

Total Quality Management - I
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Lecture – 33
PCR calculation and Gauge Capability

A warm welcome to my friends, my dear students; very good morning, good afternoon, good evening to all of you, and as you know this is the thirty third lecture for the TQM One lecture series under NPTEL MOOC I am Raghunandan Sengupta from IME department IIT Kanpur.

So, if you remember we were discussing the charts in details as we change that like charts like \bar{x} bar charts, r charts, p charts, c charts, np charts, g charts, h charts and technically in all the charts what you actually need is an average value and on the average value can technically come from the population or from the sample and when you pick up a sample you have to find out the best estimate with respect to the population and there I did mention very fittingly the concept of unbiased estimates.

And the minimum variance concepts also, I just mentioned them in what I did not go into details. So, which should be the best estimate from the sample to mimic the population parameters so the parameter was basically the mean. And then that is a central line then you have an upper control and the lower control. So, the upper control line on the lower control line would basically depend on the case that what is the overall level of confidence you want to have. So, if it is plus minus 3 sigma, then the multiplication coefficient would be 3, plus and minus being coming for the positive side and negative side. It was 4 sigma so; obviously, plus minus 2, and it was 2 sigma it would be plus minus 1.

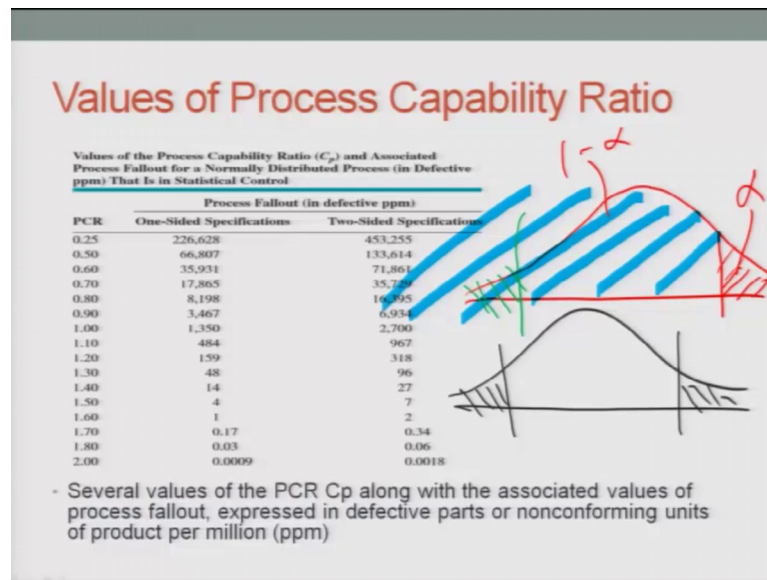
Now, the sigma value which we will find out in the initial case when you started what basically the range or r , because in that case the range would give us the average value and also a plus minus range values and then you can find out the range values given the average value of the range multiplying by the values of a_1, a_2, a_3 or b_1, b_2, b_3 or c_1, c_2, c_3 whatever the case be and those constants this a_1, a_2, a_3 or $b_1, b_2, b_3, c_1, c_2, c_3$ would depend on the sample size.

Later on, next we saw that that we used the concepts of the standard deviation being not known, then we went into the c charts then we went to the proportion charts and the proportion charts and the c charts and later on we saw that how the central limit theorem could be utilized considering that the actual distribution was binomial, how you can take the mean value and the standard deviation on the binomial distribution as the mean and the standard deviation of the normal distribution based on which you are trying to draw the charts. And later on we also saw that in case proportions or not available we took the n into p values, which is the total number.

Then we also saw that if the samples towards themselves changing like in the first set we to have n 1 sample, second set we have n 2 sample so on and so forth how we can do the calculation. The calculation remain the same only the division part in one of the calculations where you want to find out the standard deviations it would be divided by n i depending on whatever the i is, and in case you want to take the average we find out the average of all the n and is and add them up divide by the number of such samples and weak bar to find out n bar.

Then considering all these things we also later on considered different problems in a very small set and how this r programming could be utilized to find out the graphs the process control charts and in case if n is also changing like you are illuminating n or adding one n. So, that would also be coming under the concept of n ; so, to continue the discussion further.

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So, now, we are considering the process capability ratios and the associated process fallout and depending on the overall errors which you have. So, the process capability ratios are given on the first column PCR starting from 0.25 till to, then one sided specification 2 sided specification would be given.

Now, the reason is that say for example, there is let me let us discuss 2 separate examples which will be better for us to understand. Let us consider one tie rod and the tie rod has to be fitted in in some machine parts, now considered the machine part has a length of say for example, 2 feet. So, technically it would mean that the tie rod has to be exactly a little bit less some microns or some nano millimeters less than the 2 feet. Now if that is the case then; obviously, the fitting would be exact, but in case say for example, the length is much more so obviously, the tie rod would not be fitted because then then some material extra material has to be again adjusted and then fitted.

