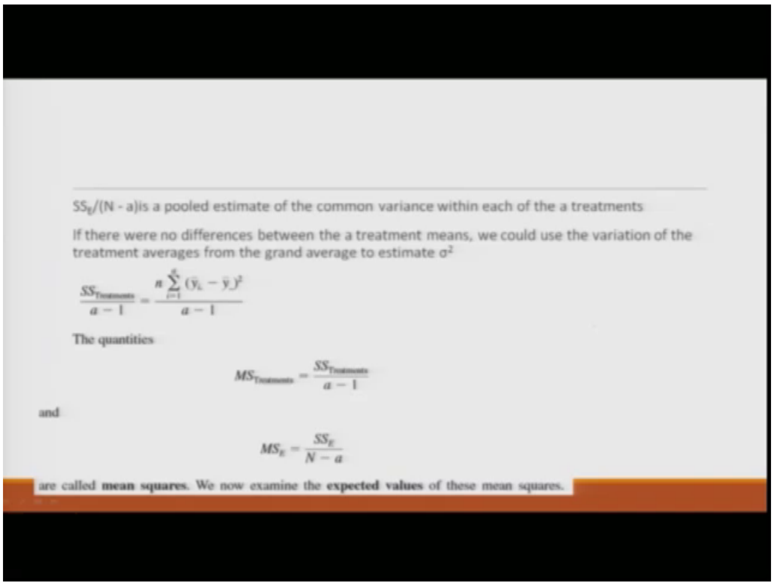

$SS_E/(N - a)$is a pooled estimate of the common variance within each of the a treatments

If there were no differences between the a treatment means, we could use the variation of the treatment averages from the grand average to estimate σ^2

$$\frac{SS_{\text{Treatments}}}{a-1} = \frac{n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2}{a-1}$$

The quantities

$$MS_{\text{Treatments}} = \frac{SS_{\text{Treatments}}}{a-1}$$

and

$$MS_E = \frac{SS_E}{N-a}$$

are called **mean squares**. We now examine the **expected values** of these mean squares.

So, some of the square of errors divided by its degrees of freedom which is capital N minus a is a pooled estimate of the common variance within each of the treatment. If there are there were no differences between the treatment, we could use the variability variations of the treatment averages from the grand average to estimate sigma square. So, a sigma square can we found out accordingly.

So, SS treatments is given by the formula which is the difference between the averages which is happening in the sample and how many such etchings which you have etchings of the experiment which you have which is a in number minus the grand average which is y bar bar y bar dot dot corresponding to the sum of all etching and all the samples.

The quantities which you find out would be basically giving be the mean square or corresponding your treatments. And the mean square for the errors would be given by SS E suffix capital N minus a and the mean square for the treatments would be given by SS treatments divided by a minus 1 which is the degrees of freedom which we have.

So, they are called the mean square we know example examine the expected values of this mean square and do our studies accordingly.

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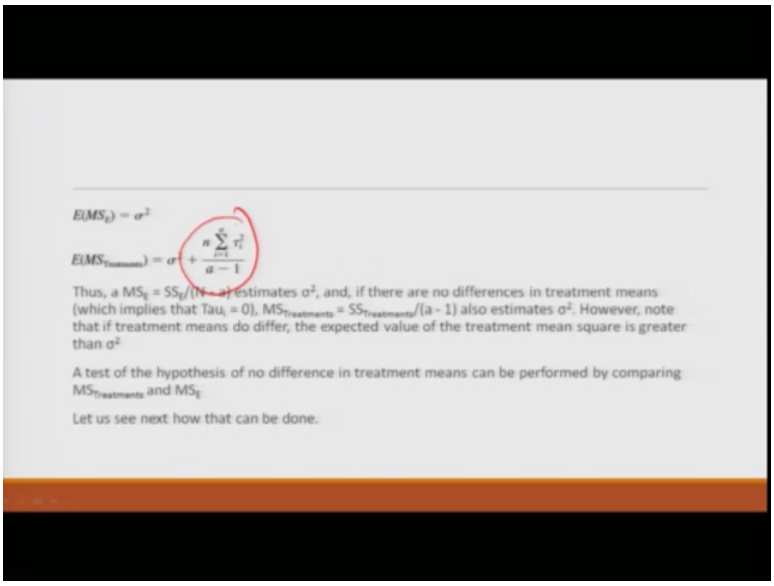

$$E(MS_E) = \sigma^2$$

$$E(MS_{Treatments}) = \sigma^2 + \frac{n\sum_{i=1}^{a}\tau_i^2}{a-1}$$

Thus, a $MS_E = SS_E/(N-a)$ estimates σ^2, and, if there are no differences in treatment means (which implies that $Tau_i = 0$), $MS_{Treatments} = SS_{Treatments}/(a - 1)$ also estimates σ^2. However, note that if treatment means do differ, the expected value of the treatment mean square is greater than σ^2

A test of the hypothesis of no difference in treatment means can be performed by comparing $MS_{Treatments}$ and MS_E

Let us see next how that can be done.

