Total Quality Management - I
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Lecture - 29
Estimating control limits with varying sample size

Warm welcome all my friends very good morning, good afternoon, good evening. This is the total quality management one class and I am Raghunandan Sengupta from the IME Department IIT; Kanpur. And we are starting the 29th lecture which would be the second last in the sixth week.

Now, we were discussing the charts and if you remember to recap for the last three four lectures; we did discuss about the x bar charts, the central line for the x bar charts being the x double bar which was the expected value of the distribution; form where you are taking. Then we had the concept of the R charts which was something to do with range, then we had something to do with the standard deviation of the sample; where information is known about the standard deviation of the population and not known.

So, you consider that then we consider the depending on the coefficients now coefficients would be what level of efficiency you want or what is the level of confidence which you want you basically have those coefficients as in all the examples we consider as 3; plus minus 3 because you wanted to cover about 99 percentage of the overall coverage area which is plus minus 3 and that is the total six sigma.

Then we considered the np charts, p charts p was the proportions np was the again something to do with the proportion. But when multiplied by the sample size gives you the total numbers in the sample; like it is something to do with x being the total number indirect way of calculating. Then in all the cases we considered the so called central limit theorem to be true and we considered the distribution to be normal and we took the mean and standard deviation for the actual population based on which you are trying to in perform the central limit theorem and we completed.

So, continuing our discussion; so, variable sample size, so we are going to consider the variable sample size.

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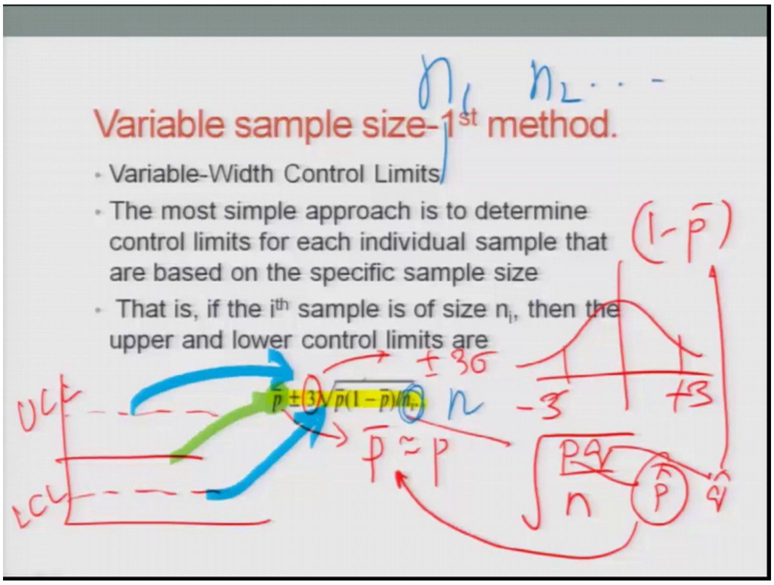

Till now we have considered sample size to be fixed; like n and number of samples we took was m; m as in mango or mangoes. So, depending on that we found out what was the average or the double average x bar bar or R bar whatever it was. So, in the variable sample size in the first method is variable with control limits. So, you would have some control limits for the variability and variability is there for the sample size.

The most simple approach is to determine control limits for each individual sample that are based on specific sample size. So n is the sample size we will keep taking n observations set of observation each time and do our studies accordingly. In case if it is not like where the number of such sample space if the product which you are producing is very less in number; like you are producing some very unique component for a space craft which your company is manufacturing or you are producing some electronic gadget for the fighter aircrafts or say for example, for a some tanks which will be only produced only once.

Obviously, the number of samples which will get for those products which will be used as a component in those machines which will be a part of either the tank or the space craft or the fighter planes a limited it is an; obviously, you will take the study or take the samples in such a way that the variability of sample size would definitely come or say for example, you have a production process where products are being produced that is true, but you have to do some destructive sampling and that becomes very costly for you.