Now, it may so happened that if the length of the tie rod is much much less than some few estimates then the microns, mean microns or nano centimeters whatever it is with respect to 2 feet then obviously, fitting that that tie rod into that gap would be difficult because you cannot basically add any materials, because the materials have been removed using the lathe machine or cnc machine or grinding machine whatever it is, such that it become so waste. So, in that case you would be only interested in to find out the errors on one sided provided which less then; then obviously, it becomes a wastages

if it is more than obviously, you can find out some method to remove the extra amount of material raw material waterways and then basically fitted. And these type of examples can be thought of in such a way that the one sided and the 2 sided process control and the charts would be applicable; so having said that I will try to draw both the figures using the normal distribution which will make some sense.

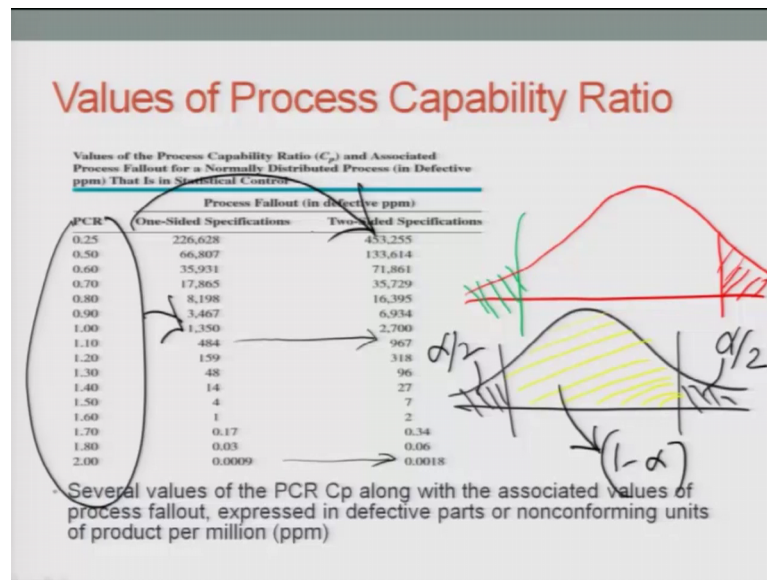
So, consider the one sided it would be either the side of an upper tail or the lower tail. So, I will use some other diagrams. So, this is the lower tail and another case you have the both side is it would basically be. So, it would be this side and this side also, now where the graph look same. So, they have a left tail right tail in both the cases. So, where the interesting part is now I will try to highlight using a different colour. So, let me basically take the violet one.

Now, in this case if there are left then type then obviously, the whole area let me try to find out the colour scheme. So, the whole area on to this is yellow. So, whole area on to the right hand side would be included here. So, in case if my probability values are say for example, α here, in this whole area becomes $1 - \alpha$, in the other case let me use an eraser to remove the yellow colour removed removed removed removed removed this is also removed this is also removed ok sorry we I did not basically bring the distribution as per say I will try to draw it yes back into action.

So, now if I am looking at the distribution on from the right hand side, it this will become I will use again a different highlight colour not yellow, let me use the blue one for the time yes. So, if I have this the area now would be on the right hand side which technically means that this is now α and this is one now $1 - \alpha$. So, they are divided in in in proportions of α $1 - \alpha$, but they are divided on to the left part or to the right part not in between. So, why what I mean by non in between I will come to that. So, let me erase this blue part because then it would be less confusing for us.

Now, consider the second diagram.