So, the mean square for the values for the mean of the errors would be sigma square and mean square for the treatment would be sigma square plus a factor which is n summation of the Tau square divided by a minus 1. So, thus mean square of the errors estimate sigma square and if there is no difference in the treatment which is the means which implies basically Tau suffix I is 0 while mean square of treatments also estimate sigma square.

However, note this that if the treatments mean do differ the expected value of the treatment mean square is greater than sigma square corresponding to the fact that the second term which you have may not be 0. A test of the hypothesis of no difference in treatment means and we perform by comparing the mean square of the treatments and the mean square of the error.

So, if there is some difference; obviously, the second component which I just highlighted in the red may be not 0 or maybe 0 depending on what is your hypothesis our. So, let us see how this can be done; so, you have assumed that the errors epsilon i j are normally an intermediate distributed mean 0 and the variance sigma square.

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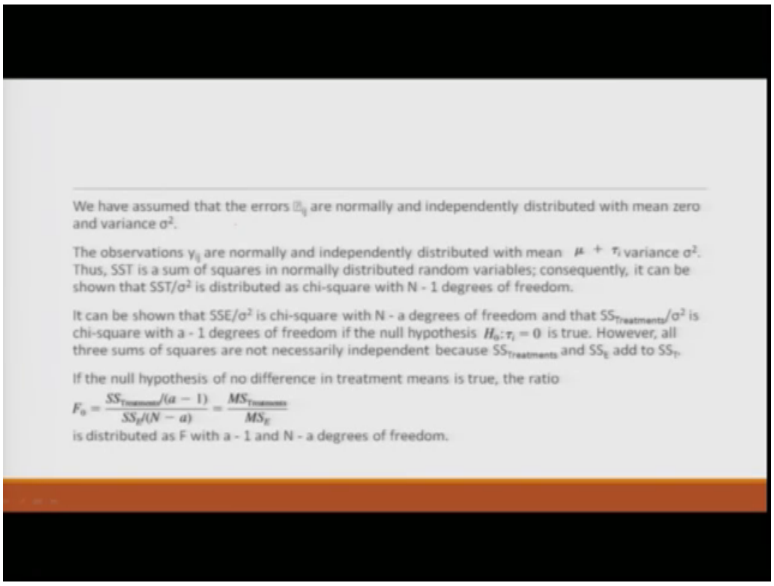

We have assumed that the errors □$_{ij}$ are normally and independently distributed with mean zero and variance σ^2.

The observations y_{ij} are normally and independently distributed with mean $\mu + \tau_i$ variance σ^2. Thus, SST is a sum of squares in normally distributed random variables; consequently, it can be shown that SST/σ^2 is distributed as chi-square with N - 1 degrees of freedom.

It can be shown that SSE/σ^2 is chi-square with N - a degrees of freedom and that $SS_{Treatments}/\sigma^2$ is chi-square with a - 1 degrees of freedom if the null hypothesis $H_0: \tau_i = 0$ is true. However, all three sums of squares are not necessarily independent because $SS_{Treatments}$ and SS_E add to SS_T.

If the null hypothesis of no difference in treatment means is true, the ratio

$$F_0 = \frac{SS_{Treatments}/(a-1)}{SS_E/(N-a)} = \frac{MS_{Treatments}}{MS_E}$$

is distributed as F with a - 1 and N - a degrees of freedom.

And I did mention time and again I will mentioned again here the details are independent of each other thus the observations y i j are normally an independent distributed with mean mu plus Tau y. So, Tau y is basically the fluctuation which is happening between the mean of any treatment with respect to the grand mean.

Then the variance is sigma square thus the sum square of t the treatment is the sum of the squares is in a normal distributed random variable consequently it can be shown that SS suffix T sigma square is distributed at chi square; this is again if I if you do remember I did mention that the normal distribution concept has a very unique property that the mean value of the sample and the variance of the sample which is the square of the standard errors are independent of each other and; obviously, you will have the chi square when you are trying to compare something to do with standard error squared with respect to the sigma square of the population.