So; obviously, you like to reduce the sample size because the observation which you take need to be destroyed in order to basically study them. So, in that case sample size would also vary. So, the most simple approach as I said is to determine the control limits for each individual sample that are based on specified sample size.

Now if the ith sample is of size ni. So, n suffix i is basically for the sample which you are taking at what time depending on that stage your sample size will change. Like say for example, 9'o clock in the morning you take the first sample and the sample size is 20. Then again 10'o clock in the same day morning, you take a sample size which is now end to, but say for example, the sample size is 19. Again at 11'o clock in the morning you take a third sample which is of size n 3; which is the third sample you are taking and the size is say or example 10.

So; obviously, that will change the sample size based on which you will try to recalculate. So, let me highlight the formula first. So, this p hat bar which you have is the best estimate for the proportions. Proportion means how many some number of defects and so on and so forth. They are there these parameter 3; which is coming for plus minus 3 sigma. Now, for the proportions, for the binomial distribution we knew this would be the standard error or the best estimate for this population standard deviation.

Now, in case p is not known you replace p with p hat; q with q hat and this p hat is basically p bar and this q hat is basically 1 minus p bar. Then if you have the average value central line given by; I will change the color is this one, this will be for this one with the minus sign. Now, if the ith sample size as I was saying that is n i formula anyway is the same only thing which changes is this n i. So, technically it should be n because sample size from reading to reading is same, but if they are changing; obviously, will have n 1 for the first case n 2 for the second case and so on and so forth.

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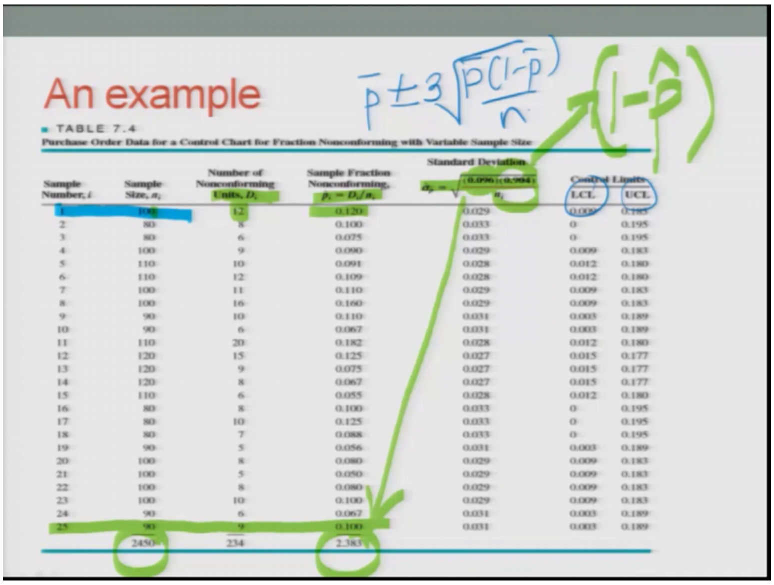

Now, let us study in details with an example again from Montego (Refer Time: 07:19) the first column are the sample numbers which is i 1, 2, 3, 4 till 25 points. So, you are taking 25 such samples now was a fix sample size then the sample size would be fix say for example, a 20. So, the total number of observations would be 25 into 20; now see what you are doing here in this case.

The sample size are changing. So, in the first case let me highlight again it is 100 in number; while in the last case twenty fifth one which is 19 number. So, the totals such additions would be basically the sample size comes, but to be 2450 the number of nonconformity defects are given by d 1; proportions are given by 12 by 100 which comes out to be 0.12. Similarly for the last one 0.18 becomes 9 by 90; from there you have the total set of of proportions defect which is the so, called would be the average values which can find it out.

So, total 2.3; 2.283 would be the proportions which is d i by n i and; obviously, you have to find the average of that. Now from this standard deviations you find out the values which is sigma hat is basically the variance of the standard error. So, standard error square or variance; so, you want to find out the standard error or standard deviation you need to find out square root of the variance. Now if you are thinking from where this value is coming; so this would basically be calculated from here. So, you have 2.8383 as the p hat sum sum of the p hats for all; i going from one to 25; you add them up and divide by 25; which is the number of readings then q would basically be given by 1 minus p hat and then you; obviously, you go into the formula. So, I should remove the highlighter go to the sketch; use the sketch pen.

