Total Quality Management - I
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Lecture – 25
S chart and OC curve

Very good morning, good afternoon, good evening my dear friends and students, this is the Total Quality Management-1 course. And I am Raghunandan Sengupta from IME department IIT, Kanpur I am sure you know my name by heart by this time, but still as an introduction I start with that. So, we are as you know considering the study of the charts which are basically x bar and R charts.

And we did discuss in some details that if mu I am just going a little went back to the 24th lecture and something to the 23rd lecture it just a recap we did discuss about the charts only two charts. One - based on the fact that mu and sigma are not known; and in other case we consider the mu and sigma are known. And if they are not known and if they are not known; what are the best estimates we can use from the sample. And also the coefficients which were basically either three or two, it is not exactly two, but or one and they would depend on what is the overall coverage's which we have with this plus minus 1 sigma, plus minus 2 sigma and so on and so forth.

And based on that you will find out that why we had taken those values of A suffix 1, 2, 3 or B suffix 1, 2, 3 or the Ds values which we saw from the tables. That means again I am repeating, the values of sample size you had on the first column and all the rest values were depending on the parameter values of the coefficients depending on the sample size. Later on in the fag end of the 24th lecture, I did spend about 10-15 minutes of trying to basic here we give you some feel that where the patterns can be understood this those I mentioned from the so called theoretical point of view, but obviously, if you do are working on the shop floor you will understand. So, those set of information would be useful to understand that how the patterns cyclicity or stratification not trends would be there which will give you some information of both the x bar and the R charts.

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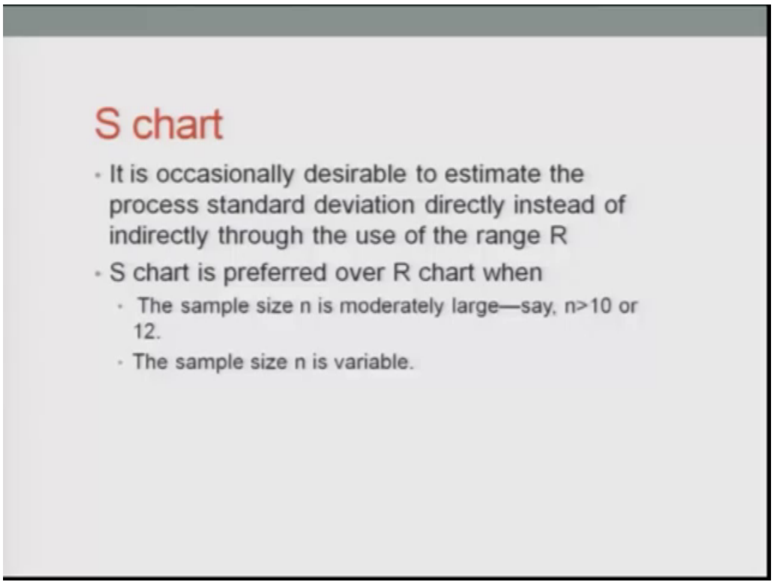

So, now, we will consider the S charts. So, it is occasionally desirable to estimate the process standard deviation directly instead of in or indirectly using the R charts. So, if you remember I did mention that if mu and sigma were not known, then what we did was replace mu with the concept of x double bar; and if sigma were not known we use the range and used the calculations to how you can find out the best estimate of the standard deviation using the range. Later half, we consider mode mu and sigma are know and we did the calculations accordingly. But what is more important to understand is that whether we can use the sample standard deviation of the sample error or standard error from the sample as the best proxy or the best set of information, based on which you can find out the variability of the total population and draw some meaning full conclusions on that, so that is what we are trying to understand now.

So, let me continue reading it is occasionally desirable to estimate the process standard deviation directly if directly instead of indirectly using the range. So, S chart is preferred over R charts, when the sample size is n is moderately large. Say for example, greater than 10 or 12 and the sample size n is variable in the sense that in the first case due to some reason in the early at the 9 ‘o’ clock in the morning I am able to take 7 observations. Then again in the about 1 hour from then which is 10 ‘o’ clock in the morning I am I do not take the same set of observation, but the sample size has increased to say for example 20. So, if it keeps fluctuating then trying to use S would bake make much better sense then trying to use the R. But the actual S as remain the same, I want to understand the variability of the overall production process whether I use R or S would basically be the question.