So, here it is basically a chi square with n minus 1 degrees of freedom it can be shown that the sum of square of the errors divided by sigma square is also a chi square with N minus a degrees of freedom. And that SS T sigma square is a chi square with a minus 1 degrees of freedom. So, all this would be following up considering that the independence factor which I did not mention degrees of freedom.

So, hence if you are null hypothesis Tau I is 0 is true so; obviously, it they can be unaltered in hypothesis, but it is not true; however, all the three sums of squares errors squares are not necessarily independent because SS T and SS E and SS case capital T may not be independent because the reason is very simple because SS suffix treatments plus SS suffix errors is equal to SS suffix the total.

If the total hypothesis have no difference in treatment means is 0 then the ratio would be given which is basically. And now when you are comparing the variances again coming back to my repeated statements which I did mention till the fag end of the seventh class is something to do with the comparison of the variances then you definitely try to utilize the F distribution with a certain degrees of freedom.

So, this is what is again company happening you have the F distribution where you compare the mean square of the treatments divided by the mean squared are the errors. So, it is distributed with f d a degree distribution with errors being a minus one and capital N minus a.

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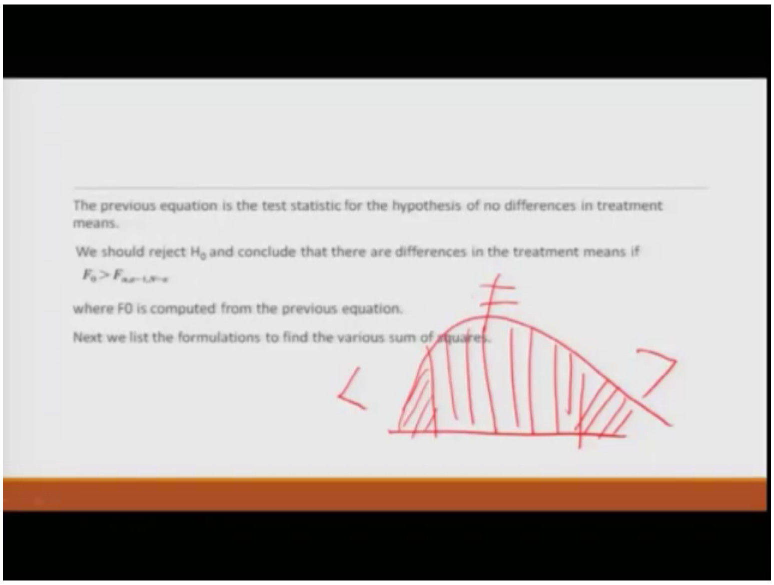

The previous equation is the test statistic for the hypothesis of no differences in treatment means.

We should reject H_0 and conclude that there are differences in the treatment means if

$F_0 > F_{\alpha,a-1,N-a}$

where F0 is computed from the previous equation.

Next we list the formulations to find the various sum of squares.

The previous equation is the test statistics for the hypothesis of no difference in the treatment means, we should reject H naught and control that there is no difference between the treatment if the value of the of F value which I find out.

So, if you if you remember if there is no difference less than greater than time I will try to draw a graph which is a symbolic way of trying to represent for the F distribution because you do not know remember the F distribution is not symmetric. So, on this part it will be less than type, on this part it would be greater than type and any error; any value in between would be not equal to; So, based on that the f distribution would be done accordingly.

So, if you remember the corresponding the hypothesis testing for less than type greater than type and not equal to are basically coming from the same fact what we have discussed further the seventh first seven classes where we did to go into the details about the probability statistics and the corresponding point estimation hypothesis testing. So, continuing reading we should reject H naught and conclude there are differences in the treatment if F naught that is under null hypothesis is greater than F.

Alpha is basically the degrees of freedom and this would be and remember alpha it is not 1 minus alpha by 2 or alpha by 2 that is a very important point; which again I am saying with apologies I am repeating it time and again. So, where F suffix naught is computed from the previous equation. So, now, we list the formulation to find out the variation very various some of the squares.