So, basically it would be p plus p hat or p bar plus would be for the upper control and minus would be for the lower control. This 3 value would come because you want that level of confidence and this would be p bar into 1 minus p bar by n and you will do the calculations accordingly ni's sorry ni's now here is the calculation.

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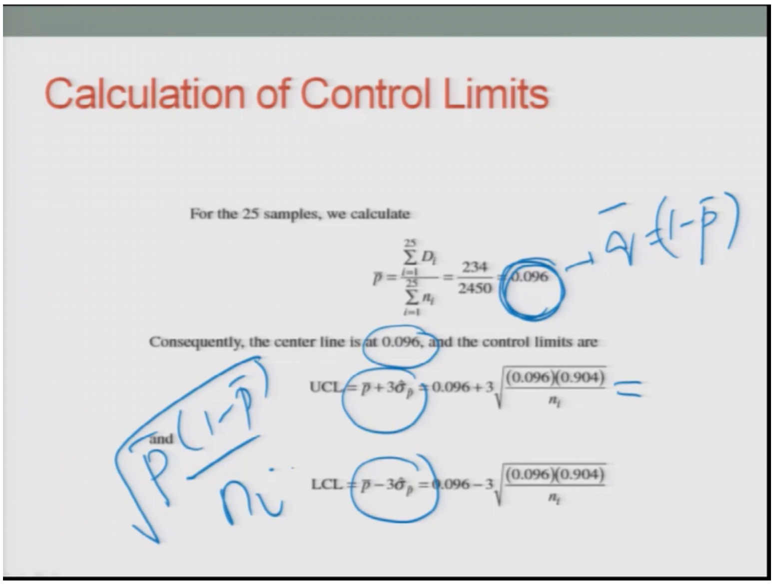

For the 25 sample size; we calculated the total sum of number of defects is 234; total number of observation which you are taking is 2450; hence this value comes out to be 0.096; the value of q bar would be 1 minus this you can find out.

So, here you can find out the central line comes out to be 0.096; the upper control will be 3 plus sigma that comes out to be the value as given here; you can calculate it the lower control limit would be the p bar minus three sigma hat which is the estimated value this sigma hat you know again I am repeating and you can find it out accordingly.

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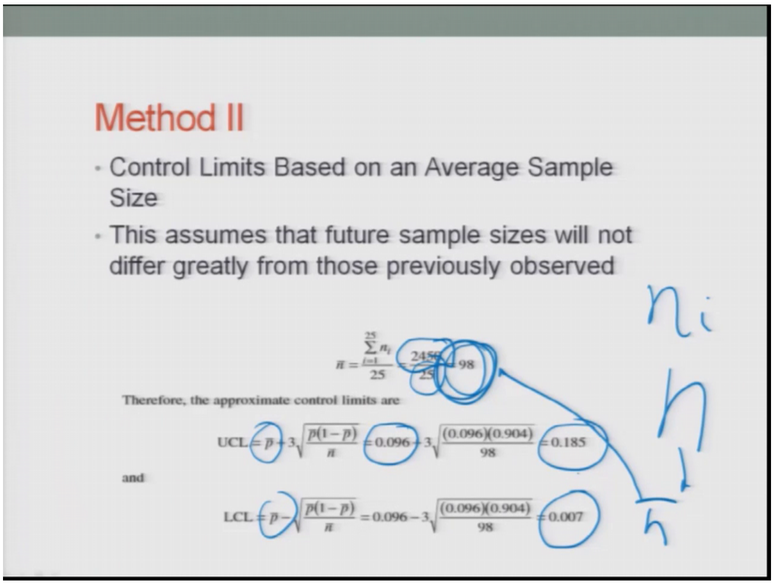

Now, you want to find out the control limit based on average samples size. So, sample size is changing and you want to basically use the average sample size this assumes that future sample size will not differ greatly from those previously observe. So, there would be difference, but the difference is not large.