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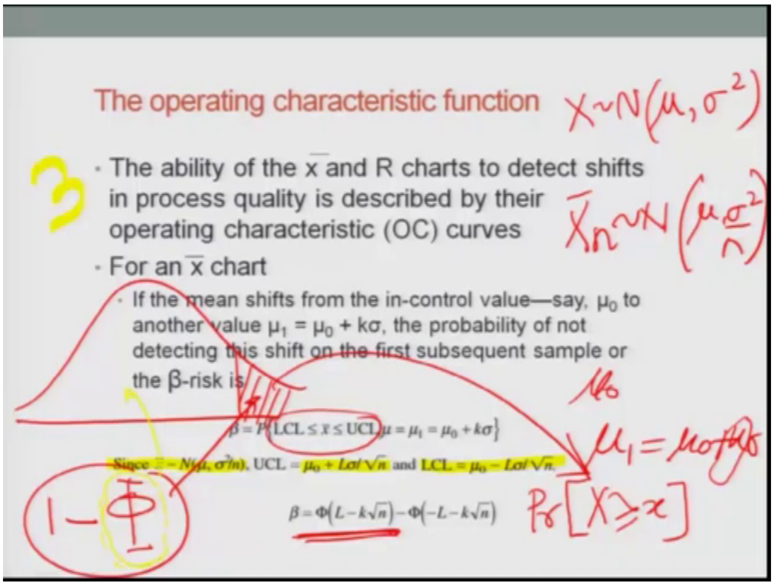

So, the operating characteristic functions would be important for us to study the ability of the x bar and the R charts to detect shifts in the process quantity is desirable by the operating characteristic curves, the OC curves where we had the LTP, a, q, l, alpha beta and all those things. For an x bar charts, if the mean shifts from in control value say for example, is change from mu naught to mu 1, so there they can be either positive or negative forget about that. The probability of not detecting this shift in the first subsequent samples is basically a type of error, which we did not generally term as the risk which is alpha and beta.

Now, what we want to find out is that whether the pattern of shifting based on the change of the mean values, and does have an have an effect on the overall risk. Now, the word risk we use about the total loss, but that risk can come either from the positive sense or the negative sense. What I we mean by the positives and second negative says is we have already discussed. But what I want to say is that it can be either a positive risk for me in the sense that some bad items have been produced at my end, but the pass the quality test so obviously, it is a loss for the other person. But it may also happen that they are some good items which are rejected so obviously, is a loss for me that is why I am saying is a positive or negative risk.

So, what we want to find out is basically the beta value and that would be given by the probability. So, now this is the probability that their x bar which is the sample mean is between the lower control limit and the upper control in the limit that is less than upper control, but more than lower control. Provided that the mean value has changed which means initially here mu naught, now it has basically changed to mu 1, where mu 1 is basically mu naught from some k sigma. This sigma is basically the standard deviation of the overall population.

So, now, this k technically we will consider can be plus or minus that means, if the process is going very good obviously the mean value would basically I would not say the word very good. Consider due to some reason or the other, I have slowly purchased a machine and where the feeling of capability of say for example, can of juice that is decreased. And I want to basically change the prices also obviously, if the amount of juice in the can is changing, and I want to find out whether it is meeting my process capability. And in case say for example, I give some extra percentage like if you see in the bottles 33 percentage extra at the same price. So, if you consider those obviously, in k that in that sense the k value would be positive. Now, this k value would also have an implication from the point of view the variability the alpha value if you remember z alpha by 2 minus z alpha by 2, so they would make sense here also.