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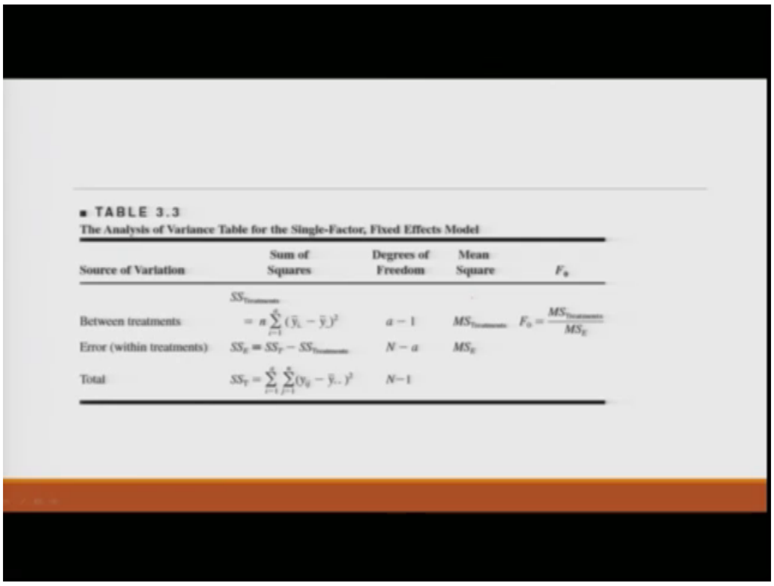

TABLE 3.3

The Analysis of Variance Table for the Single-Factor, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	$SS_{\text{Treatments}} = n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2$	$a - 1$	$MS_{\text{Treatments}}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Error (within treatments)	$SS_E = SS_T - SS_{\text{Treatments}}$	$N - a$	MS_E	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$	$N-1$		

So, this is the table which will give you an idea. So, in this is the analysis of variance table and ANOVA. So, sources of variations if I read it on the first column they are between treatments, error within treatments and this is the total. So, between treatments if you remember is SS suffix treatments errors within treatment which is SS suffix E which is the difference between SS T minus SS treatments.

And; obviously, SS T would be the sum of both of them you can double check that the values are coming separately as well as count them together. Then the degrees of freedom they have been mentioned accordingly. So, for the case between the treatment will be a minus 1 because your are collectively comparing them and your are losing one degrees of freedom. And errors within the treatment; obviously, that you are losing 1 degrees of freedom in each of the treatment hence it will be capital N minus a. And if you sum them up total errors degrees of freedom is capital N minus 1.

Similarly, the mean square is are found and if you compare the F-statistic less than type greater than type it is given by the ratios of the mean squares of the treatment divide by means square of the errors and it can be done the other way round also.

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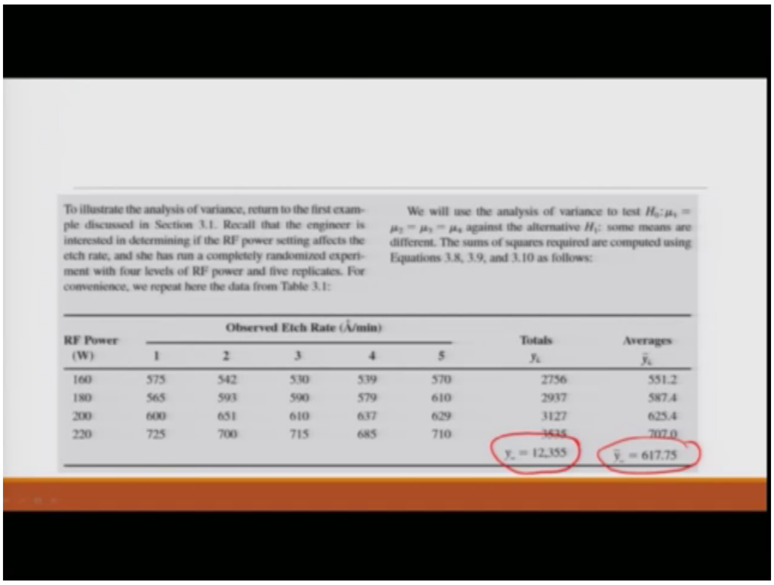

To illustrate the analysis of variance, return to the first example discussed in Section 3.1. Recall that the engineer is interested in determining if the RF power setting affects the etch rate, and she has run a completely randomized experiment with four levels of RF power and five replicates. For convenience, we repeat here the data from Table 3.1:

We will use the analysis of variance to test $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ against the alternative H_1: some means are different. The sums of squares required are computed using Equations 3.8, 3.9, and 3.10 as follows:

RF Power (W)	Observed Etch Rate (Å/min) 1	2	3	4	5	Totals $y_{i.}$	Averages $\bar{y}_{i.}$
160	575	542	530	539	570	2756	551.2
180	565	593	590	579	610	2937	587.4
200	600	651	610	637	629	3127	625.4
220	725	700	715	685	710	3535	707.0
						$y_{..} = 12{,}355$	$\bar{y}_{..} = 617.75$

So, illustrate the example of analysis of variance or ANOVA. So, we will return to the first example recall that the engine is interested in finding on the power setting affecting the etch rate the and he or she runs a completely randomized experiment with respect to the power and the 5 replicates which are being done for each power powers are what one is 160, 180, 200, 220 and each has basically 5 samples on under them.