So, what you need to do is that if you check here you had taken 2450 total number of observations how many such times you are taken the observations was 25 in number. Because in one time you took 100; another time you took 90, sometime you took something else. So, there are 25 such. So, on an average you may have taken at each (Refer Time: 11:39) 98; observations based on that you again.

But now in that case ni's do not come actually n would come where n is n bar which is what you have calculated which is 98; based on that you find out p bar or p hat comes out to be 0.096 and the upper control and the lower control values are 0.185 and 0.007.

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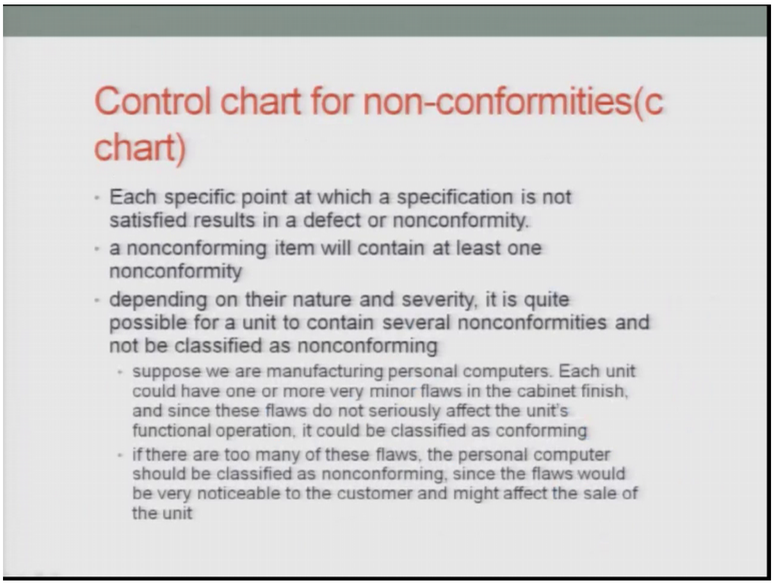

Control chart for nonconformities or c charts. So, now, initially if you remember we have considered the x bar as for the average. For the dispersion, we first took the range and then basically replaced it with either the standard error or the standard deviation. Then we took the concept of n into p charts and all these things now we will try to use the concept on nonconformities, which is also in some way the concept of proportions.

So, each specific point at which a specification is not satisfied results in a defect or nonconformity. So; obviously, it would be good or bad or black and white whatever the outcomes or there would be two outcomes and one has is as per your satisfaction, other is basically not nonconforming; which would be otherwise. Nonconforming item will continue at least one nonconformity. So, say for example, you are checking three types of defects.

So, consider them to be length as some product is being manufactured you want to find out the length, the breadth and say for example, the color, quality or of whatever the item is. So, if one of them is not according to the standard; obviously, you will not consider them to be conforming to your specifications, so it would be a nonconforming product.

Depending on the nature and severity; it is quite possibly for unit to contain several non conformities. So, it may be possible that the first object which you took; the example which I give few seconds back length, breadth and color and color type where like the you consider this; you are a producing some types of high quality tables or chairs and the table has basically the very specific specifications; which will be use say for example, in a in a high star nine; 7 star hotel and; obviously, the hotel owners want the specific coloration to be exactly matching to one color sheet.

And you as a company are producing those tables which have to basically fit it in either in their high class restaurant of this hotel chain or wherever it is and you basically checked the length, breadth because the dimensions has to be specified as given by the hotel owners of the company which is ordering those tables and the color conformity also has to be there.

So, it can be either the table if it is confirmed as per the standard so; obviously, length, breadth and color would match. In case if it is not it can be that one of the non confirmative is there; which may be either the length or the breadth or the color or it can be more than one also. So, this is what is mentioned; a nonconforming item will continue at least one nonconformity. Depending on the nature and severity is quite possible for a unit to contain several nonconformities and not be classified as a nonconforming once.