Now, I want to find out the probability that x bar I am again repeating the statement which I have highlighted which basically the x bar the mean value of the sample is between lower control and upper control provided the mean value of the whole process has change. Now, what we need to would understand is that; what is the overall distribution we are looking at. Now, in the last lecture, I did mention and let me write it down clearly in the red colour. So, technically x which is the random variable is distributed um normal with mu mean and the sigma square variance now when I basically replace mu with x bar suffix x n, n is basically sample size. So, this is the sample mean, sample mean we know is distributed n with mean mu sigma square by n. So, this part we have already discussed if you remember. So, this is what we are trying to understand.

So, since x bar which I am now trying to highlight using yellow colour since x bar is distributed with mean mu and variance of sigma square by n. So, the upper control limit now would become mu plus L, L is the coefficient into sigma by square root divided by square root of n which was already discussed. So, in that case L in that I other example when mu and sigma one know one known we had taken as 3, so depending on the alpha value. In this case the lower control limit is also minus part I am only discussing is minus L into sigma by square root of n. And the beta values which you see here is basically the standard deviates which you consider when you converted into a standard normal distribution. So, let me draw it with red colour. So, this is the x-axis, this is the distribution with mean value of 0.

So, what it means is that this capital phi means what I would like to highlight is this. So, I am using x as a general random variable. So, if I go here, so I consider x is here. So, this is the whole probability which is on the left hand side for this case. When I basically erase is it let me erase it, when this is not right and I am using this which means I am considering this part of the graph. So, in this sense, the calculations will be different, how it will be different.

So, in the second diagram, which I have just finished. This capital phi would basically mean the cumulative distribution for all the values of the probabilities starting from minus infinity to till x. So, this will give me the total probabilities from the left to the right. And if I am I am finding out one minus that that will give me the values on to the right that means, these values and this one let me highlight for ease of understanding this one, this will give me the values of the left. So, if I understand that from the standard normal deviate capital phi L minus k by square root of n and capital phi minus of L minus k by square root of n would give me the total standard division of the cumulative distribution values added up from to the left or the right depending on how you look at the standard normal deviate.

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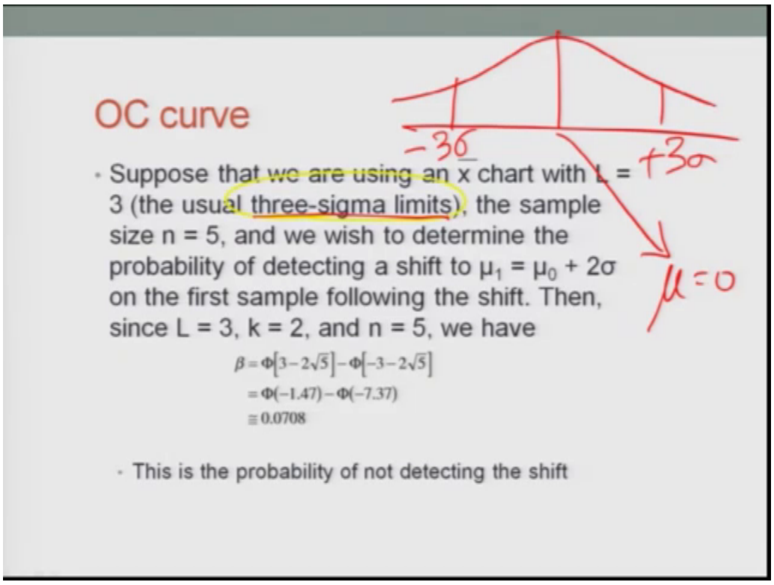

So, OC curves which is the operating characteristic curve, suppose that we are using an x bar charts with L, L is the coefficient which we have is 3 that is usually I did not mention that. So, this is being highlighted maybe for the first time. So, this is the 3, sigma limits which means that if I draw the curve in fact, let me use the red colour for ease. So, these values are minus 3 sigma; I am using the plus minus 3 sigma considering the mu value is 0 for the standard normal deviate. So, this is the 3 sigma limits. The sample size is n and we want to which is phi. And you wish to determine the probability of detecting a shift from mu naught to mu 1, which is basically mu naught plus 2 sigma that means, 2 sigma is the total width plus minus 2 sigma which we are considering, but it is moving onto the positive side on the first sample following the shift.

