Total Quality Management - I
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Lecture - 30
OC curve, c and u charts

A very warm welcome to all my friends and students: good morning, good afternoon good evening. This is the TQM-1 course under the NPTEL MOOC program and I am Raghunandan Sengupta from IME department, IIT, Kanpur. And this is the thirtieth lecture which is basically the end lecture for the sixth week.

So, if you remember we are considering the variable sample size and for the non conformity concept, how the charts could be drawn. And for the variable sample size I did mention, the upper control limit and the lower control limit would keep changing depending on the n i size n i suffix n with suffix i, where i means the sample size. And obviously, if you do not want, you can take average sample size also which is in the last example, it was total observations divide by the number of times you took the observations which was 25 in number. So, it would be divided by 25 and total number of observations was 2450 and from that we got the average sample size to be 98 any way those are the values I just mentioned in order to recapitulate.

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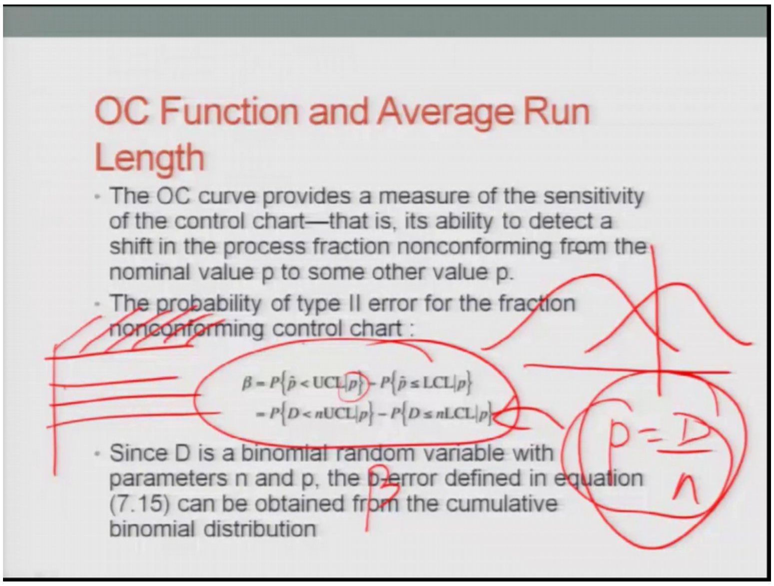

Now, I will consider the operating characteristic functions and average run length the oc curve provides a measure of the sensitivity of the control charge. So, what do you want to do is that that is it is ability to detect a shift in the process fraction nonconforming from the nominal value of p. So, if you p remember p is the total proportions of the probability now if there are defects and if I say the probability of defects is 15 percentage it means that if I pick up continue pick up 100 such observations fifteen of them would be defective. And obviously, if you do not know the population average that 15 will basically take a sample and do the work accordingly.

The probability of type two error, now coming back to long discussion which he had quite long time back, it was basically the consumer risk and the and the producer risk, so those alpha and beta values. So, the probability of type two error for the fraction nonconforming control charts would be given that provided p is known you want to basically find out that what is the probability that other type of defect would be occurring. So, now, I will pause and again draw the one of the diagrams which I may have discussed, but I want to basically discuss it again that will make things much clearer to all of us.

So, consider that let us take a very simple example of our bank. And you are a banker and as a banker manger you basically need to say what is the score based on which you will decide to advance a loan to the person who is applied for the loan or deny the person the facility of the loan. Now, if you remember that of you have seen that basically the bankers or the mangers whoever it is of the officers basically sees whatever profession you are in, what is your paying pay back capability, whether you have a house, whether you have a car, what is your age, number of dependence and all this things. So, they are cumulatively collated, and when you collating it you give some marks the banker gives some marks. And if the total marks total points let us use the word points, the total points is basically 60, and above in this case 60, I am taken as an hypothetical example. If it is 60 and above you the person is given that loan if it is 60 or less than 60, 59 and below the person is denied the loan.

Now, what you what to understand is that whether there is a chance the person who should be given the loan is denied and vice versa. So, what the vice versa case is that person who should not have been given the loan is basically given the loan. So, the OC curve provides a measure of the sensitivity of the control charts that is it is ability to detect a shift in the process fraction conform, nonconforming from the nominal value p to some other value p, p 1, p 2, p 3 as the averages are changing. Now, which means that if it is shifted from p to p 1 or its shifted from p to p 2 or vice versa, you would basically try to find out whether the shift gives you some error or not. So, here is where the type one and type two errors would come.

Now, the probability of type two error for the fraction nonconforming charts is basically given by the fact that given p you want to find out, so this value basically means given p. Given p you want to find out that the upper control limit is basically more than p hat. And here what you want to find out is basically there are two type of errors. One is that if I draw the table, and if the upper control limit is greater which means it lies here; and if the lower control limit is greater; obviously, it goes it is up. So, what you want to find out is basically- what is the difference in the probabilities that both of them happening provided p is basically the average value.

Now, that would also mean that you were basically are able to find it using the concept of the total numbers which is one was basically the fraction nor is the basically the total proportions or numbers that you know would be given p is given by say for example, D by n. So, D is the number of defects. So, if you basically replace p in this formula and try to find out the beta level of risk obviously n and D would be come into the picture, and p would be out because that has been replace. And once you basically find on the formula corresponding to the probabilities, it is also again the same case that beta is the error, a type of error depending on the difference. What are the difference is that given p or the proportions, it could also have been say for example, rather than p, I could have denied denoted by D by n, but it I am not doing it in this calculations.