So, consider these suppose we are manufacturing personal computers; each unit could have one or more very minor flaws in the cabinet which you try to basically finish and make it and see these flaws do not seriously affect the units functioning, it could be classified as confirming. So, the example which is being given is that they would be many different type of nonconformity.

So, now, you have to basically decide on the severity of the problem that what are the set of nonconforming errors and what are the set of confirming errors. Errors means that even if say for example, the error is within a particular non conformity norms; still you will basically consider it to be applicable and you can pass it. Consider these example you are measuring say for example, a very simple lawn mower and the lawn mower would definitely have some amount of error either in the shape and size of the grass cutter or the shape and size of the handle.

So, any plus minus centimeters or millimeters in change in those specifications are allowed and you definitely you will consider them that even if they are not exactly meeting the standards; you will consider them to be under the purview of conformity and you will pass it. But, if such problems or such issues comes out to for a very high standard product consider like this a very high valued product or component which will use in the airline industry for the aero plane to function; or say for example, the type of pace maker which you will supply to the heart patient. So, in those cases not a single error would be there and; obviously, the non conformity in the long run should definitely be 0.

So, continuing in this; if there are too many of this flaws the personal computer should be classified as nonconforming since the flaws would be very noticeable to the customer and might affect the sale of the engine. So, may be the number of non conformities are very less and those can be passed, but if the number of non conformities or the type of nonconformities are very large or they are very serious; obviously, there is a problem and you have to basically take corrective actions; later on to rectify that and; obviously, those consideration should be studied when you are doing the non conformity or the conformity calculations and try to draw the charts.

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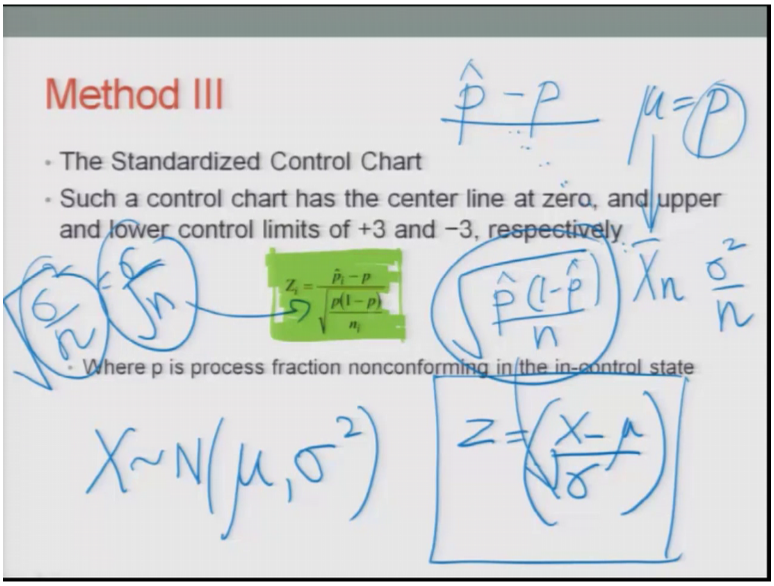

So, as I did mention there may be many errors less; errors for the computers. The standard control charts which will consider is that such a control charts; if whatever the information is we will first try to convert in to a standard normal table and use the standard normal distribution.

Now, coming back to the standard normal distribution and the concept which is there; generally we know this is true where x is a distribution which is normal with certain mean which is this mu and variance is sigma square. Now when you want to convert into a standard normal; obviously, use this transformation which I just put in a box.

Now for the case of this binomial distribution, where there is either good or bad; black and white yes or no whatever. What you want to actually understand is that your actual mu value is basically the proportion which is p. Now in case you want to basically utilize the value of mu from the sample; obviously, the sample estimate would come up which is x bar n considering as the normal distribution; that is the best estimate um vu e; which is uniform minimum variance unbiased estimator.