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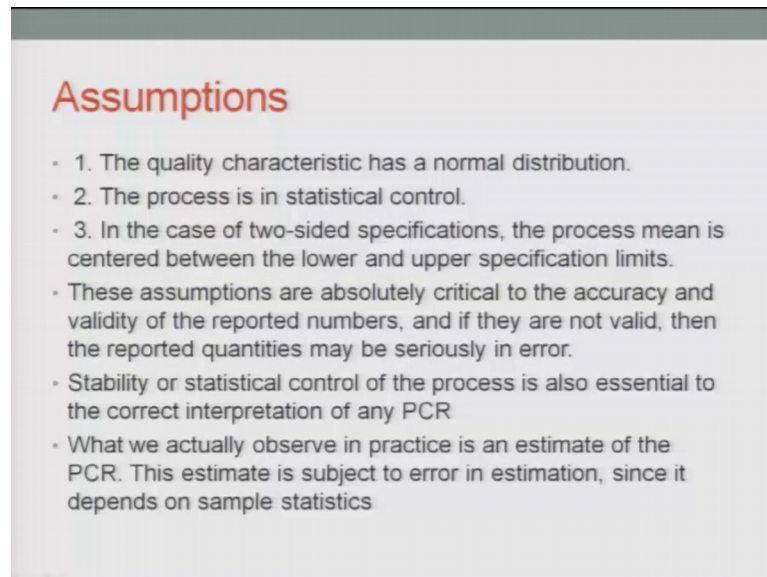
In this case the highlighted part would be let me use the colour is yellow. So, now, it will be this portion inside and this would be highlight. So, this would be again it let me not continue the sorry for that because the colour is changing. So, please bear with me. So, I will remove this yellow colour, bring the red colour into the picture, this will be alpha by 2, this will be alpha by 2, and this would be basically 1 minus alpha; which means that because this being a normal distribution both the upper tail and the lower tail would be proportionally divided in case if it was not a normal distribution; obviously, the middle portion would be one minus alpha and the rest alpha would be divided in some proportion on to the right hand side means above the red one and below the black one those vertical lines.

But that this being a symmetric distribution alpha for that value of alpha is equally distributed on to the right and the left hence it is alpha by 2. So, that is why it mentions one sided specification 2 sided specification. So, I spent about ten minutes talking about that drawing different diagrams was because this was the reason. So, you have the process capability ratios are given, then the one sided and the both and the 2 sided specifications are basically given accordingly.

Only one thing remember the values which you have would be almost twice of each other. So, if you have basically considering the values of 0.0009. So, for the 2 sided one it will just would be about double of that. If you are considering say for example, 484 it

should be about 968, but it is 967. So, that those values of plus 1 has one has been adjusted to make it approximately values exactly. So, the PCR value would basically dictate either the one sided or the 2 sided depending on how the case is.

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Assumptions

- 1. The quality characteristic has a normal distribution.
- 2. The process is in statistical control.
- 3. In the case of two-sided specifications, the process mean is centered between the lower and upper specification limits.
- These assumptions are absolutely critical to the accuracy and validity of the reported numbers, and if they are not valid, then the reported quantities may be seriously in error.
- Stability or statistical control of the process is also essential to the correct interpretation of any PCR
- What we actually observe in practice is an estimate of the PCR. This estimate is subject to error in estimation, since it depends on sample statistics

Now, considering the assumptions the quality characteristics has a normal distribution as we have been discussing time and again the process is in statistical control. So, depending on the points which are above the upper control, lower the control will basically take actions accordingly. In the case of 2 sided specification the process mean is centered between the lower and upper specifications as mentioned so; that means, as a central line you have an equal proportional above equal proportional is below. So, if you are looking at the charts. So, this is the this dotted one is the central line. So, if you have a upper control lower control, the charts would be accordingly done.

So, these assumptions are absolutely critical to the accuracy and the validity of the reported numbers and if they are not valid then the reported quantity is maybe seriously in error. So, stability or statistical process control of the process is also essential to correct to the correct interpretation to have a correct information about the PCR, what we have actually observe in practice is an estimate of the peace the probability the ratios which we were talking about, this estimate is subject to the error in estimation since it is it depends on the on the sample statistic.

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Minimum Values for PCR

■ TABLE 8.3
Recommended Minimum Values of the Process Capability Ratio

	Two-Sided Specifications	One-Sided Specifications
Existing processes	1.33	1.25
New processes	1.50	1.45
Safety, strength, or critical parameter, existing process	1.50	1.45
Safety, strength, or critical parameter, new process	1.67	1.60

So, it recommends minimum values of process capability ratios for existing process if it is 1.3, one sided would be 1.25 if a new process if 2 sided is 1.51 sided would be 1.45 and so on and so forth values are given.

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Process Capability Analysis using control chart

- Histograms, probability plots, and process capability ratios summarize the performance of the process. They do not necessarily display the potential capability of the process because they do not address the issue of statistical control
- The control chart should be regarded as the primary technique of process capability analysis.
- The \bar{x} and R charts should be used whenever possible, because of the greater power and better information they provide relative to attributes charts.
- However, both p charts and c (or u) charts are useful in analyzing process capability.

So, process capability analysis using control charts are done accordingly, histograms probably plots and process capability ratio summarize the performance of the process, they do not necessarily display the potential capability of the process because they do not address the issues of statistical control per c. The control chart should be regarded as the

primary technique of process control capabilities analysis, the \bar{x} and R chart should be used whenever possible because the greater power and the better information they decimate and the overall information which you get.

However, both p or the u charts are useful in analyzing process capability and trying to find out the proportionally concept which is in intrinsic part of say for example, and go no go gauge or bad and good items and all these things.