So, you want to find out that; what is the probability that that D is less than equal to n into UCL which is the upper control limit and in the other case that D would be less than also equal to the n into the lower control limit, based on that you will do the calculations. Since D is a binomial random variable with parameters n and p n is the parameters and p is also the parameters. The beta error, so this is what the beta error defined in equation the above which is 7.12 here can be obtained from the cumulative binomial distributions and the calculations maintain accordingly.

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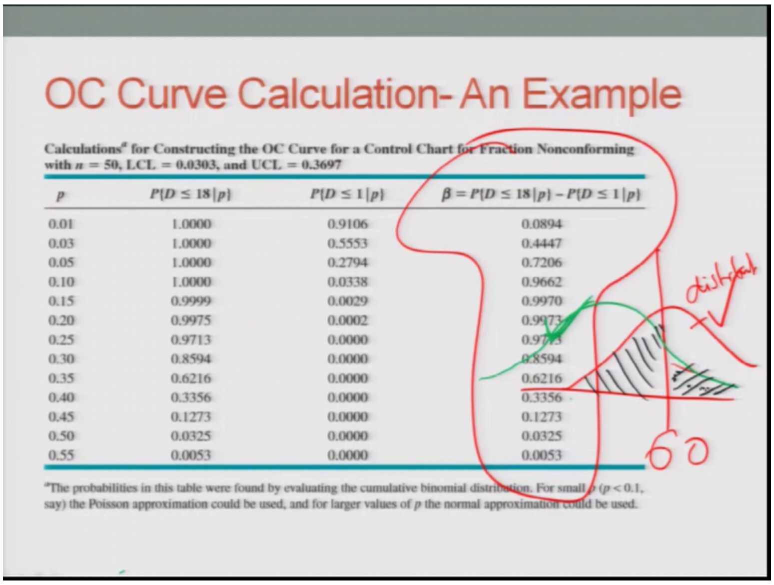

So, you are trying to find out calculations for the constructions of the operating characteristics curve for control charts for fraction non-conforming considering that n is 50, upper control limit is 0.03, lower control limit is 0.03 which is about three percentage and upper control limit is basically about 0.03697 which is about 40. Now, the p values are given along the first column the probabilities that D is less than 18, and the probabilities that D is basically less than 1. So, this is basically the total numbers depending on the lower control and the upper control values which you have found out yourself. So, the probabilities are given in the second column of D being less than 18, which is the upper value and in the third column it is given that D is less than less than some value which is which is related to the lower control values.

So, once you find out you find out the differences between them and calculate the value of beta which is given in the last column. So, this I am highlighting. So, it will give you basically the beta value or the errors. Now, if you remember I have to do the diagram which I drew was this; and you as the banker are willing to give a loan for a set of persons whose scores are greater than 60 less than obviously you are not.

Now, I also mentioned that there are errors on both the count in the sense that you deny loan to person who is eligible and you give a loan to a set of persons or a person who will basically default. Now, consider this one. Now, I should basically use a different color to highlight the information with, then I will use the black one to mark the probabilities. Now, consider this area and also consider this area. If you consider the area onto the right, which is here where I am pointing my style as here it basically means that you are giving your loan to a set of persons who would not payback and what is the error there. So, why I am saying that let us think it very rationally.

60 is the line which basically is the score over and above that you will give lower that you would not give, so obviously, the average value are of the set of persons distribution wise who would basically be willing to pay the loan or have the capability of the loan would definitely be more than 60. So, the distribution which is shown in red which I should again highlight, this is the distribution for the set of persons who will keep the loan that is why I put a tick mark. And if I see the color using green, so this is the set of people oh sorry the color should definitely be same type in order to make our life simple.

So, where I tick mark the green distribution, which incidentally both the distribution is normal. So, this set of people have a score less than 60. Now, as saying the right hand part where my stylus is the probability or so called loss. So, for those set of people who would definitely be denied the loan, but you give him them the loan. And the area onto the left now where the style is now hovering which is the black area, black marked area are the set of people who would be given the loan technically, but they are denied the loan. Now, which means that there are definitely losses, so both negative losses in one case you will face a bad debt, the people would not retain you the loan. And in second case that means, where to which set of people you are denying the loan, but they should definitely get the loan is basically a loss of business.

Now the interesting proposition is that say for example, you think that both should be decrease; obviously, that should be true now which means that if we keep shifting the 60 line, I make it 55 or 70 you will see that individually you cannot reduce them at the same time. So, what is needed is basically a reduction of the overall loss which is basically the alpha and beta or type one to type two radar on a combine sense. That means, you will try to basically make the some of this errors as low as possible because in that case you will be certain that the overall loss you are going to make. In case for this two cases or examples is that denying the set of people who should definitely be given the loan and giving a loan to set of people who would never repay you back would basically be the least in this case; that means, the overall quantum of loss would be least in this case.

So, this was an example which I tried to give you considering that alpha and beta cannot be reduced at the same time, then again coming back to the example based on say for example, the proportions p the probability of D being less than equal to eighty in the probability of d being less than equal to 1. So, the first case is for the upper control limit second one is for the lower control limit based on that you find out beta which is the overall loss and this loss probabilities are given as 0.0894, 0.4447 and so on and so forth.