Now, of that case the; obviously, the variance would be see for example, in the normal case would be sigma square by n; which is n is the sample size. So, if we use this concept; obviously, it will become now the best estimate for p would basically be p hat. So, which is one to one correspondence to x bar; now that what we have (Refer Time: 19:35).

Basically that the estimate and we find out the difference between the estimate and the parameters. So, this p value is basically mu here; now in the denominator if you see u v we divide it say for example, with the value of n which is the sample size. Now if I consider the idea of trying to find out say for example the sample variance which is the best estimate for the population 1; obviously, you had sigma square by n.

Now, in this case sigma square by n would be these whole terms would be replaced; obviously, you have to find out the square root of that would be would be sigma by square root n. So, this becomes technically p; obviously, in that case p hat would be there; p hat into 1 minus p hat; which is q divide by n whole square; this is the formula for the binomial case. So, this comes as replication of dividing it by sigma; which is your trying to basically transform and normalize it.

So, once you normalize it; you have the formula which I am trying to basically put as a green color. So, this would be the standard normal deviate; now what you will do is that u will try to use the standard normal deviate tables and utilize this. So, based on that you will draw the diagram; find out the values and basically conclude accordingly. So, in that case the central line which will try to draw would be the p hat or the actual value; if it is possible for you to known would be the corresponding deviations plus and minus whatever coefficients multiplied by sigma would be sigma value is that what you have in the denominator which is square root of p into 1 minus p by n.

So, if you were want to basically have six sigma limits; obviously, it will be plus minus see if you want to have four sigma limits; again it will have basically plus minus 2. So, continuing the discussion here p means the process fraction nonconforming and in the control state. So, based on that you will try to find out that what are the values of the mean and the values of the standard division based on which you will do the calculation and before that you will convert that into a standard normal distribution of the values.

So, for the problem which we does discussed; so, some of the sample fractions nonconforming are given by p hat; which is the best estimate for p as I told and the sample numbers are given along the X axis; 1, 2, 3, 4 so on and so forth; till the twenty fifth one.

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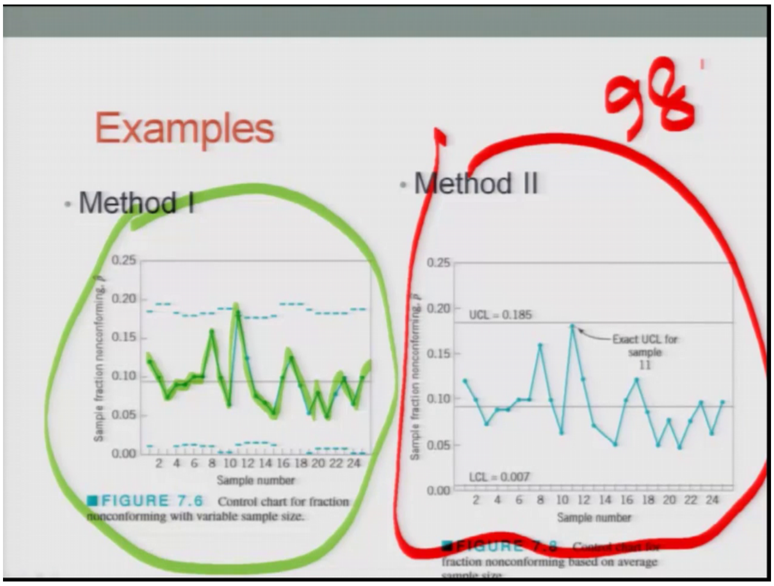

And once you when you basically charted the values are given as the diagram here. So, I will try to utilize the green one; so, where I am highlighting, this is basically the chart which gives you the control charts for the fraction non conforming with variable sample size; and when you try to draw the control charts from fraction nonconforming based on the average sample size; if you remember the average sample size was 2450 divided by 25 while in the variable sample size I need and the average sample size came out to be 98.