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An example

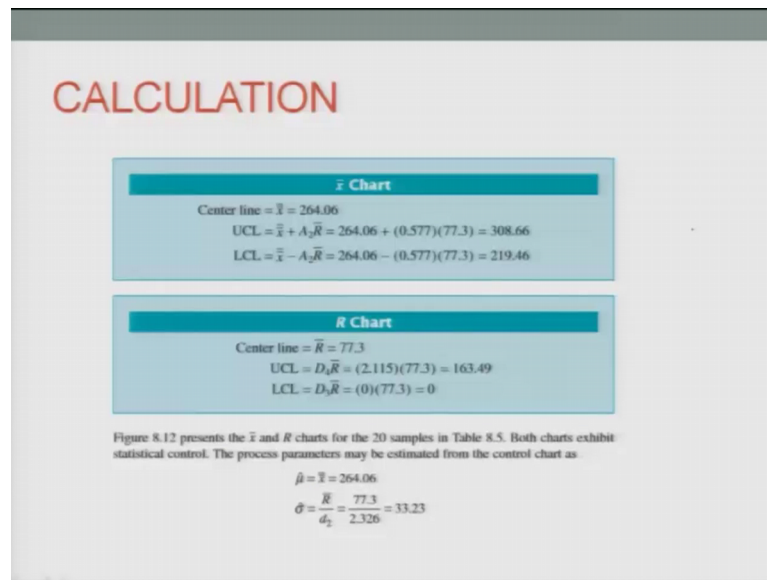
■ TABLE 8.5
Glass Container Strength Data (psi)

Sample	Data					\bar{x}	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	290	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	282	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	269	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{R} = 77.3$

So, let us consider an example. So, glass container strengths are given, you have you take one sample size is basically 5 in number. So, the first row gives you 265, 205, 263, 307 and 220 are the values and you have basically taken 20 sample. So, 20 into 5 is the total number is 500, for each row you for each sample you find out the averages which comes out as given in the second last column which is \bar{x} . So, the first value is 252, the last values is 253. So, from summing them up divided by 20 gives you the $\bar{\bar{x}}$ which is actually the average of the sample or technical if you have taken a huge number of samples repeatedly with each sample size being large, then this value of 264.06 would be exactly equal to the population average.

You also find out the range. So, range are given on the last column starting from 102 to 70 and the average range value is given as 77.3.

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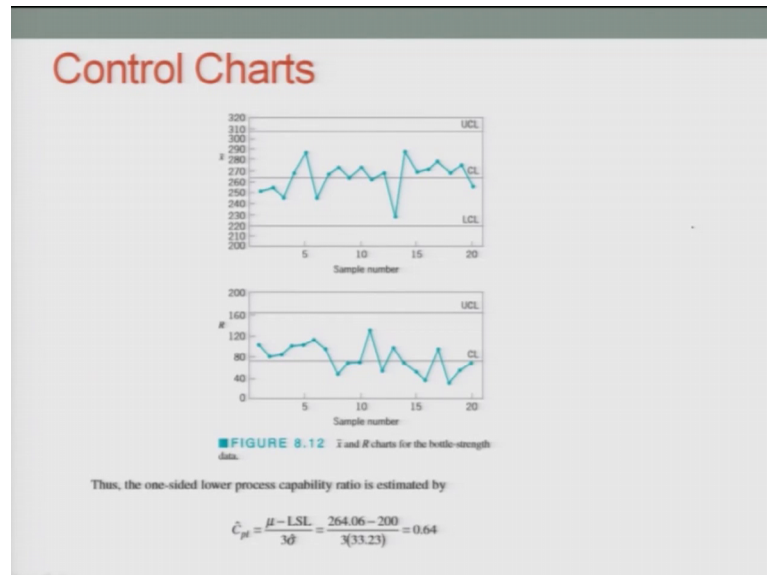
So, if you have the calculate do the calculations find out the x bar charts, upper control is given by this x x double bar plus A2R bar. So, A 2 values which have discussed and I just discuss when we are in the initial slide which was showing the thirty third slides, I did not mention that they depend on the sample size. So, depending on the sample size I find out the upper control has about 308.66, 66 and a lower control has 2 hundred and nineteen point four six similarly for the R charts the values average values 27.3, the upper control is 163.49 and the lower value is basically 0.

So, they once you find out you can find out the process parameters and then basically if you compare the process parameters if they are actually really close to the x double bar value for the mean value and for the sigma value if r bar divided by the d 2 values small d 2 value whatever it is depending on the sample size which comes out to be 33.23, if it is closure to the overall population standard deviation we know that the calculations on the process capabilities have been done as per the norm.