So, now, consider this 3 sigma limits now L is 3, k is obviously 2, and n is given as 5 because that is the basically the sample size we are taking. If you put that values the beta value comes out to be about 0.07, which is about 0.0708. So, this is the probability of not detecting any shift which is happening because out of the total number of such hundred pickings or hundred such observations we are taking they would be charged that we are not able to detect the change. So, whether that is very positive or negative would depend on how your problem has been formulated.

Why I am saying that consider this like you are manufacturing or a testing pacemakers so obviously any 0 amount of fluctuation would have a devastating effect. But see for example, you are manufacturing a chair or some of small machines, so not being able to detect the those much shifts would not have much of over consequences concept, but negative consequences now fear on the oh overall process. So, obviously, the value of beta are not detecting the shift would have different implications depending on how the problem is being formulated and what type of testing you are doing.

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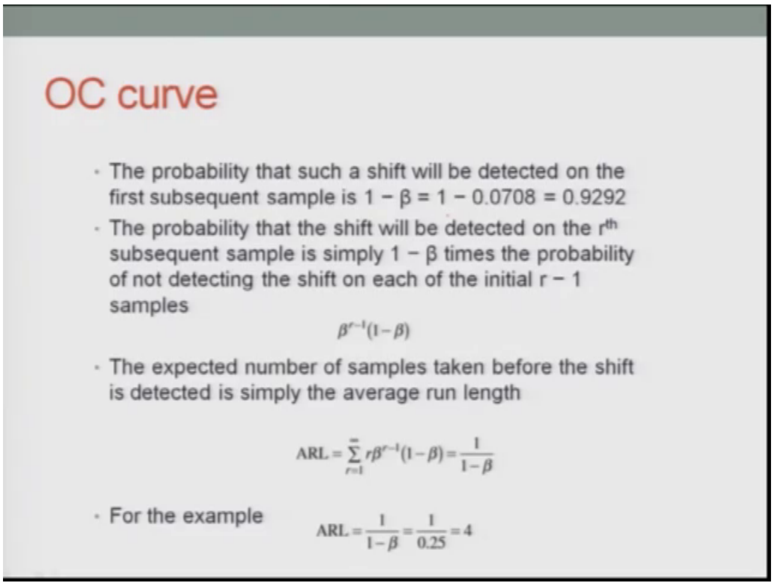

So, continuing this, the probability that such a shift for the OC curve the probably that such a shift would be detected on the first subsequent sample, it would be given by 1 minus beta. Beta would be not defecting, 1 minus beta would be the defect they were detecting. So, if it is lost from one side it will be positive from the other hand side, which comes out to be about 92.9 percentage. So, this gives the probability that the shift will be defect detecting than the subsequent sample is simply 1 minus beta times the probability of not detecting that in each trial.

So, why it is that because consider there are R trials. So, there are basically R such places to be filled up R seats, now you have detected the shift only on the rth trial; that means, in the R minus 1 trials which have already happened you have not been able to detect the seat the shift. So, what is the probability of not detecting is beta in each trial. So, how many such betas would come, it will be beta for the first one beta, for the second one beta for the third one till the Rth minus 1. So, beta would be multiplying R minus 1 terms. So, it will beta to the power R minus 1. And the last place consider as I was saying there are chairs seats to be filled up all of them have been filled up with a value of beta, how many number times R minus x times minus 1 times. And the last one which is the other place has been filled up by 1 minus beta which is a sort of distribution which we already known, I will come to that later on. So, the hence the total probability would be beta to the power R minus 1 into 1 minus beta.

So, the expected as number of samples taking before the shift is detected by simply taking the average run length so that would be. So, in each case I have the probabilities as R beta to the power R minus 1 into 1 minus beta and R would be the set of the number at which stage we are detecting. So, if it is beta only, so it will be beta into one because first stage you have to not been able to detect, it would basically becomes beta 2, so R 2, R 3, R 4, R 5 you will basically multiplying the terms of 2 into; say for example, beta to the power 1 into 1 minus beta plus R now becomes 3.