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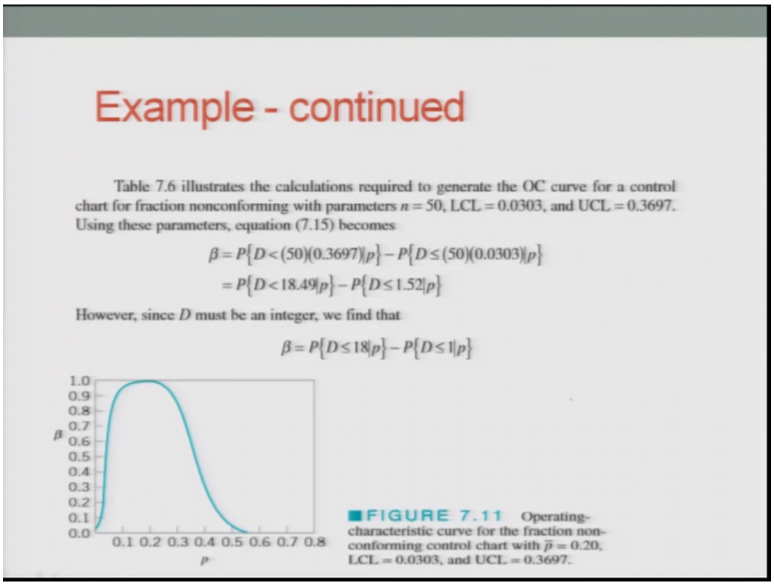

So, this table 7.6, which we have does discussed illustrates the calculation required to generate the OC curves for a control charge for fraction nonconforming with parameters and n is equal to 15, the lower control value comes out to be 0.03 approximately. I am not talking and mentioning all the decimal points after what they are to a level of significance. The upper control limit is basically comes out to be 0.03697 using this parameters which is the equation we have already discussed, we can find out the value of beta.

However, since D must be a integer, we basically try to find out and what is the value of beta. So, the operating characteristics curves for the fraction non-conforming control charts with the value of p hat or a p bar which is 0.20 and the lower control limit is 0.03 and upper control limit is 0.03697, they can be commented accordingly depending on the sample size and how the work can be done.

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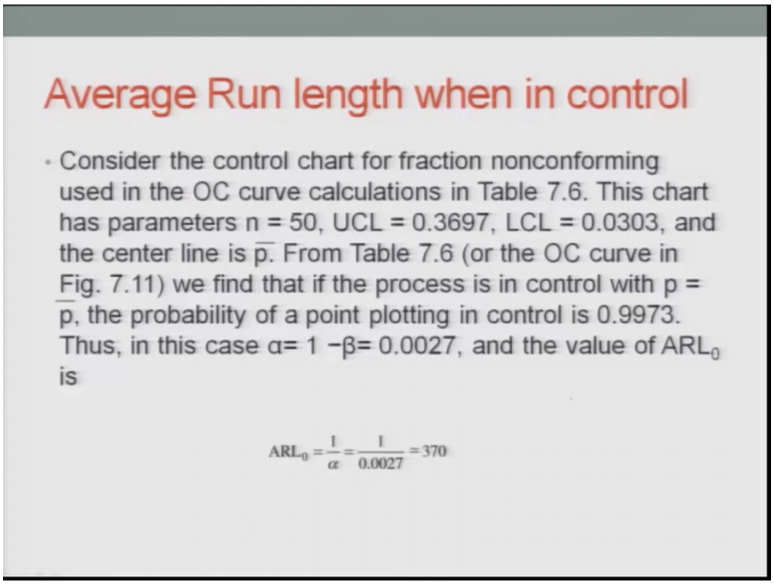

Now, you want to find out the average run planned when the process is in control. So, consider the control charts of fraction nonconforming used in the OC curve calculations in table 7.6. Now, what you want to do is that you want to find the average length time based on which you can take a decision whether that actual processes is in control and out of control. This charts has the parameters as already discussed as n is 50, upper control limit is 0.03697, lower control limit is 0.0303, the central line we obviously, for the calculations would be p bar. And from that if I can find out the value of beta comes out to be 1 minus 0.0027 based on that you can find out what is the average length run time which comes out to be about 370.

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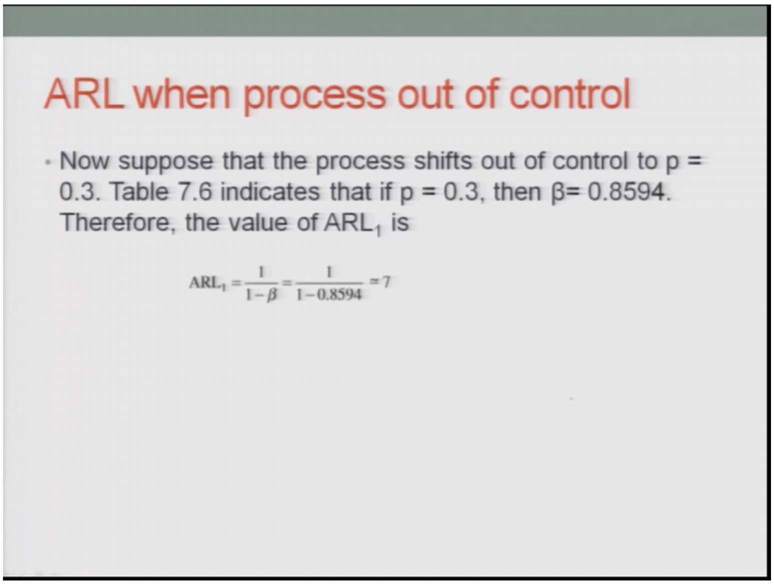

Now, suppose that the process shifts out of control to p value of 0.3, then again doing the calculation the average run length basically comes out to be about 7. Now, we will consider the control charts for average number of nonconforming conformities.