Now, if I am trying to utilize the variable sample size the diagram is shown which in figure to 7.6 which I am just circled in green color and if I use the concept of say for example, fix sample size which is in method two; where the sample size was basically 98; the upper control limit, the lower control limit, then central lines are drawn based on that I get the table which is given in figure 7.8 and here for all this cases; the actual sample size remember that would be fixed; it is not varying much, but in the initial diagram which is given in figure 8.6; they are varying because the sample size in each greeting is waiting so; obviously, from there you can draw meaningful conclusions based on which how they are performing.

Now, remember one thing; the which I think everybody would have understand, but I will still highlight it and I will try to highlight it using say for example, the violet color; so see if you see many of the where the stylus is now of focusing this one; this small which are drawn these are the changing limits which you have for the upper control limit and lower control limit before.

If you remember ni’s are changing; if ni’s are changing the ps hat which is the p double bar or the p single bar or whatever the proportions you have; that is central line is fixed, but the upper control limit and the lower control limit; whatever change which you have is plus minus the square root of p into 1 minus p divided by n. So, that n is changing from reading to reading from sample 1, sample 2, sample 3. sample 4; it is changing as sit is changing the upper control values are also changes.

So, that is coming out here; so, I will try to basically remove the highlight in order to basically make it clear. So, this is what is; so, if you consider where I am basically now erasing you see small-small horizontal blue lines. So, these are the upper control values for each and every single change of the sample size. And similarly when I go to the upper lower control limit; again these values are shown by small, small a horizontal line which basically signifies the lower control limit for different sample sizes.

In method three which you use as we discussed if we use the standard control charge for fraction and conforming and based on that if we draw it; fraction conforming would also be somewhat similar to the p charts. Here the upper control limit and the lower control limit would be plus and minus 3. Why they are plus and minus 3; let me make it very.

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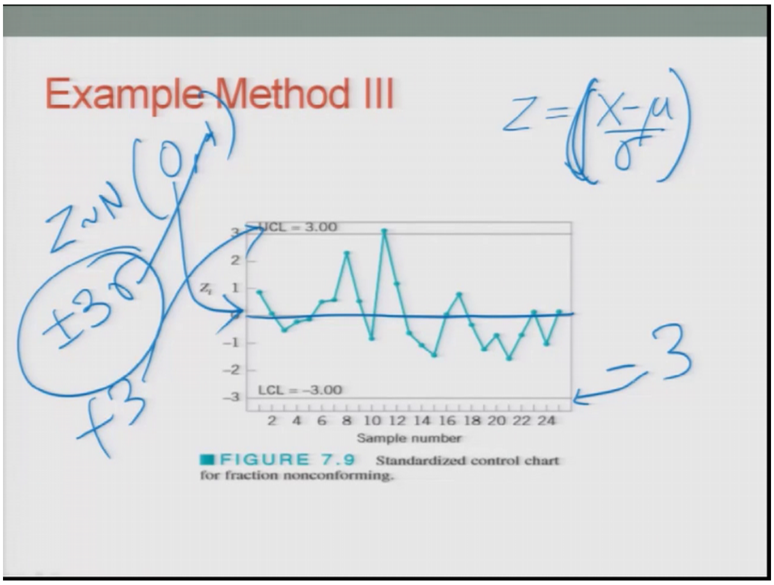

Now, you have already converted Z using the x into Z using the standard normal deviate.

Now, the interesting issue is Z has a distribution normal with 0 mean and 1 standard deviation and 1 error. So, if you with this information go back to this table which is shown; which is figure say or that figure which is shown is 7.9; the central line is 0 because this is the mean value as already we know. Now, if I consider plus minus 3 sigma; the sigma value is already 1; hence the upper control limit would be plus 3 as shown here and the lower control limit will be minus 3 as shown here.

So, any calculations further to try to highlight why those values are centerline, upper control, lower control would basically be rudimentary because we already know we had converted that distribution using the standard normal concept to a standard normal deviate and drawn the charts accordingly.

Thank you. And, I will end this 29th lecture and continue with the 30th, and the further on for the TQM 1. Have a nice day.