Total Quality Management - I
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Lecture - 27
Attribute charts - The p chart

Good morning, good afternoon, good evening, my dear friends. I am Raghunandan Sengupta from the IME department, IIT, Kanpur. And this is the total quality management one course under the NPTEL MOOC program. And we are continuing our discussion about the moving range charts corresponding to the fact that whether normality holds or normality does not hold, and this is the 27th lecture depending on the discussion which we have just finished in the 26th lecture.

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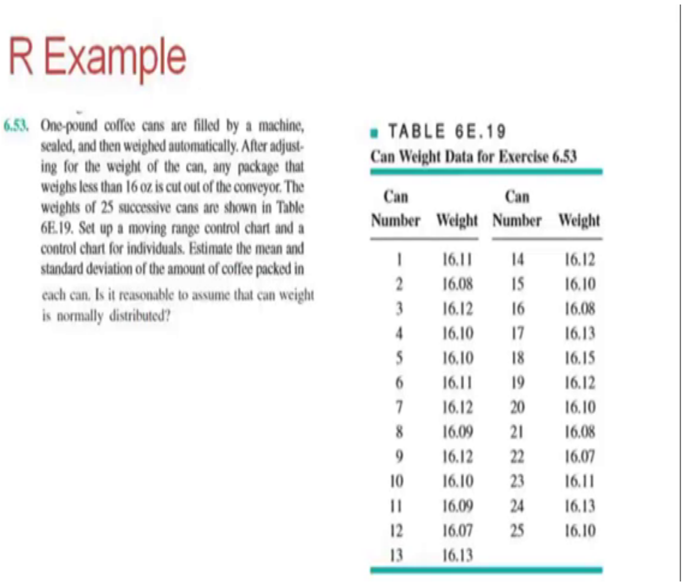
R Example

6.53. One-pound coffee cans are filled by a machine, sealed, and then weighed automatically. After adjusting for the weight of the can, any package that weighs less than 16 oz is cut out of the conveyor. The weights of 25 successive cans are shown in Table 6E.19. Set up a moving range control chart and a control chart for individuals. Estimate the mean and standard deviation of the amount of coffee packed in each can. Is it reasonable to assume that can weight is normally distributed?

TABLE 6E.19
Can Weight Data for Exercise 6.53

Can Number	Weight	Can Number	Weight
1	16.11	14	16.12
2	16.08	15	16.10
3	16.12	16	16.08
4	16.10	17	16.13
5	16.10	18	16.15
6	16.11	19	16.12
7	16.12	20	16.10
8	16.09	21	16.08
9	16.12	22	16.07
10	16.10	23	16.11
11	16.09	24	16.13
12	16.07	25	16.10
13	16.13		

So, consider an example for this discussion, which you already had. So, one pound coffee cans are filled by a machine. So, they are process not a production process. They are sealed and the end when automatically. So, production process I do not mean like manufacturing a tie rod or trying to basic manufacturing electrically equipment, but they are just filling up bottles. After adjusting for the weight of the can, any package that weighs less than 60 ounce is cut out of the container and removed. The weights of 25 successive cans are shown and they are shown in the figures are shown in 6E.19 which is the table on the right hand side when I am pointing my right hand. We have to set up a moving range control charts and a control charts for the individuals estimate is required for the mean and standard deviation of the amount of coffee, which is packed in.

So, we want to basically give our opinion whether is reasonable to assume that the can weight normally distributed. So, the can numbers are given in the first column in the third column starting from 1 to 25. The weights are given from the first can is 16.11 till the last one which is 16.01 so obviously, there are different type of variations also.

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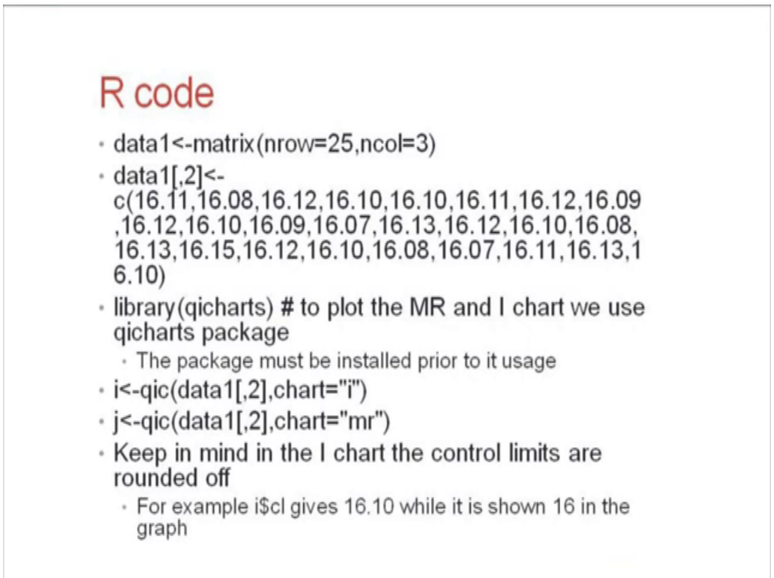