So, those values are given in that table. So, the table has the first columns are the powers the second third, fourth, fifth, six are basically the reading numbers in a matrix and the totals are given in the second last column. So, if you add up the row corresponding to 160; that means, add up 575 plus 542 plus 535 plus 539 plus 570; you have a value of 2756; the average of which is given in the last column which is y bar i dot.

So, it becomes 551.2 similarly you find out all the averages find out the averages of these averages which is comes out to be 617.75; which is here and the average of the total sum is basically this 12355.

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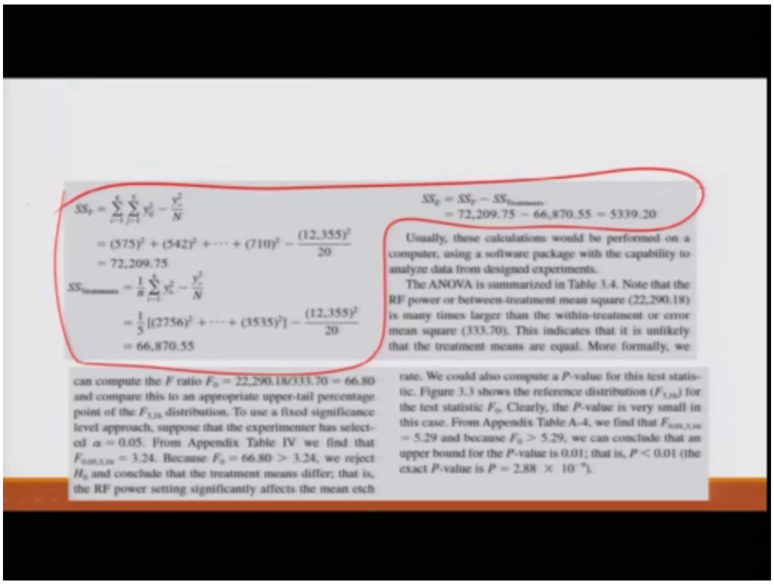

$$SS_T = \sum_{i=1}^{4}\sum_{j=1}^{5} y_{ij}^2 - \frac{y_{..}^2}{N}$$
$$= (575)^2 + (542)^2 + \cdots + (710)^2 - \frac{(12{,}355)^2}{20}$$
$$= 72{,}209.75$$
$$SS_{\text{Treatments}} = \frac{1}{n}\sum_{i=1}^{4} y_{i.}^2 - \frac{y_{..}^2}{N}$$
$$= \frac{1}{5}[(2756)^2 + \cdots + (3535)^2] - \frac{(12{,}355)^2}{20}$$
$$= 66{,}870.55$$

$$SS_E = SS_T - SS_{\text{Treatments}}$$
$$= 72{,}209.75 - 66{,}870.55 = 5339.20$$

Usually, these calculations would be performed on a computer, using a software package with the capability to analyze data from designed experiments.

The ANOVA is summarized in Table 3.4. Note that the RF power or between-treatment mean square (22,290.18) is many times larger than the within-treatment or error mean square (333.70). This indicates that it is unlikely that the treatment means are equal. More formally, we can compute the F ratio $F_0 = 22{,}290.18/333.70 = 66.80$ and compare this to an appropriate upper-tail percentage point of the $F_{3,16}$ distribution. To use a fixed significance level approach, suppose that the experimenter has selected $\alpha = 0.05$. From Appendix Table IV we find that $F_{0.05,3,16} = 3.24$. Because $F_0 = 66.80 > 3.24$, we reject H_0 and conclude that the treatment means differ; that is, the RF power setting significantly affects the mean etch rate. We could also compute a P-value for this test statistic. Figure 3.3 shows the reference distribution ($F_{3,16}$) for the test statistic F_0. Clearly, the P-value is very small in this case. From Appendix Table A-4, we find that $F_{0.01,3,16} = 5.29$ and because $F_0 > 5.29$, we can conclude that an upper bound for the P-value is 0.01; that is, $P < 0.01$ (the exact P-value is $P = 2.88 \times 10^{-9}$).