So, let me continue reading it, figured the figure which will see later on represents the values of x bar and R charts for 20 samples, both charts exhibit statistic and control the process parameters can be estimated and they are stated which I am again repeating, x hat mu hat basically means the best estimate for the population which comes out to be again I am repeating is 264.06, and sigma hat which is the estimate for the standard

deviation comes out to be 33.23. So, if you find out these charts and draw them for the x bar and the R charts they are given as shown in this slide.

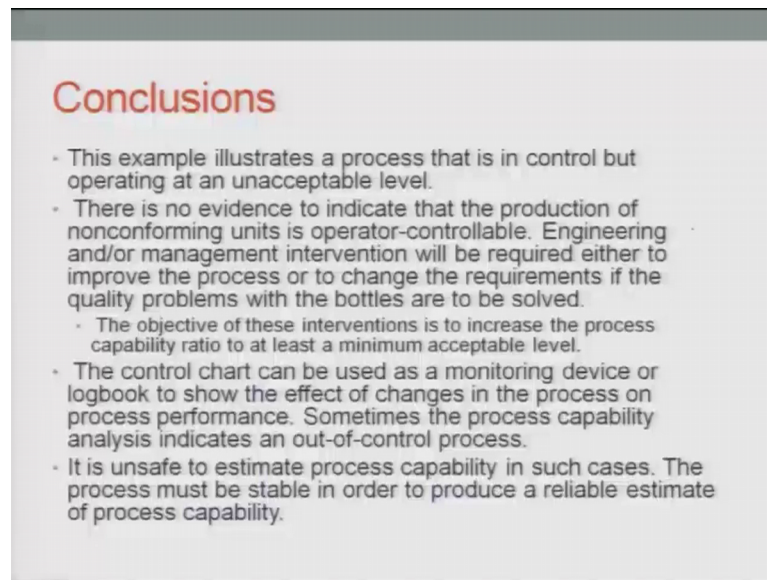
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So, the first one basically measures find out the x bar, the second one basically find out the range. So, thus if you can find out one sided and 2 sided process capability would be basically you are trying to find out the difference between the mean and the lower control values divided by 3 sigma, and if you find trying to find out the both sided 2 sided process capability it will be the upper control minus the lower control. So, in case if it is a normal one what you are trying to do is that for both side you are taking the whole so called difference between the upper control lower control value, depending on what the value level of significance or level of confidence which we have. An in case if it is one sided obviously, it will one half of this total 2 sided concept which you have.

So, in one case technically it is you are trying to find out the overall and the distances between the mean value and the lower control or the mean value, and the upper control depending on whatever the case is such that you are able to cover that that area in such a way that it will give you the one sided values and in another case you can find out the difference using the 2 sided such that this one sided and 2 sided example which I gave would makes much sense to find out the process capability and the process characteristics of the charts.

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Conclusions

- This example illustrates a process that is in control but operating at an unacceptable level.
- There is no evidence to indicate that the production of nonconforming units is operator-controllable. Engineering and/or management intervention will be required either to improve the process or to change the requirements if the quality problems with the bottles are to be solved.
 - The objective of these interventions is to increase the process capability ratio to at least a minimum acceptable level.
- The control chart can be used as a monitoring device or logbook to show the effect of changes in the process on process performance. Sometimes the process capability analysis indicates an out-of-control process.
- It is unsafe to estimate process capability in such cases. The process must be stable in order to produce a reliable estimate of process capability.

Conclusions this example illustrates what we have just shown, illustrates a process that is in control, but operating at an unexpected levels so; obviously, corrective actions needs to be taken. There is no evidence to indicate that the production of nonconforming units is operator controllable, engineering and or management intervention would required either to improve the process or to change requirement if the quality problems are there with the bottles and they need to be solved.

The objective of this interventions is to increase the process capability ratio at least to a minimum unexpected acceptable levels, the control charts can be used as a monitoring device or logbook to show the effect of changes in the process on process performance. Sometimes the process capability analysis indicates an out of control process and the last point which is mentioned here in the slide it is un safety estimate the process capability in such a case, the process must be stable in order to produce a reliable estimate of the process capability and thus find out what are the values of the parameters or better still we want to find out the values of the estimates from the sample which would be the best estimates for the parameters or the population.