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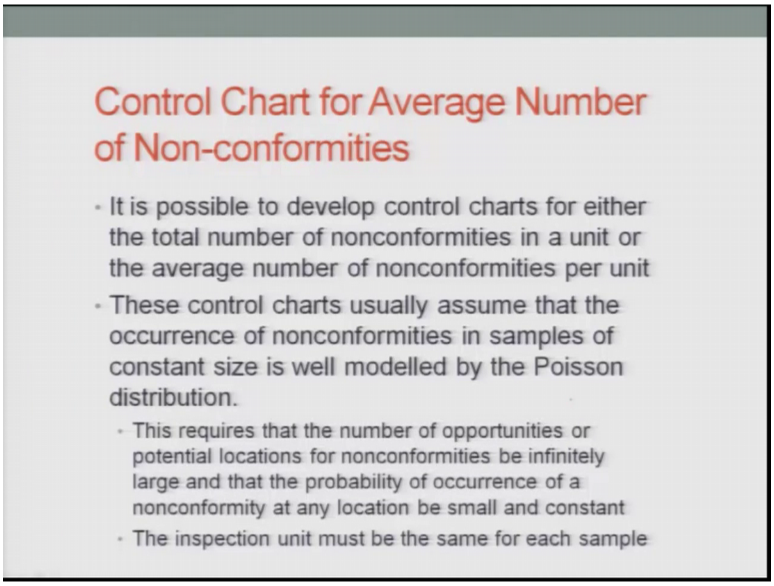

So, it is possible to develop control charts for either the total number of non-conformities in a unit or the average number of non-conformities per unit and do the calculations accordingly. These control charts usually assume that the occurrence of nonconformities in sample the constant size is well modeled using the Poisson process. So, this requires the number of opportunities or potential location for non conformities may be infinitely large and that the probability of occurrence of a non-conformity at any location would be small. So, collectively it is a large, but at individually it is low.

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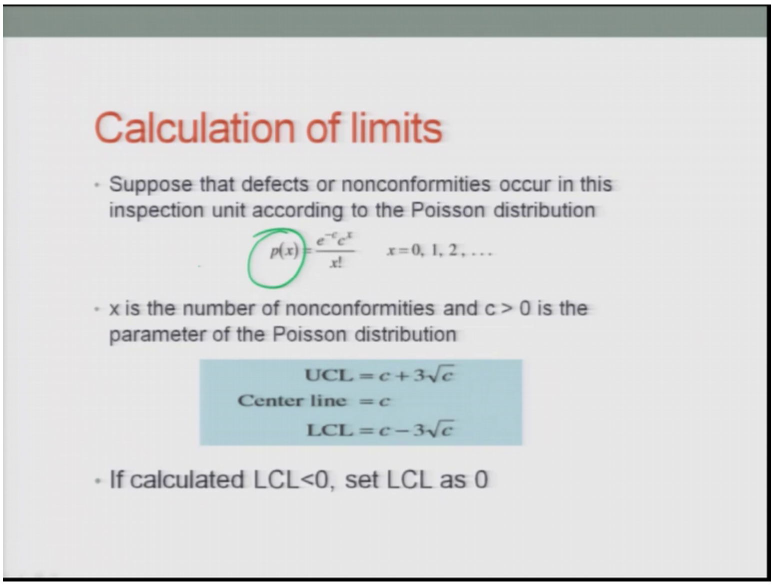

Now, the inspection unit must be the same for each sample and you basically follow the same policy. So, suppose the defects or non-conformity occur in this inspection unit using the Poisson distribution, and the Poisson distribution is given which is the probability for f of x or p of x is given as e to the power c or lambda, whatever the parameter is. Lambda to the power x or c to the power x divide by x factorial and the x values can be 0, 1, 2, 3, 4 and so on and so forth.

Now, if I want to find out the number of nonconformities and considering the fact that c is greater than 0, then obviously, there would be a central line there would be upper control line and there would be a lower control line. Now, for the Poisson distribution one should know what is the mean and the variance, and based on that we do the calculation. The mean central line value is coming out to be c or lambda in whichever you basically try to define. Another upper control values and the lower control value would be again if you remember would be the mean value plus minus the coefficient to what level of efficiency you want for the calculations plus minus 3, multiplied 3. You are multiplying it only for the case when it is basically there is 99 percent coverage multiplied by the so called square root of the standard deviation, but the standard deviation for the Poisson distribution is as given which is c and you do the calculations.

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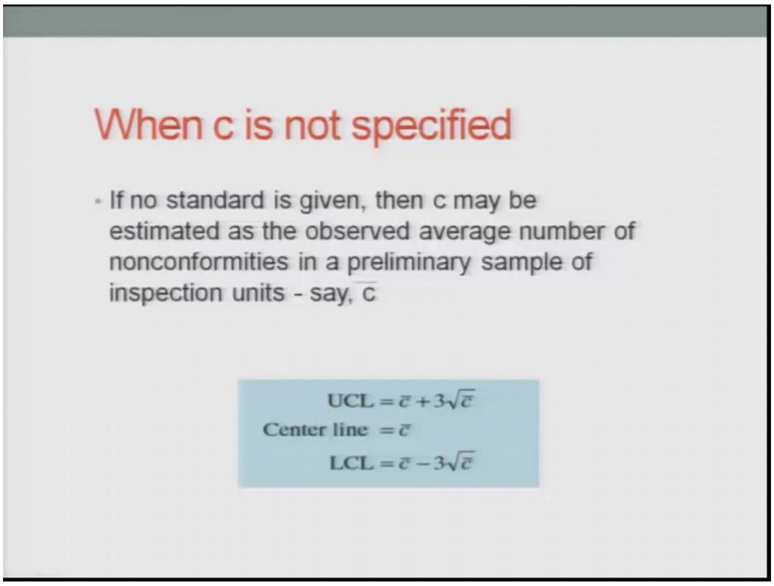

So, if you find out the lower control limit values coming out to be less than 0, you set them to zero based on that you do the calculations. If no standard is given then c may be estimated at the as the observed average number of nonconformities in a preliminary sample of inspection. So, based on that you find out c bar or c hat and do the calculations accordingly, where now in place of c you will basically using the c bar or the c hat. So, again the upper control and the lower controls are given which I am circling. So, c bar plus minus 3 of square root of c bar and you do the calculations again.