So, we basically write on the R code. You basically say between an matrix the matrix is basically of size 25 cross 3. So, it will basically with the serial number the can and the weights accordingly. So, the data if you call basically the weights are given staring form 16.11 to 16.01 and you want to basically plot the moving range charts. So, the moving range charts would be basically calculated by finding of the mod of the difference. So, if I concentrate on the first two readings, it will be 16.11 minus 16.008, so that would be mod and for the last value; obviously, it will be 16.13 minus 16.10 again you try to find out the mod of the weights.

The charts from when we draw, obviously, you will try to find out the average the upper control lower control and corresponding to that and then based on that you have to basically give our judgment whether it is normal or not. So, we should keep in mind in the chart the control limits are rounded off according. So, they can be either put to two places of decimal, three places of decimal, but we are trying to round off two basically give a fast result and do the calculation, but that would not matter about conceptually would not matter whatever we are giving.

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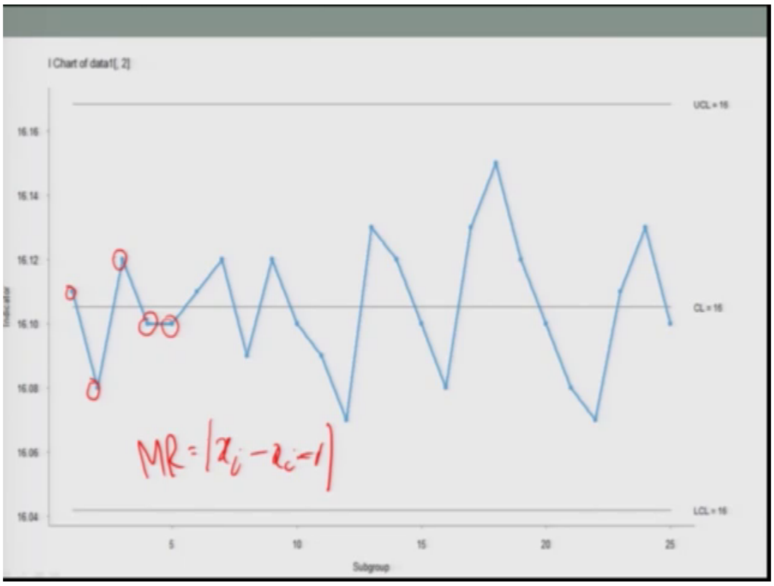

So, if you basically give the values, so your upper control lower control and the central line all are given as 16, but they are not 16, obviously, in order to make it understand in the graph I have given the values of central values as sixteen point something upper control sixteen point something local control sixteen point something. If you find out the moving ranges, so this value which you have the values which you have here or here or here or here or here are basically the difference of, so this is a x minus 1 mod of that. So, the moving range is given by this mod of this and we do the calculations according. So, that is why it is not negative. So, once you do the calculation, then you do the data concept for the finding out the range, difference of the range and do the calculations.

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So, the ranges are given in this way the central line is 0.024 based on the calculation of the central line the upper control values is given by 0.078 and the lower control value is given by 0. So, obviously, you have the overall charts for each and every moving range values are like this. So, you know to basically need to comment about intelligently about the normality.

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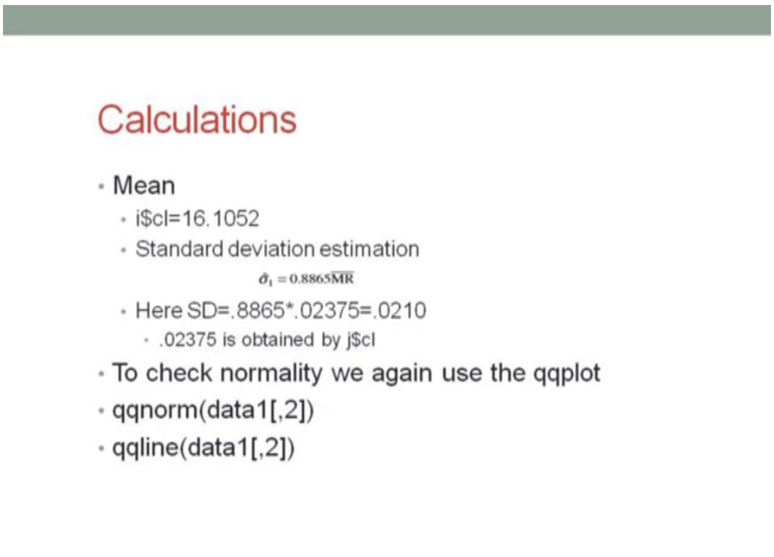

