

**Total Quality Management - I**  
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**Lecture – 24**  
**Pattern in Charts**

Welcome back my dear friends; this is the 24th lecture for the total quality management course. I am Raghunandan Sengupta from IME Department IIT Kanpur. So, if you remember; we are discussing the control charts and how the change of the sample size from  $n_1$  to  $n_2$ ; which is from  $n_{old}$  to  $n_{new}$  would give you different set of information's both for the  $\bar{x}$  charts and the  $\bar{R}$  charts.

And if you remember a very intuitively, I had mention and that should also be available to all of you then the  $\bar{x}$  would not  $\bar{x}$  double bar, which is the center line would not change, but; obviously, the upper control and the lower control change depending on whether the sample size is increasing or decreasing.

But when we come to the rain chart; so, you will find out the mean value  $\bar{R}$  would also change; similarly the upper control and the lower control would change. Later on the fag end of the 23rd lecture; I did mention that those ranges would also depend on the type of error; alpha and beta which I mentioned about the consumer disc producer disc and also it will depend on what are the values of alpha and I did draw the diagram.

So, you can understand that as alpha; the coverages has changed  $1 - \alpha$  would be the coverage; as the coverage change, how the name is changed. And for a value of alpha as 0.002; you saw that the z value was coming out to be in the right hand side; that means, from your side if you look at the Z value on the positive side it was plus 3.09.

And for a value on the left hand side which was in the negative values; because we are considering the central value or the mean value as 0 for the standard normal distribution, the value would be minus 3.09. So, with this I will continue and I also try to discuss that how I will just very briefly discuss that how these alpha and beta is basically affect your the central lines of the upper control and the lower control.

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**$\bar{x}$  and R chart when  $\mu$  and  $\sigma$  are known**

- The  $\bar{x}$  chart
  - $UCL = \mu + 3\frac{\sigma}{\sqrt{n}}$
  - Center line =  $\mu$
  - $LCL = \mu - 3\frac{\sigma}{\sqrt{n}}$
- Defining  $3/\sqrt{n}$  as A, we get
  - $UCL = \mu + A\sigma$
  - Center line =  $\mu$
  - $LCL = \mu - A\sigma$
- For R chart
  - $UCL = d_2\sigma + 3d_3\sigma$
  - Center line =  $d_2\sigma$
  - $LCL = d_2\sigma - 3d_3\sigma$
- Defining the constants  $D_1 = d_2 - 3d_3$  and  $D_2 = d_2 + 3d_3$  we get:
  - $UCL = D_2\sigma$
  - Center line =  $d_2\sigma$
  - $LCL = D_1\sigma$

Now the  $\bar{x}$  and R charts when  $\mu$  and hence  $\sigma$  are known. So, we know that if they are known. So, till now we consider that they are unknown not known and hence we replace them with the characteristics from the sample. So,  $\bar{x}$  double bar in the long run as the more more observations you take, it basically becomes equal to exactly equal to the  $\mu$  value; similarly it should be the case or the standard deviation.

The  $\bar{x}$  bar charts if  $\mu$  and  $\sigma$  are known; I should highlight this because this is very important to understand. So, in the initial cases we did discussed the  $\mu$ , where I am having my pen. Let me check whether the color highlighter yes where I am having my pain this  $\mu$  and  $\sigma$  were unknown, but now we are considering them to be say for example, known.

So, if they are known. So, how would you do the calculations? So, in the  $\bar{x}$  bar chart; the UCL is basically becomes  $\mu$  plus minus. So, UCL and LCL I am just repeating the same thing. So, the limits would be plus minus 3;  $\sigma$  by  $n$ . Now, this is important to understand, so I will spend some time here and I use the color black and has required will.

Now, consider this is the central line which is equal to  $\mu$ ; the upper control lower control we are mentioning in this  $\mu$  plus; this 3 value is depended on the factors which was there may not be 3; it may be something else also I will come to that. So, first let me

may reemphasize change the color in order to make it more expressive. So, we reemphasize on this and later on we will consider the value of why 3.

So, let us basically pay attention on; now what is important to note is this, now the  $\mu$  is I am using blue color to make you understand that why this blue colored;  $\sigma$  by square root  $n$  have an important. So,  $\mu$  is best estimate when you have is actually the sample mean. Now, what new to understand is that what is the overall variance when you are trying to utilize the sample mean.

So, that the variance would be  $\bar{x}$ ; so, this would be the variance for the for the sample. See if you are aware of basic statistics, the overall variance would be coming out with using some simple calculations would be coming out to be this. Now, this the variance if it is the variance; obviously, you have to find out standard deviation.

So, what you do you take the square root this becomes; so that is why. So, this calculation part I am just leaving it for the interested readers; here you can basically study this; I am just highlighting it and bolding it. So, you will understand how it can be used. So, this is  $\bar{x}$  understand the I am trying to find out the variance of the sample mean with respect to which population expected value.