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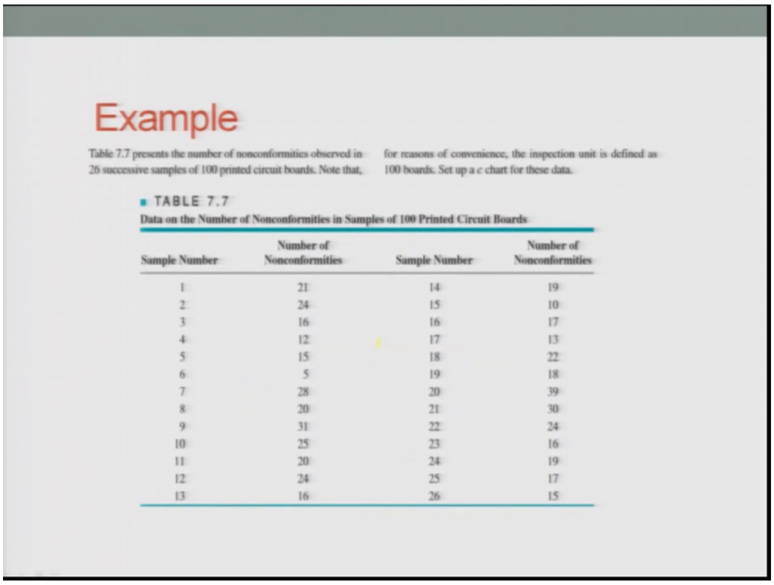

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

So, now let us present an example it presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. So, basically I have continued taking observations one at a time. Now, the question is how many number of times did I take it is 26. And if I want to find out the number of defects, it basically means that you have to find out the value of D, but when I try to find out the proportions you have to also find out what is the sample size in each. So, here it mentions very categorically the sample size is for each one group is basically given as 100.

So, based on that, I have in the first column and the third column the sample number. The second column and the fourth column, basically gives you the number of nonconformities based on which you will do the calculation. So, we would like to measure that a mention that for reasons of inconvenience the inspection unit is defined as 100 and will continue using that 100 as the sample size for reading to (Refer Time: 19:21). The sample numbers are given on the first column, sample numbers again I am repeating and given on the third column the number of nonconformities based on the sample size are given in the second and the fourth column.

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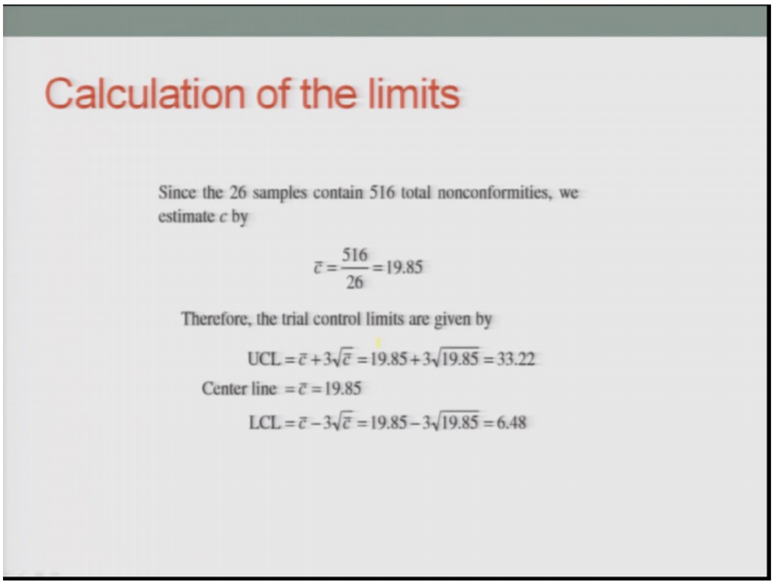

So, hence if I want to find out C s bar which is the best proxy of c that value comes out to be 19.85 and corresponding values of upper control and the lower control limit basically comes out to be about 33.22 and 6.48.

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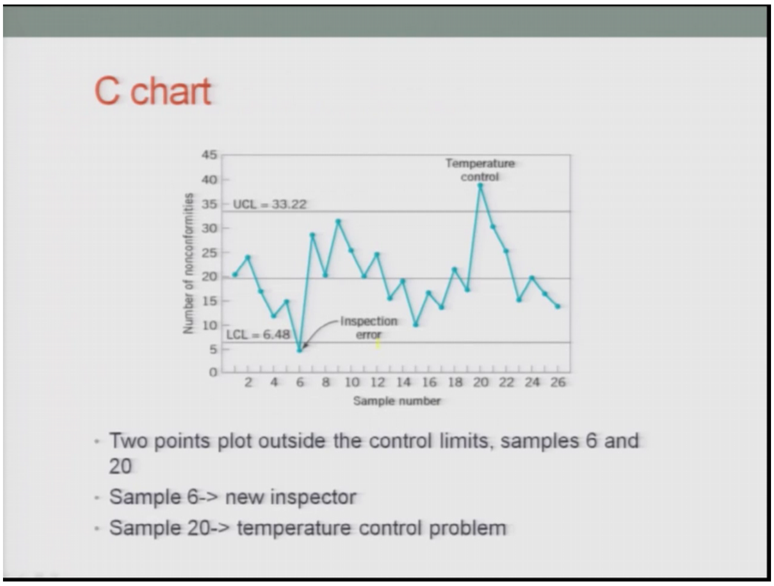

So, when we plot it two points plot outside the control limits, which are basically sample 6 and sample 20. So, say for example, so hypothetical excuses or reasons not I would not use the word excuses the reasons for which the non conformity for those set of values came out may be that the broker on the new machine was a new operator. And obviously, there has been some change in how somebody wants the machine to work or the coolant was too large or say for example, the drill bit was not properly fit or the person was doing the very slowly.