So, the mean value is about 16.052 which if you remember I did not mention; obviously, I know that more a explicit drawing of the diagram would definitely be much more exciting. But I want to give you the feel that how you can do that using the actual information which is there in either in the book or in the actual processes, which can be available. And then obviously, you will try to utilize some of the concepts of R, if you remember I did mention that we are trying to utilize R in our work.

So, here the value comes out to be 16.052 the standard deviation comes out to be say for example, 0.8865 multiplied by the mean value of the range. So, mean value of the ranges are basically the mod of first reading would be mod of the difference of x 2 minus x 1 mod of that. Second would be x 3 minus x 2 mod of that, add up all these values divide by the number of readings, obviously that will give you the average which is if you look in this in this chart it is about 0.02375. So, multiplying that with the value of 0.8865, it comes out to be about 0.0210. So, once we do that the plotting of the graph you will basically find out the upper control lower control.

Now, if you remember where we are closed the discussion in the last lecture it was mentioned very expressively that it you need to do basically the finding out the Q-Q plots or some do some plots corresponding to the distribution and to for to find out whether is normal. So, one of the very good plots where you can find out where the distribution what type of distribution is basically the Q-Q plot which is the quantile-quantile plot. So, I will come to that later on with discussions.

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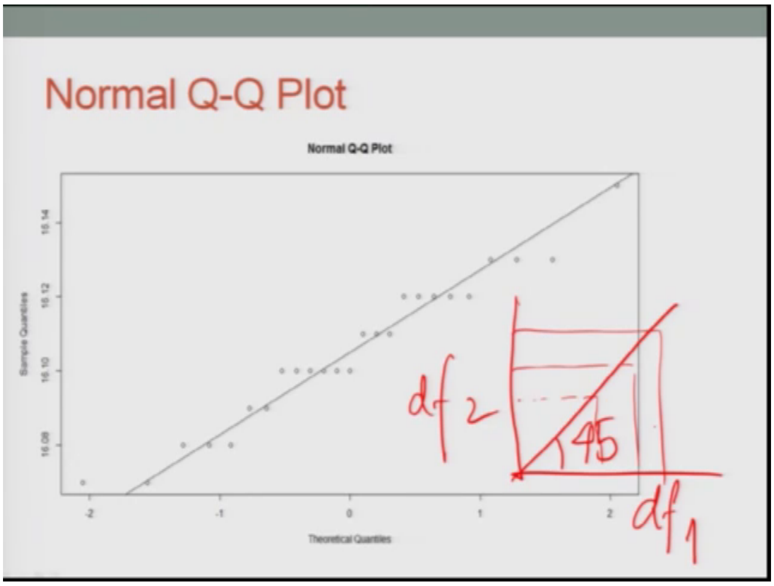

If you do the quantile-quantile plots, the quantile-quantile plots for the for that for the information set or based on the range values and the mean values for filling up the coffees are given in the diagram which is there in front of you in the slide. Now, here I will pause to give you concept of what we mean by quantile-quantile plot. So, I will try to basically explain it using the words and I will repeat slowly. So, you will understand.

Now, consider there are two distributions; and this quantile-quantile plot is very intuitive method by which you can understand what type of distributions actually is and you can make some actual very nice intelligent and very simple judgment based on the information which you get from the quantile-quantile plot. Quantile-quantile plot is basically to do with two different distributions, what are the distributions I will come to that later, based on the overall coverage of the probabilities of the quantiles. So, if I cover 10 percent, so corresponding to the 10 percentage coverage of the total set of values which is there for the overall observations I should have covered some overall probability. So, obviously, remember the probability for the overall coverage is 1, that means when I draw the CDF value the overall coverage of the area under the cumulative distribution function or the distribution function is 1.