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Process Capability Using Designed Experiments

- A designed experiment is a systematic approach to varying the input controllable variables in the process and analyzing the effects of these process variables on the output.
- Designed experiments are also useful in discovering which set of process variables is influential on the output, and at what levels these variables should be held to optimize process performance.
- One of the major uses of designed experiments is in isolating and estimating the sources of variability in a process. For example, consider a machine that fills bottles with a soft-drink beverage. Each machine has a large number of filling heads that must be independently adjusted.
- The quality characteristic measured is the syrup content (in degrees brix) of the finished product. There can be variation in the observed brix (σ^2_B) because of machine variability (σ^2_M), head variability (σ^2_H), and analytical test variability (σ^2_A). The variability in the observed value is $\sigma^2_B = \sigma^2_M + \sigma^2_H + \sigma^2_A$

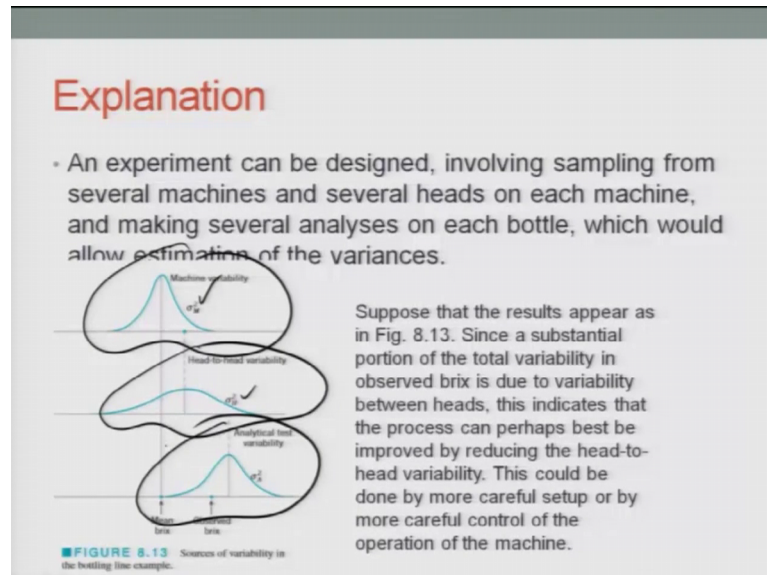
So, process capability using design experiments which will consider this concept of design of experiments will consider in more details in TQM 2 course, just for the information I did mention when we started TQM 1 course, a designed experiment is a systematic approach to verifying the input controllable variables in the process and analyze the effects of this process variability on the output. Design experiments are also useful in discovering which setup process variability is inferential on the output and what level this variable should be held to optimize process performance. One of the major uses of designer experiments is to isolate the values and estimate the sources of variability in a process for example, consider a machine that feels bottles with soft drink beverages.

So, each machine has a large number of filling heads that must be independently adjusted and you can basically find it out accordingly. The quality characteristics measured is the syrup content in the bottles of the finished products. So, they can have a variability due to the observed values, due to the machine variability, due to the head variability. So, they are different type of not problems variability can occurred. Anyone to find out basically the variability of the observations with technically would be the sum of all the variabilities.

Now remember one thing adding the various would give you a better estimate of trying to find out the variability not adding the standard deviations. So, this should be mentioned I am just mentioning it as an important point, whenever you are trying to

study in variability adding the variance is the right way of doing in mathematical, it doing it the right way rather than adding the standard deviation.

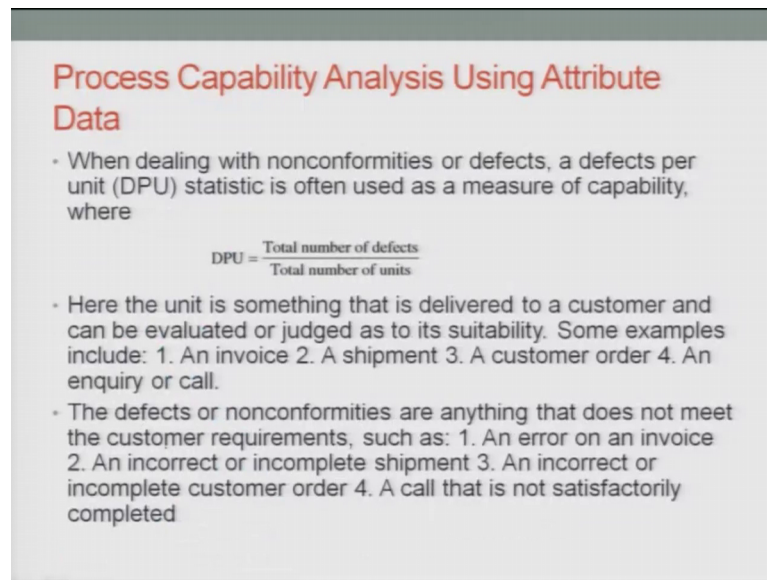
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So, few explanations to continue that an experiment can be designed involving sample from several machines and several heads on in each machines and making several analysis or each bottle, which would basically allow us for the estimation in the variances. So, if you consider this problem let me use the colour as this. So, in this case you have the machine variability and then you with the variances is given by sigma squares of x_m , in the third second case you have the head to head variability and the variance is given by sigma squares of x_h , and the last one basically be the analytical test of the variability and you want to find of the variability considering the test serving propounds. So, what is the variability for them? So, this variability or the variance is given by sigma square suffix a.