So, obviously, there would be some reasons based on which them the decision was taken to do a check of the sample number 66 and 20. So, here the information comes out in this example is that in sample 6 our new inspector joined. And basically the new inspector would have been be either very stringent or very relax based on that you were able to gather the data as it has given. And say for example, in the twentieth set of reading may be the temperature or the humidity which would you are trying to control suddenly basically either increase or decrease.

So obviously, would have a negative impact on the actual attributes of the characteristics, which you are trying to understand for the proportions of the probabilities based on which you will try to find out c hat or c bar provide a c s not known and the corresponding standard error or the variance.

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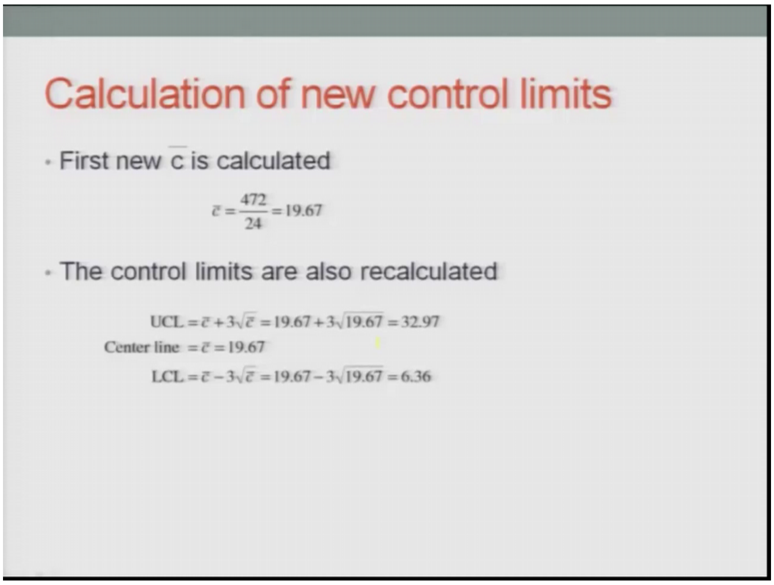

So, the new calculations first calculate the new c star or the c hat and the value comes out to be 19.67. Once you find out the value of 19.67, the control charts are drawn which means the upper control limit the lower control limit you find out basically UCL and LCL which would be c or c bar plus minus 3 of basically square root of c or c bar. And do the calculations the values comes out to be 32.97 for the upper control limit and 6.36 for the lower control values or the lower control limit.

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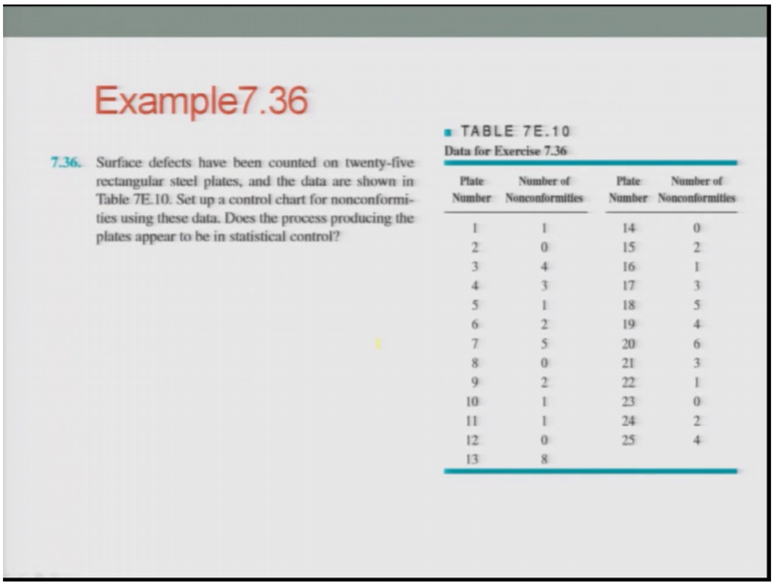

Now, can let us consider a surface defect that has being counted on 25 rectangular small plates and the data are shown in table 7E.10. What are the data let us basically read it, it gives you the plate number which is in the first and the third column it gives a number or nonconformities which is given in the second and the fourth. So, you are required to set up control charge for nonconforming them using this data. So, we want to basically pass on the judgment whether the process producing the plates appear to be such statistically under control and whether if they are in aberrations how they are been able to take care of that.

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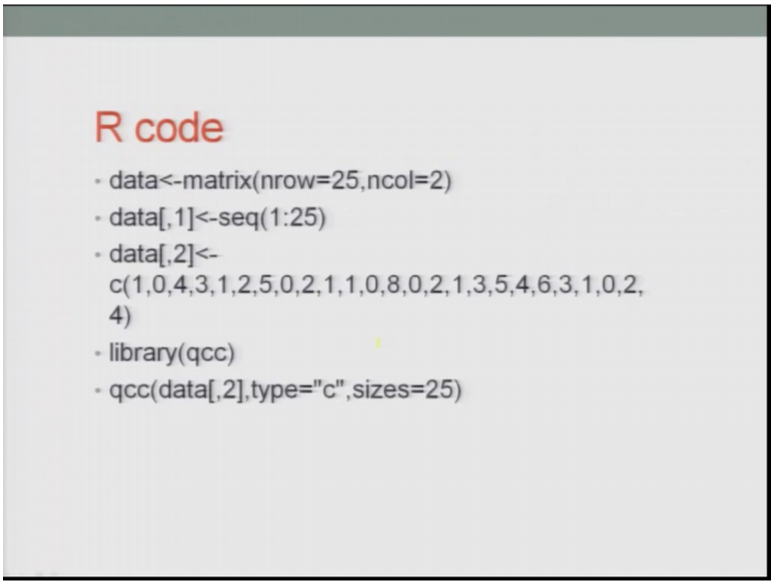

So, again this is a small R code, the data matrix is of size 25 cross 2 and then we put the data which is basically the so called conformities. So, is basically start with 1, 0, 4, 3 and goes out to till 1, 0, 2, 4. So, from there you find you recall the library functions based on that you do the calculations and then you can find out c or c hat and the corresponding upper control and the lower control values.