Now, consider there are two different distributions. For the time being consider both of them are normal. And the number of readings are different and they are being taken from two different normal distributions. So, what I do is that I basically take the first normal distribution, rank it from the minimum to the maximum, do the same thing for the second distribution which is also normal from minimum to maximum. Now, what I do is that that I cover a certain quantile range for the first case and certain quantile range or some number of observation for the second case. Now, as it is normal, the overall coverage which is happening at a certain rate for the quantiles would be of equal values technically in theoretically run.

So, if I basically covered half of the set of values for the first and the second, the overall coverage of the probability would be 0.5, 0.5 in the both the cases. If I am been able to consider say for example, 25 of the area then; obviously, the number of readings which are there for the first distribution and the second distribution considering both are normal would also be the same. Now, if we intuitively plot it on an x y scale where along the x direction I am plotting say for example, normal distribution one whatever the mean whatever the stand deviation is and along say for example, the y distribution I basically draw the second normal distribution whatever the mean distribution whatever the variance is that is immaterial.

So, if I am basically able to fit a 45 degrees line in those y and x axis, it would mean the quantile-quantile plots or the range of coverage range not the overall area range of coverage of both the set of values which are normal would be such that the set of points would always be along this 45 degrees line. Now, think intuitively if I am trying to basically draw, so the sorry this is not the way I should be basically do. This 45 degrees this is the quantile for distribution function one this is distribution function two. So, if I a I am on this line always on this 45 degrees line, it means that the overall coverage which is happening both for the distribution are proceeding at the equal rate and their covering the equal area which means that underlying the distributions are same.

Now, along the fact if I replace one of the distribution as normal, and another distribution whatever data which I am getting and if both the quantile-quantile for the normal distribution which I know and for the other distribution which I do not know match. Which means that I can convincingly say that the actual distribution, which I am taking and trying to fit and trying to plot against the normal distribution is also normal. So, now, obviously, you will ask that why normal distribution cannot it be any other distribution along the x-axis, the answer is yes.

Say for example, I am taking the technically I am taking the exponential distribution theoretical exponential distribution along the x-axis I am trying to basically take another distribution from nature or from some reading and trying to plot it plot the distribution function of that unknown distribution along the y-axis. Now, if the quantile-quantile plots for both of them happen along the 45 degrees line then obviously, we are much more convince that the actual distribution based on which we are trying to study and trying to compare with the known exponential distribution is also exponential whatever its mean and variance be.

Now, when I am trying to do that study for different above distribution the main of focus basically becomes that let me try to understand the Q-Q plots for the quantile-quantile plots for the unknown distribution based on the fact that the known distribution which I am trying to plot along the axis is normal. So, if you basically find out that the normal quantile-quantile plots is as shown in this slide in front of you, you will just note down what are the x and y-axis. The x-axis basically gives you theoretical quantiles, which I am trying to cover for a theoretical distribution; and the y-axis basically gives me the sample quantile based on the readings which I am just taking. So, if the fit is 45 degrees line it means the reading from where I am trying to plot is basically are type of distribution, which exactly matches theoretical distribution. And in the theoretical distribution is normal then the sample observation which I am trying to take and trying to study is also normal, whatever the mean whatever the standard divisions be.

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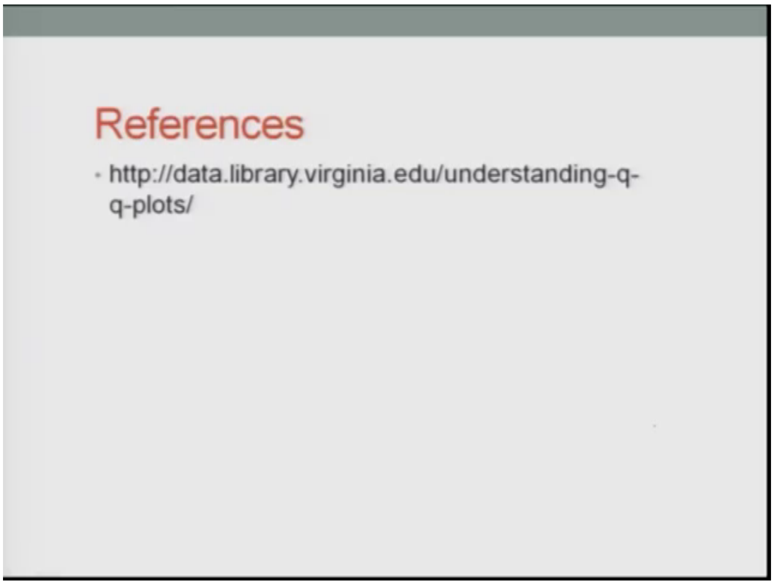

So, few of the references are you can basically understand the Q-Q plots from as I mentioned the before the starting of this course, there are very good data sets either in in (Refer Time: 13:44) university of Florida. So, one such very good source apart from these two universities, Virginia University; and basically you can have a look at this information set for the library functions or the data from the library can be utilized for trying to understand the Q-Q process.