So, from this if you find out the sum square of total you basically add them up the value comes out to be 72209.75; as sum of square of the treatment comes out to be 66870.55. We can compute the F value comes out to be 66.80 by finding out this the ratios and compare this to the to the approval upper tail percentile depending on whether this right hand side on the left hand side which is F 3 comma 16 depending on the degrees of freedom. To use a fixed significance level approach we do that suppose that the experiment has selected alpha is equal to 0.05 depending on the level of confidence.

So, from there we find out F 0 as coming out to be 66 as to be our F 0 coming out to be 3.24. So, as 66.0 is greater than 3.24; we reject H naught and conclude the treatment means differ that is the power settings significantly affect the mean of the etch rate.

So, why how we find found out this values? So, they are very clear in this table one can have a look what I am basically trying to highlight based on that that experiment or that analysis can be done. Usually these calculations would perform on a computer using a software package with the capability to analyze the data. The ANOVA is summarizing table in the corresponding table. So, note that their power in between the treatment means is many times larger than within the treatment errors. So, this indicates that is like unlikely that the treatment means are equal more formally we can compute the values as added.

So, continuing that so, the power setting significantly affect the mean in each rate etch rate we can also compute the values P values for the test statistics and do the corresponding tests accordingly.

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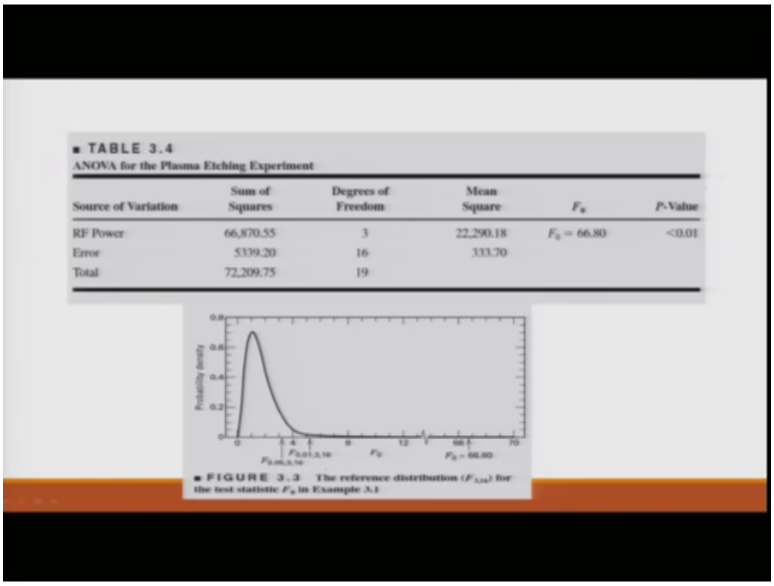

TABLE 3.4

ANOVA for the Plasma Etching Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
RF Power	66,870.55	3	22,290.18	$F_0 = 66.80$	<0.01
Error	5339.20	16	333.70		
Total	72,209.75	19			

FIGURE 3.3 The reference distribution ($F_{3,16}$) for the test statistic F_0 in Example 3.1

So, these are the tables which I have ANOVA for the plasma etch experiment, the sources of variations, the powers are given are the sum of the squares is given 66 and 870; I am not talking about the decimal values the errors are given 5339; the total is given addition of that is 72209, degrees of freedom are corresponding to 316 and 19, sum of it; obviously, 3 plus 16 is 19 mean squared as I are given for power which is basically the H factor or the value corresponding to a because you remember I had that that here a was 5; 160, 180 to 200.

So, sorry for it is 160, 180, 200, 220; so, the mean square is 22290 again I am not mentioning the decimal the mean square corresponding to the errors which is is 33 and 33 the F value if we F naught this comes out to 66.8 and; obviously, we take the decisions accordingly.

So, the reference distribution based on which we do and if I remember I did draw our distribution. So, here it is this is the F distribution their values are given. So, the reference distribution F 3 by 6 comma 16 depending on the degrees of freedom for the test statistic F 0 is given and basically based on that we reject not or accept that.

Now, we see the answer would change if we somebody is not if you basically frame your H naught accordingly. So, with this I will close this lecture and thank you very much for your attention and being the part of this NPTEL MOOC course. We will continue discussing the NPTEL and in later on in the lectures in a tenth and so, on and so, forth.

Have a nice day thank you very much.