It will be 3 into 1 minus beta into beta to the power 3 minus 1 and you will continue finding out till the last stage which technique can we infinites you find out all these values multiply them sum them up that will give you the basically the information that what is the average run length which is given by ARL. And for this example if you have beta as given as 0.0708 then you find out the average run length as four; that means, by whether within the fourth one or that of that trials of the experiment we are able to detect that there is a shift.

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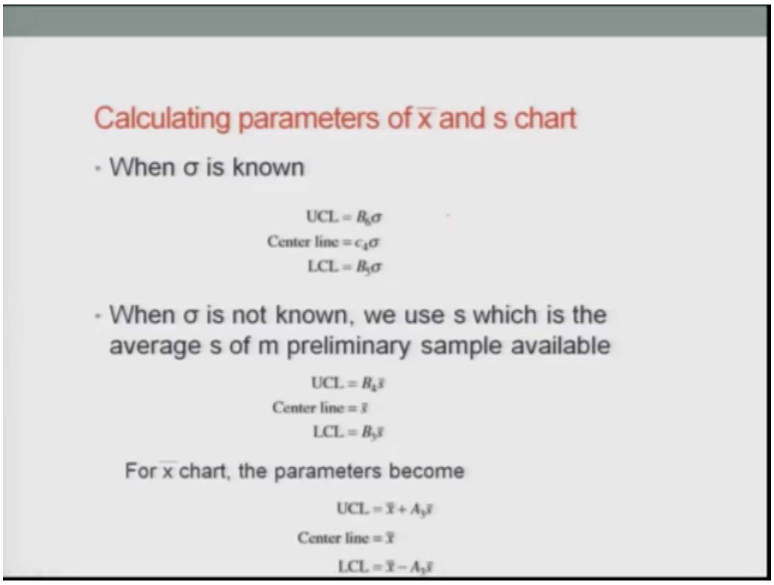

Now, you want to find out the calculating the parameters of x bar and S charts. So, it is not r. So, when sigma is known, the upper control and the lower control values would be calculated from the tables, it depending on 9 values. So, the central line is c 4 into sigma which is the value over and above you will have the upper control and the lower control. The upper control limit would be given by multiplying sigma would B 6 and lower control would be given by multiplying sigma would B 5. So, these B 5, B 6 would definitely be dependent on the sample size.

When sigma is not known, what you do is that you replace that sigma by the corresponding sample standard deviation or the sample standard error. So, this is which is S so but now obviously, you will try to be replace by the average value of the standard error. So, let me continue by reading it. When sigma is not known, we use S which is the average of S bar which is the average of S for m preliminary such samples you have taken. So, you have taken first sample, second sample, third sample, fourth sample four in each case the standard error would be S, you try to find out the average of this four subsamples you have taken.

The word subsample I mean that first you have taken five you find out the standard error; second you again you have taken five, you find out the standard error; third you have find out this set of observation is again five. So, you have basically m is three considered m is 1, 2, 3 for the set of observations you have taken find out S find out the average of this with this you will use as S bar for a subsequent calculations. So, based on that when you find out the central values would basically B S bar. So, in the initial case c four into sigma was basically depending on the sample size.

Now I will use the actual S bar and the corresponding upper value and the lower values would be basically be B 4 and B 3 which would be the multiplying factor based on the sample size which you are taking. For the x bar chart, so now, would be initially you had x double bar plus minus I am using the plus minus board for the upper control and the lower control as k. So, this k can be either 3 or 2 or 1 as I have already discussed multiplied by sigma by square root of n. So, now, this sigma m by square root of n would not be coming into the picture, but it will be replaced by S bar with a multiplying factor. So, this is what is mentioned here.

So, the upper control limit becomes x double bar multiplied by the value of A 3 into x bar and the lower control limit would be basically replaced that plus sign would replaced by minus and you will find out the lower control limit. And obviously, the central value continues to be x double bar because our main emphasis is to study the x bar charts. So, here the average value does not change only the stand the sigma value has been replaced initially by R values if there range values. Now, we are trying to replace that with the S bar values or the S value, which is the standard error.