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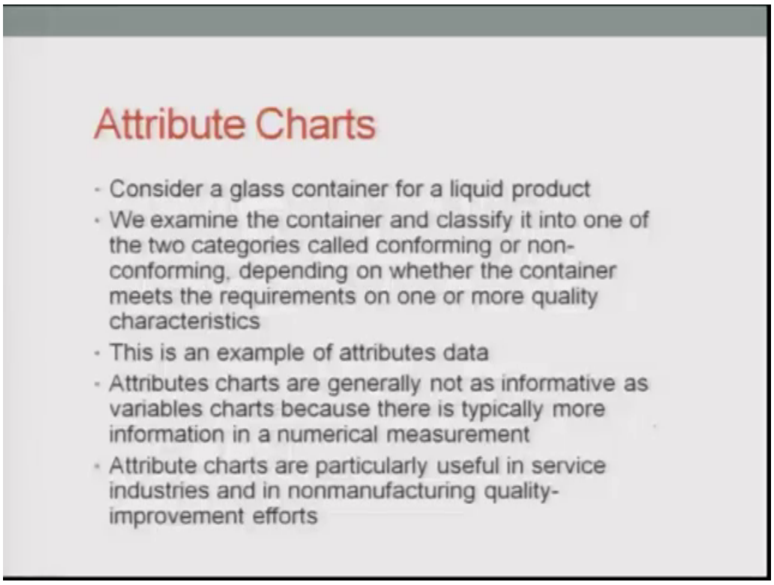

Now, having a discussed at length for a about two three lectures or more than two three lectures about the variability charge based on the random variables, we will try to basically discuss may be to that extent the concept of attribute charts. So, consider that attributes are the characteristics, which I did mention. So, consider a glass container for a liquid product, we examine the container and classify it into one of the two categories which basically I considering confirming and non confirming based on the attributes or characteristics.

So, which depends on whether the container meets the requirements whatever the requirements may be color, may be shape, may be the crushing power. Like say for example, I want to manufacturing some cans, and the cans after you usage should be crushed very easily such that they cannot be reused so obviously, my main focus would be from the point if the attributes would be how easily they are crushed. So, depending on and let me continue that depending on the whether the container meets requirement on one or more quality or quality characteristics, so this is an example of the attribute data.

Attribute charts are generally not as informative as variable charts because there is typically more information in the numerical values. So, numerical values gives you actual and understanding that how you are trying to measure it, and this readings would give you an actual feel that how you are going to analyze. But characteristics would be a little bit subjective, I am not saying that your losing as any information you have to basically do a lot more understanding of the attribute characteristics.

So, there would be subjectives and based on the subjectivity you have to basically do your analysis accordingly. So, attribute charts are generally not as informative as variable charts because there is a typically more information is a in a numerical measurement. Attribute charts are particular useful in service industries and in nonmanufacturing quality improvement efforts, where they are important, they would be utilized.

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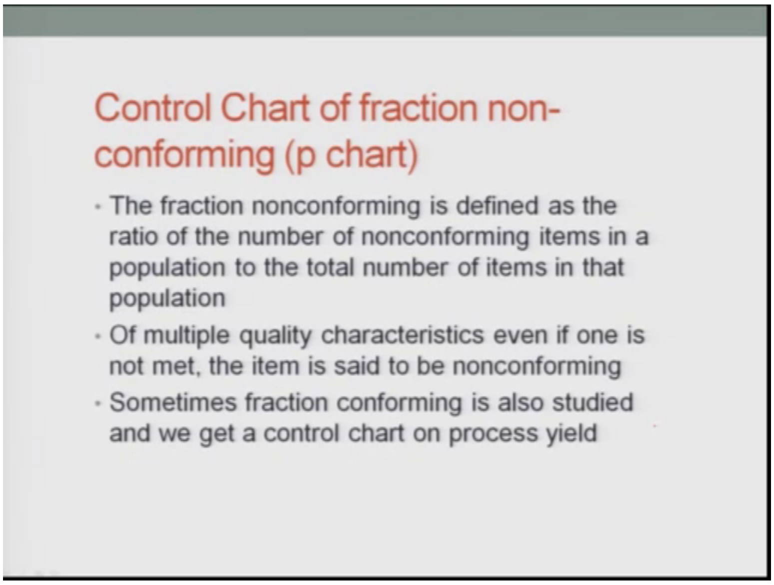