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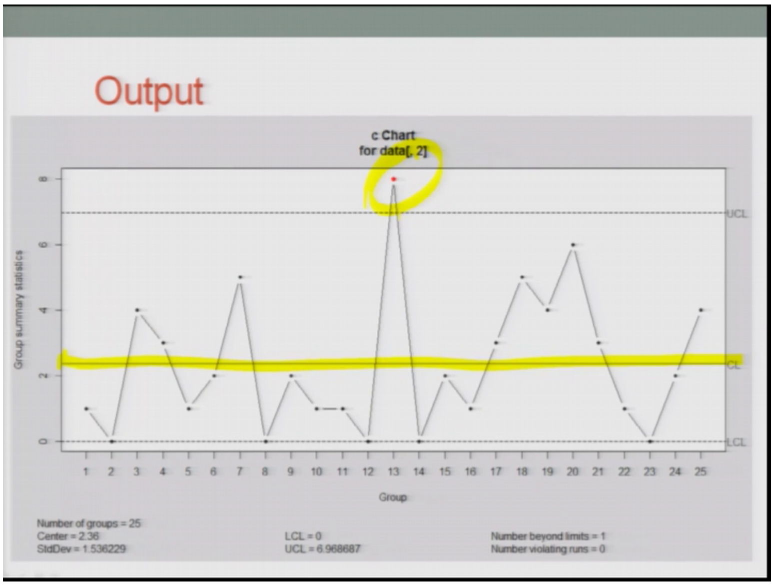

So, once you do that, so here is the chart which we have the number of groups is given is 25. If you to remember the central line which is this one is about 2.36 standard deviation values are given as 1.53 I am only reading till thus two places of decimal. The upper control and the lower control values comes out to be respectively 6.96 and 9. And based on that if you find out then you will definitely be in a much better position to come in that one of the reading basically was I would not say use the words spurious, but it was out of the limits. So, some corrective action needs to be taken or you want to basically study where the problem occurred.

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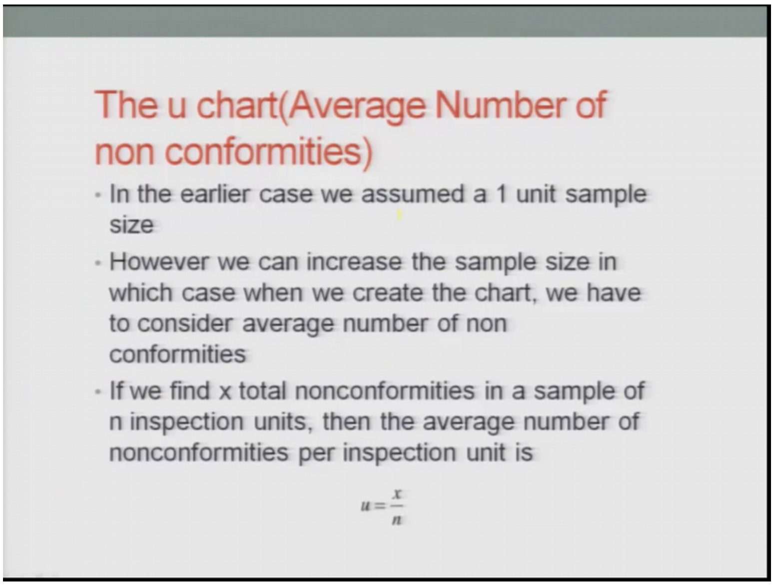

Now, I will use the u charts which is the average number of non-conformities. In this case, we assumed a one unit sample size; however, we can increase the sample size in which in case when we clear the charts and we have to consider the average number of nonconformities. So, if you find x which is the total non-conformity in a sample size of any inspection then the average number of nonconformities per inspection is given by the value which you find out basically that is basically x by n.

Now, if you remember so x is the total nonconformities, n is basically the sample size. So, may be possible that in the first set of observation or sample size n, you found out two defects. In the second case, when it was again to do with the number of defects say again its say for example, comes out to be 5. So, if I basically find out the total number that number basically is given by x.

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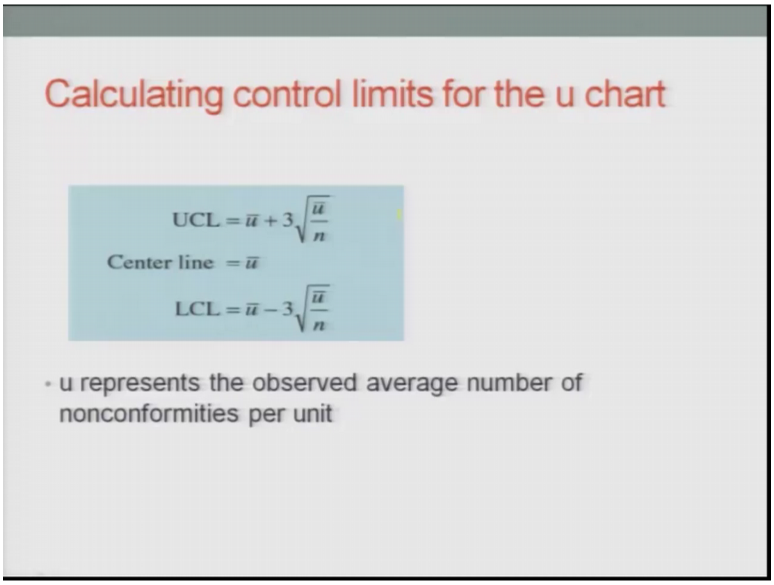

Again continuing the discussion if I want to find out the centerline, the upper control and the lower control the concept given is the same we use the centerline to be u bar or u whatever it is. And obviously, u bar would be the case for the examples where we consider the sample and try to basically draw some meaningful conclusions from the sample onto the population, the corresponding standard deviations are given as. So, this three again I am repeating is basically the overall level of confidence you which you want to have and they would be multiplied by square root of u bar by n and do the calculations accordingly. So, here you represents the observed average number on nonconformities per unit and you do your calculations accordingly.