Now as I mentioned that add up the variability in finding out the total variability. So, to continuing that, suppose that the result appeared as given in in figure since a substantial portion of the total variability observed and is due to the variability between the heads, this indicates the process can perhaps best be improved by reducing the head to head variability and we will take corrective actions accordingly. This could be done by more careful setup or by more careful control of the operations of the machine.

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Process Capability Analysis Using Attribute Data

- When dealing with nonconformities or defects, a defects per unit (DPU) statistic is often used as a measure of capability, where

$$DPU = \frac{\text{Total number of defects}}{\text{Total number of units}}$$

- Here the unit is something that is delivered to a customer and can be evaluated or judged as to its suitability. Some examples include: 1. An invoice 2. A shipment 3. A customer order 4. An enquiry or call.
- The defects or nonconformities are anything that does not meet the customer requirements, such as: 1. An error on an invoice 2. An incorrect or incomplete shipment 3. An incorrect or incomplete customer order 4. A call that is not satisfactorily completed

Now, we will further continue our discussion about process capability analysis using attribute data. So, technically we had considered whatever mentioned was really variability, now it is basically due to the attribute of the characteristics. When dealing with nonconformities or defects per unit, per thousand, per hundred or per lakh whatever it is, often used as a measure of the capability and that is given by deep you which basically means defects per unit and that is given by the total number of defects by divided by the total number of units.

Here the unit is something that is delivered to the customer and can be evaluated adjust accordingly some examples include several example preparation of invoice a shipment a customer order and so on and so forth. The defects or nonconformities are anything that does not meet the customer requirements as given, they can be in error in invoice they can be say for example, in correct and incomplete in shipment has occurred, on incorrect or incomplete customer order has been done or process or for the enquiry of a call which is the fourth example it could be a call that is not satisfactory completed and not solved according to the customers feedback.

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DPMO

- The DPU measure does not directly take the complexity of the unit into account. A widely used way to do this is the defect per million opportunities (DPMO) measure

$$DPMO = \frac{\text{Total number of defects}}{\text{Number of units} \times \text{Number of opportunities}}$$

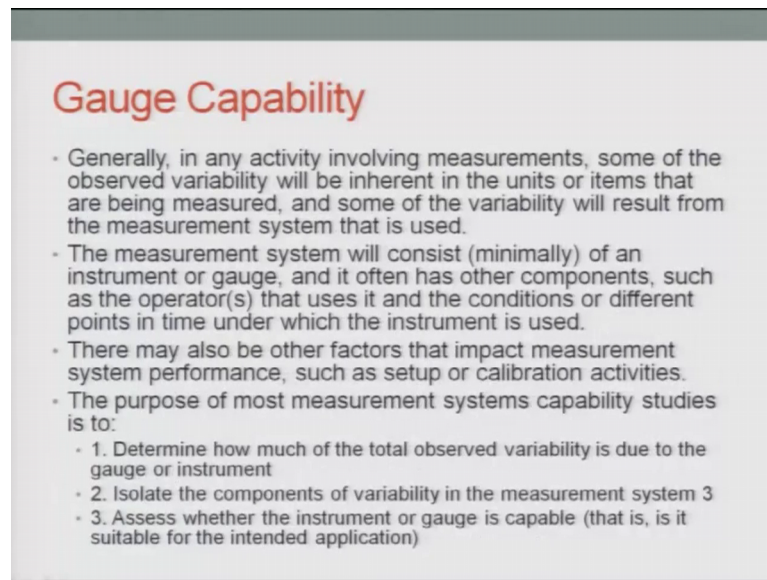
- Opportunities are the number of potential chances within a unit for a defect to occur. For example, on a purchase order, the number of opportunities would be the number of fields in which information is recorded times two, because each field can either be filled out incorrectly or blank (information is missing).
- It is important to be consistent about how opportunities are defined, as a process may be artificially improved simply by increasing the number of opportunities over time.

So, considering the defect per million opportunities concept that DPU measurement does not directly take the complexity of the unit to be studied or more widely used measure is the DPMO which is defect per million opportunities. So, that basically means very simply like this. You had the total number of defects, now you have divided by the total number of units now rather than dividing only by the total number of units, you will also divided by the number of opportunities of such field cases which is I have got.

So, opportunities are the numbers of potential chances within unit for a defect to occur for example, on a purchase order the number of opportunities would be the number of fields in which the information is recorded. So, they can be different type of measurement which have been taken, because each field can either be filled or incorrectly or can be kept as blank.