Control charge of fraction and confirming which is the p charts. So, there I will consider the conformity and nonconformity for the attributes which are trying to study. The fraction nonconforming is defined as the ratio of the number one nonconforming items in a population to the total number of items in that population. So, you want to find out the fraction or the chance or the relative frequency which in the long run is basically the probability, more you do more it basically tends to assign value which is the probability like tossing a coin I find out the head and tail for 100. So, in one heads in 45, so the actual relative frequency is 0.45.

Again I do it I find out it is 47, so relative frequency is 47, 0.47. Then again I do it I find out the numbers out of a 100 C say for example, 55 the reality frequency is 0.55. So, you keep it doing it, the average basically in the long run it basically becomes exactly equal to 0.5.

So, continuing the discussing the fraction and nonconforming is defined as the ratio of the number of nonconforming items in a population to the total number of atoms in that population. Of multiple quality characteristics even if one is not met, the item is said to be nonconforming and obviously, we take actions accordingly. Sometimes fraction conforming is also studied and we get a control charts on the process yield and then we can pass some from intelligent judgment about that.

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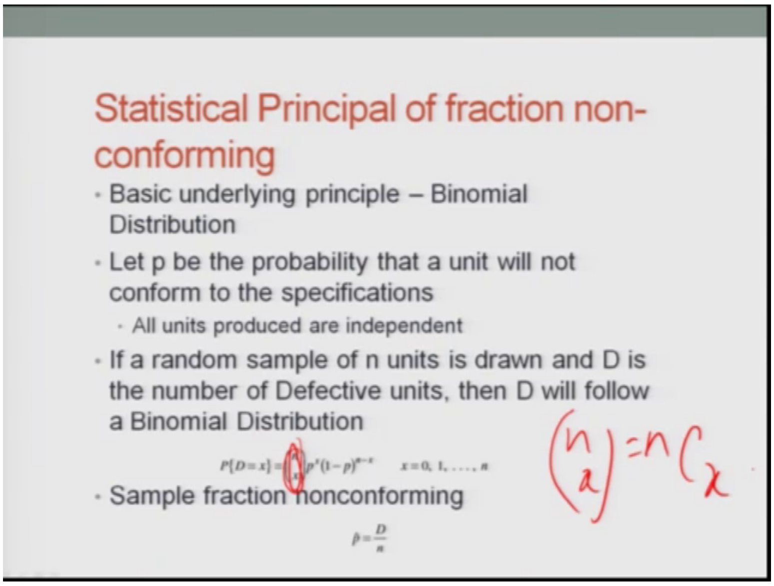

So, statistics and principle are fraction conforming would be basic underlying distribution binomial. Why binomial that would be either yes, no, good, bad. So, basically we are passing judgment based on the outcome. So, let p be the probability that the unit will be will confirm the characteristics or 1 minus p would be the probability that the unit does not meet the characteristics. It can be extended to the multivariate case also like it can be a probabilities of p 1, p 2 and another third would be 1 minus p 2, p 1 plus p 2, so obviously it would be multinomial distributions accordingly.

So, let p let p reread it again let p be the probability that a unit will not confirm to this specifications and all units produce are independent of each other. So, obviously, it will mean that the binomial trail characteristics would hold true, which means that each trial in this case for a binomial case would they would be two outcomes with probabilities p and q, where the sum of p plus q is 1 - point number one which I mentioned. Point number two would be the corresponding probabilities of p and q does not change from trail to trial point two. And point number three is that the probabilities are independent on each other.

So, if I am tossing a coin, the probability of getting ahead for that particular coin, if it is a unbiased coin remains as 50 percent throughout the experiment. And the probability of getting a head would not affect what is the probability of getting a head in the next trail or getting in the trail in the next trail. So, it basically it is independent from trail to trail and this would be the concept of binomial trails.

So, again reading it all units produced are independent. If a random sample of n units is drawn and D is the number of effective items then D would basically follow a binomial distribution with the corresponding the PMF because PMF is probability mass function would be n C x. So, this value which I am seeing here in front of me which is n x is technically is equal to n C x the number of combinations of taking x from n number of observations. P to the power x 1 minus p to the power n minus x so obviously, you have n number of chairs to be filled up, x are of one type n minus x are of other types. So, the overall combination becomes n C x.