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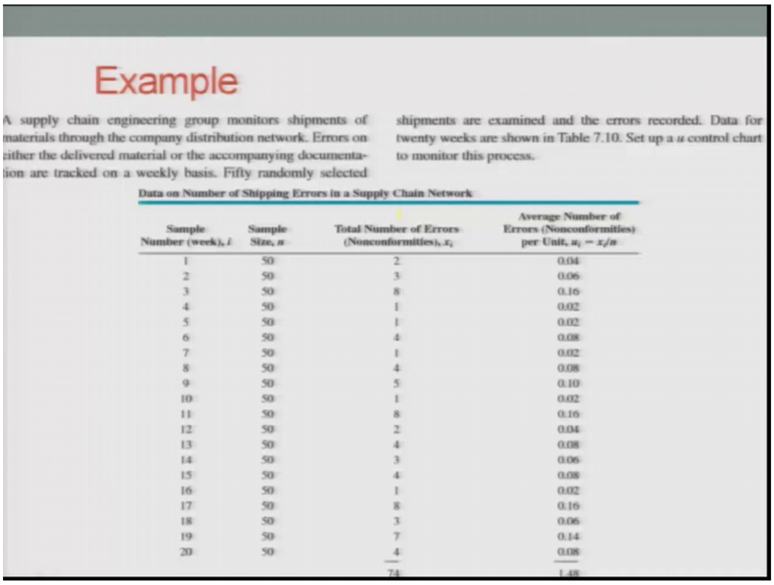

Example

A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for twenty weeks are shown in Table 7.10. Set up a u control chart to monitor this process.

Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

Now, in the next example, a supply chain engineer group monitor shipment some materials to the company's distribution network, errors on either on the delivery material of the accompanying documentations are traced on a weekly basis 50 random was selected. So, basically they are some of the observations are given here. 50 randomly selected shipments are examined and errors occurred, and they are recorded and data for 20 weeks are shown in figure, which is 7.10. And then basically you want to basically set the u charts and find out the u bar, and the basically the upper control values and the lower control values.

So, sample numbers are given along the first column, the sample size for each such samples which you are taking are fixed which is 90. From each set of 70, 50 in the first sample of the first week the sample for this next week, which is again 50 and so on and so forth. The total number of errors which you find out are given in third columns starting from 2 to 4 that means for the sample for the first week the number total number of errors was basically twos. Then if I want to find out the sample number which is number 2, the sample size is again 50. And the total number of errors or the nonconformities comes out to be 3; and based on that you basically chart down and find out what is the average number of errors provide non-conformity per units are basically reported.

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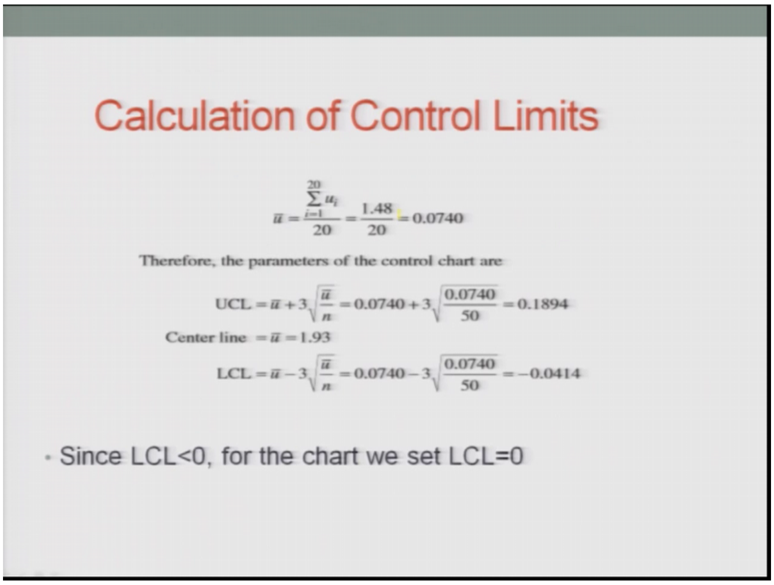

Though once you basically do the calculations for the control limits you find out u bar which comes out to be 0.0740; and based on that you do the upper control limit and the lower control limit, the values comes out to be 0.1894 and minus of 0.0414. Now, here where we have we have made as assumption the lower control limit if it is negative or less than 0, you will consider it as 0. So, based on that you find out the upper control, lower control and central line, and do the calculations accordingly and plot it; once you plot it you will be able to understand where the errors are whether the assignable causes unassignable causes and then take corrective action depending on what is the qualitative field which you find out from the quantitative field which is the charts.

So, charts give you where the problem is occurring they did do not give you the actions you should take. So, they will tell you that yes some problem is occurring, so that problem has to be tackled on your own part that can be either due to bad workmanship, that may be due to humidity, that may be due to temperature that may be due to coolant temperature and so on and so forth, there many reasons for that. So, with this I will end the thirtieth lecture and continue in the thirty first lecture till the fortieth one for the TQM class. Have a nice day.

Thank you.