It is important to be consistent about how opportunities are defined as a process and then maybe artificially improved simply by increasing the number of opportunities over time.

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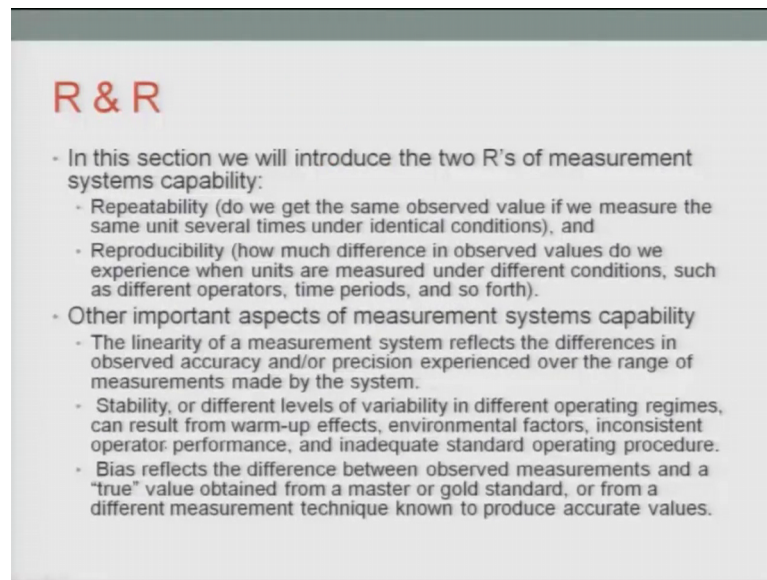
Gauge Capability

- Generally, in any activity involving measurements, some of the observed variability will be inherent in the units or items that are being measured, and some of the variability will result from the measurement system that is used.
- The measurement system will consist (minimally) of an instrument or gauge, and it often has other components, such as the operator(s) that uses it and the conditions or different points in time under which the instrument is used.
- There may also be other factors that impact measurement system performance, such as setup or calibration activities.
- The purpose of most measurement systems capability studies is to:
 1. Determine how much of the total observed variability is due to the gauge or instrument
 2. Isolate the components of variability in the measurement system
 3. Assess whether the instrument or gauge is capable (that is, is it suitable for the intended application)

Generally in any activity involving measurements, some of the observed variability will be inherent in the units or item that have been measured and some of these variability will result from the measurement system that has been used. The measurement system will consistent will consist on an instrument on or gauge and is often has the components such as the operator that uses it and the conditions or different points in time under which the instrument is used. There may also be other factors that impact measurement systems and performance such as setup or capability activities which are there. The purpose of the most measurement system capability studies are 2 number one determine how much of the total observed variability is due to the gauge or the instrument which is being used.

Isolate the components of variability in the system and assess whether the instrument or gauge is capable, what is capable of that is it is whether is suitable for the intended word which is being utilized.

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R & R

- In this section we will introduce the two R's of measurement systems capability:
 - Repeatability (do we get the same observed value if we measure the same unit several times under identical conditions), and
 - Reproducibility (how much difference in observed values do we experience when units are measured under different conditions, such as different operators, time periods, and so forth).
- Other important aspects of measurement systems capability
 - The linearity of a measurement system reflects the differences in observed accuracy and/or precision experienced over the range of measurements made by the system.
 - Stability, or different levels of variability in different operating regimes, can result from warm-up effects, environmental factors, inconsistent operator performance, and inadequate standard operating procedure.
 - Bias reflects the difference between observed measurements and a "true" value obtained from a master or gold standard, or from a different measurement technique known to produce accurate values.

Now, we will just go through the simple concept quality really about the repeatability, and the reproducibility of the process capability studies or the analysis. So, in this section we will introduce 2 rs of measurement system capabilities and understand them in detail. Repeatability means that we do we get the same observed values if we measure the same unit several times under the identical conditions; point 1 and point number 2 is which is reproducibility means how many difference how much difference in observed values do we experience when units are measures and they are being measured continuously under different conditions such as different operators time periods so on and so forth.

Other important aspects of measurement systems capabilities are the linearity of the measurement system, reflects the difference in the observed accuracy or the precision experience over range of measurements being made by the system, and stability or different levels of variability in different operating regimes are there. So, you want to basically understand that what is the results for one of warm up effects and environment factors are there inconsistent operating performance are there in adequate standard operating procedures are there. So, they can be biases which reflect the difference between the observed measurements and a true value obtained from a master or the gold standard. So, or such that from a different measurement techniques is basically able is available to us such that we can take much better readings from the same.

So, with this I will end the thirty third class and continue with the thirty fourth lecture which would continue till the fortieth one. And then try to wrap up the course accordingly.

Thank you very much. And have a nice day.