And now for each filling up of one chair of type p or probability p, obviously how many such ps are there it will be x. So, the hence it is p to the power x and the rest of the ps are filled by the opposite type which is one minus p which is q and that would be raise to the power n minus because those are the number of places it can feel. So, if I want to find out the power fraction on the nonconforming which is actually p in the long run that should be basically D by total number of sample size. So, morer number of observation I pick up n increases d in the long run would be such that the ratio of n by D, if D in the infinite case when I am taking actually the overall population should be exactly equal to p.

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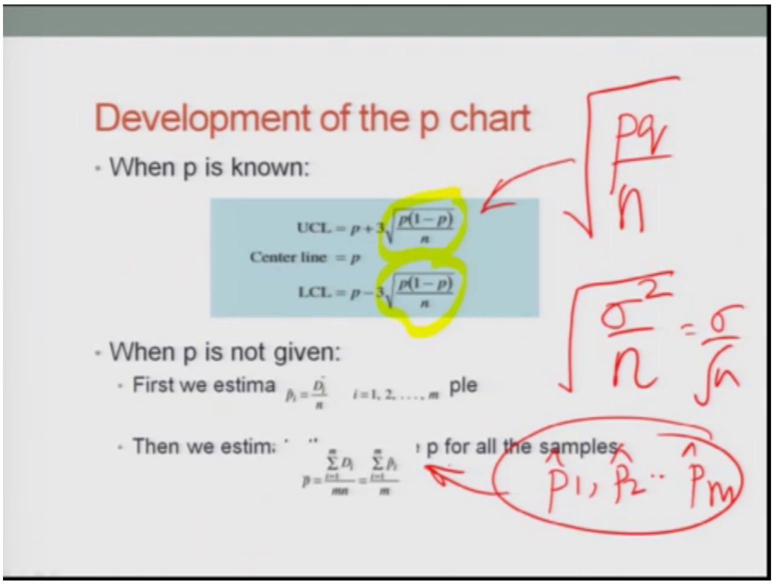

So, when p is known the proportions of defects or non defects are known the upper control limit and the lower control limits are given by upper control limit is given by p plus this three value is again coming from the overall variability of plus minus three sigma. So, this is three in the upper control limit, three in the lower control limit are those facts are taken in to consideration that I am trying to find out plus minus 3 sigma accordingly.

Now, what is important to note in this calculation is this. Now, if we remember for the binomial distribution, the mean and the variances would be calculated in a such a way that if I try to find out for the corresponding sample, the sample, wait let it be append the sample corresponding variance would be given by p q by n square. So, if I draw simply with the normal case what I had was basically sigma square by n. So, this would be wait let me try it again because, so when I am trying to find out the standard deviation. So, it would be square root of that which is sigma by square root n. When I want to find on the standard deviation, it becomes this which is p into q which is p into 1 minus p divide by n, so that you would be basically be the overall dispersion quantum, so per unit quantum of dispersion onto the left to right. Now, what is the coefficient which you are going to multiply with that dispersion quantum would basically depend on what is your a level of confidence which you want to have.

So, again coming back these three value or two value or one value would depend that whether you are trying to find out some concept relating to plus minus 6 sigma plus minus I am sorry it would be a 6 sigma which is plus minus 3 sigma or it would 4 sigma, which is plus minus 2 sigma or it is 2 sigma which is basically plus minus one sigma. So, based on that the calculation which are shown in front of you would make sense. So, when p is not given what we do is that we saw first try to find out the best estimate of p which is given by p hat which technically would be the ratio of D to the sample size. And once we find out different type of such estimate we need to find out what is the average, so that average of p hat or p bar would be utilized for r calculations correspondingly.

So, as mentioned here the p hat would basically be, so now, consider that you have taken n observations first time. So, the p hat would be p hat 1 that means, for the set of observations. Then I take another set of observation, which is the second one again n number of times then I will find out the p hat for the second set of observations would be p hat 2. If I continue doing it say for example, if I have m number of them then the average would basically be the sum of all the ratios of D by the let me write it down it will be easy for me to explain.

So, basically p hat 1, p hat 2 and so on and so forth p hat m and average of that is basically what is given here which is sum of all the ratios of p hats i. i is equal to 1 to n find out the sum and basically divided by the number of observation which is there which is m. Number of observations corresponding to the number of samples. So, in each group we have n and how much such ns we take which is m. So, then we will basically replace very simply the p’s with the p hats which basically the average values, which you have taken.