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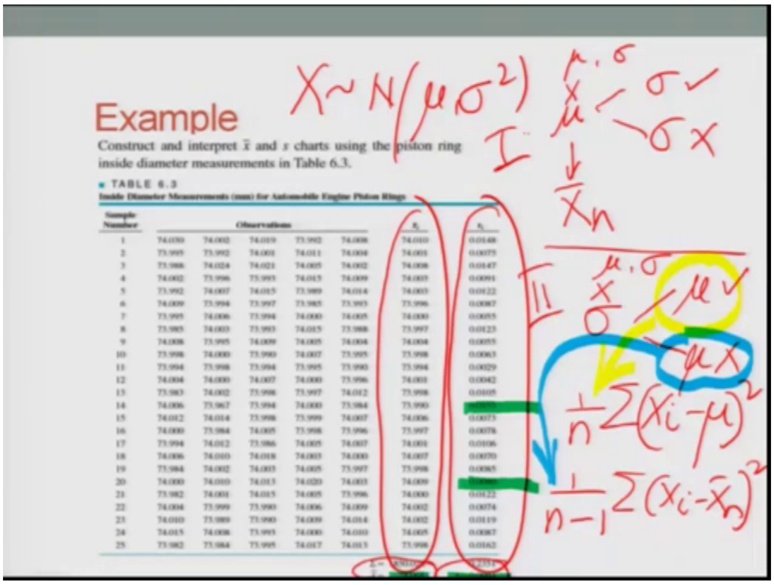

So, we are required to construct and interpret the x bar and the S charts not R remember, it is the S charts using the piston ring inside the diameter example. So, here again in the first column you have the sample number starting from 1 to 25 and the observations are given. So, what you have for the first sample which consists of five observations the values are I am just reading them 74.030, 74.002, 74.019, 73.992 and 72.008. From that what you need to find out. So, we need to find out x bar for each sample of size five, the standard error for each sample of size five. So, those are formulas are there and based on that when you go and try to find out the x double bar it comes out to be 74.001 and S bar comes out to be 0.0094.

Now, let me highlight this value of s. So, what is important to learn is that from statistics. So, this if the slide is not there, but I want to basically discuss what about 2 minutes. Now, in general in statistics we know that if I will try to use the red colour, so it will makes sense here. So, say for example, your mean and sigma are the way values you want to study. So, now, let us go step-by-step. In case your main concern is to study the value of mu, mu is not known. So, I am using a minus sign on a cross sign on above meaning it is not known to you. So, they can be two outcomes; one is sigma is known another is sigma is not known, so obviously, you have to replace it by the sample mean.

So, what is the answer, answer is the best estimate which is basically known as umvue in statistical sense which is uniformly minimum unbiased estimator would be X bar n. So, this I am talking about the normal distribution remember that. So, here let me make it much more explicit. So, in this case, X bar is the best estimate for mu either, in the case when sigma is known and also in the case when sigma is not known, so obviously, there is no problem.

The second idea which I wanted to discuss here for which I said I will take two minutes is that mu and sigma are the main emphasis of study the distribution is normal. So, let me make mark it as one, one means basically the studies for mu. Now two is basically the emphasis where I am trying to study the standard deviation of the variance. So, I want to study this which is not known, but there are 2 outcomes, again with this case mu is known mu is not known. Now, if mu is known, my actual formula to find out the best estimate from the sample, which would basically give me the maximum amount of information of standard deviation from for the population would be. And in the case when the population mean is not known, now see your difference I will highlight it; obviously, it is very obvious from the thing which I have drawn and use the highlighting as yellow one.

So, basically for the case one mu is known, I use this formula. When a mean is wait let me go to the different colours. So, it will be easier for me yes when mean is not known I use this formula. So, it looks very intuitive why. So, there are two main differences in the formulas. I am talking from the theoretical point of view from the mathematical point of view. I will come to the intuition later on and I will mentioned it very briefly. So, in the first case you see that you divide by the sample size which is n which is here where I am hovering in the pen; and the second case we divided by n minus 1. So, what is the reason for that?