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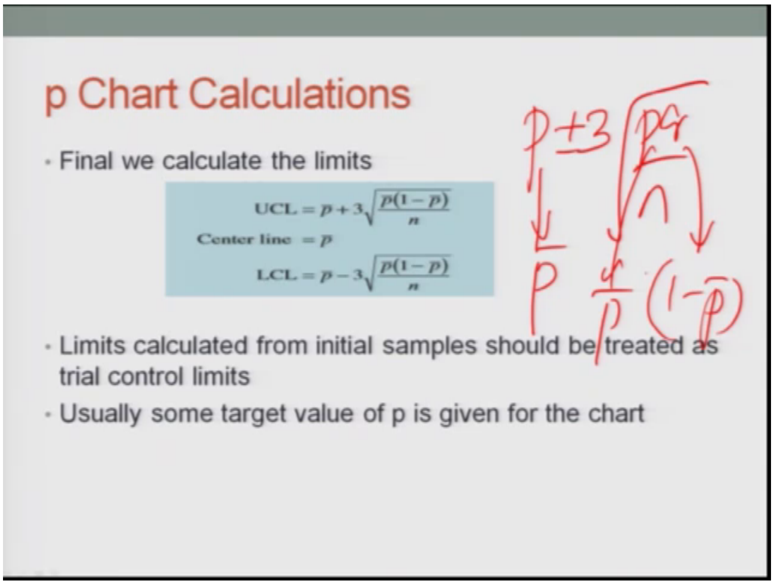

So, initially which you had is p plus minus 3 p into q divide by n square root. So, these p's get replaced by p hat; these qs get replaced by 1 by p hat, these p gets replaced by p hat p hat means p bar or the average of this p hats combined m number of times. So, finally, we calculate the limits of the upper control and the lower control and find out the central line. So, limits calculated from initial sample should be treated as trail control values, and we will utilize them for trying to basically draw the control limits. Usually some target values of p is given from the tables based on that we do our calculations accordingly.

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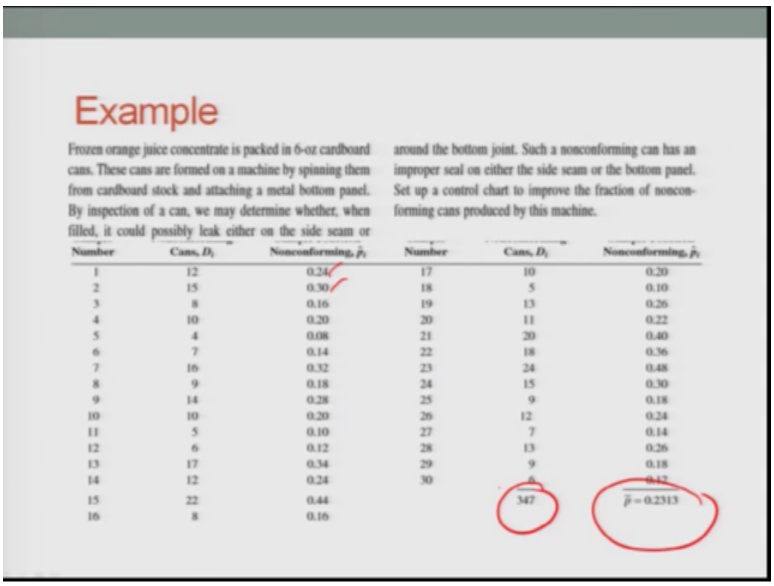

So, let us consider an example. Frozen orange juice concentrates are packed in 6 ounce cardboard cans. These cans are formed on a machine by spinning them from the cardboard stock and attaching a metal panel. So, basically you are spinning them trying to make out from the cardboard and putting the cap accordingly. By inspection of a can, we determine whether when filled is it could possibly leak on the side or the seam and around the bottom.

So, the numbers are given on the first column, the cans how many such cans are done how many defectives are given which is 12, 15, 8 and so and so forth and the non nonconforming ratios are given. So, I find out the ratio 0.24, 0.30 and so on and so forth I find out the p bar value which is from the overall average of 347. So, based on that I have p bar I find out 1 minus p bar I have the n and I do the calculations accordingly to find out what is the upper control, lower control and the central value based on that I draw the charts. I will continue that in the 28 and the other lectures. And, I wish you all the best.

Thank you very much for your attention. Have a nice day.