So, to answer that let us see the formulas and the values which are in inside the bracket square. In the first case, if mean value is known obviously, you will always use the mean value and there is where I am hovering the electronic pen. So, obviously, there is no loss of information which we have from the sample we have taken. We take pick up the sample and study the sample variance only because we are not bothered about the expected value of the population because that is already, known which is mu.

Now, in the second case where I am now hovering my electronic pen is that. When mean value of the population is not known, what do we do first we basically replace the mean value mu with the sample mean which is X bar n which we have already done here. Now, the moment you use the set of observation sample for the first time you will lose some set of information which is known that the degrees of freedom basically you lose is one that is why the set of observation which you had initially now gets replaced by not by n minus 1, because the set of observation which you have in front of you lost some amount of efficiency and it becomes n minus 1 that is why the change in the formula.

And remember these the formulas which I have given here, the first one let me use a different highlight colour to make it much more obvious. The first one and the second one are extensively used for the case to find out those sample variance provided the population mean is known in the first case; and provided this population mean is not known, where we replace the population mean by the sample mean which is X bar suffix n.

So, considering this whatever us talking about you find out the S which is the standard error for each sample of size five. Find out all of them individually find out the sums hence the value of S bar comes out to be as given here is 0.0094, I have mentioned that, but I am again mentioning it. We have already found out X bar for each observations. So, say for example, for the thirteenth one the value is 73.998, so we have basically 25 readings and sum of them divided by 25 comes out to be 74.001, which I am again highlighting.

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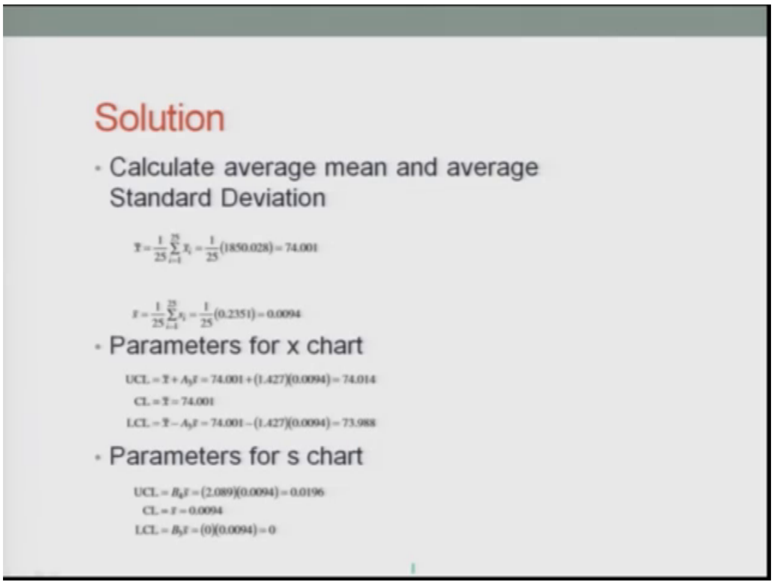

So, now I want to basically do the last bit of the calculations. So, you calculate the average mean in the average standard deviation. So, as mentioned the X double bar comes out to be 74.001, S bar comes out to be 0.0094 based on that when you go and do the calculation for the x bar charts this the x double bar can remains as 74.001. The upper and the lower control limits would be basically be x double bar plus A 3 S bar. This A 3 bar is coming from the sample size information which we have. And hence the upper control limit comes out to be 74.104 and the lower control limits comes out to be 73.988.

Similarly, when you do the calculations in place of R charts, now you are using the S charts. The upper control limit when you multiply S bar with B 4 and when you multiply S bar, B 3 which come gives you the information on the lower control limit the corresponding respective values comes out to be is 0.0196 and 0. So, using this you have basically been able to replace the R charts with the S charts which is standard error and that will give you much better set of information and better set of information about the process control. Provided one important thing which I did mention I am again mentioning is that the sample size for each such small readings keep changing depending on the about type of experiment we are trying to do. So, with this I will and this class and continue more discussions about the process capability charts later on in the later lectures.

Have a nice day, thank you very much.