So, that is why is it becomes  $\sigma$  by square root of  $n$ . So, larger the sample size technically the variance of that particular sample should in the long run decrease. So, it should be intuitive because more and more observations we are taking; actually the variance of the sample as such because you are taking all of them. So, in between the sample we will considered the variance actually becomes 0 in the long run technically.

Now, coming to the case why 3; so, let me again change the color and make it weapon. So, let me first erase it; so, it would be easier I would have a lot of space. So, I will erase this and erase this; so, this I am sure you have noted it down; when you doing the NPTEL; MOOC course service you can go back to the slides and understand that.

So, we try to use the color green yes; so now, consider the standard normal deviate; this is  $\mu$  is 0 and variance or the standard deviation here is 1. Now, when I consider plus minus 1 and covering about 67 percentage, when I consider plus minus 2; I am covering about 97, 95. So, these are approximate value you can check the table.

When I am considering about plus minus 3; then that coverage over all is about 99. So, this value of minus 3 and plus 3 is what is being used for this calculations. So, minus 3; obviously, would come from here. So, this will give you (Refer Time: 08:24) so, if I want to basically decrease the limit. So, plus minus 2 sigma so; obviously, the value of 3 would be replaced accordingly.

If I want to basically make it plus minus 1 sigma; so the value of that coefficient 3 would be replaced accordingly. So, you have to be careful what is your overall; the level of confidence you are using. Now, remember one thing you in the twenty third lecture; the last slide which you considered. We did mention; I will change the color of the highlighter in order to make you understand.

We did consider some alpha. So, this alpha if you consider; so, I will use the highlighter of yellow color to make things simple. So, these values when I am trying to; however, it. So, actually the total coverage which will have here would basically 1 minus alpha. So, as 1 minus alpha changes depending on the value of alpha, the values of Z alpha by 2 and obviously, minus Z alpha by 2 would change.

So, if I am trying to cover about 99 percentage of the area. So, then the right value which means this area on to the right and this area to the left would total sum up to about 1 minus 99; which would be 1 percent. So, 1 percentage you would be divided into 2 equal halves which would be 0.5 percentage on to the left; which is on this part where I am putting my finger and about 0.05 percentage on the right.

So, once when I calculated from the table those values which will basically will give me that information would be plus 3 on to the right and minus 3 on to the left. So, plus 3 minus 3; I am basically talking from the point of view of the diagram which I have drawn. This would be decently clear to you; if you do a decent amount or understand the standard normal tables and the distributions how they look like.

So, I remove it in order for you to write more information if I need. So, this I am cancelling for the case when mu and sigma are known. So, for x bar charts; the upper control limit is  $\bar{x}$  plus a value of 3; as you will know is change can be changed; sigma by square root of n and the lower control limit would be the minus value of 3 into sigma by square root of n.

Now, defining  $\sigma$  by square root of  $n$  as  $a$ ; you will basically have the upper control limit and the lower control limit given as  $\mu + a$  and  $\mu - a$ . Now if you go back to the initial part; where we have considered that a table where  $n$  values of the sample size were given based on that we decided on  $a$ ,  $d$ ,  $b$ . So, this would now make sense that why we are considering those values accordingly.

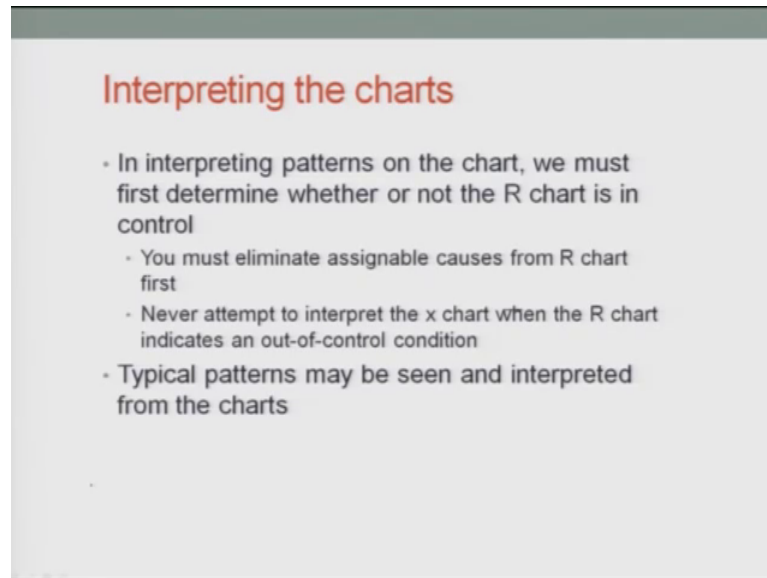
So, the more the spread less the spread; obviously, you will find out corresponding  $Z$  values  $Z$  suffix that  $\alpha$  by 2. So, what that  $\alpha$  would be dictated by what is your over coverage or level of confidence which you need. So, similarly when you go to the R charts; so, the upper control limit would be again coming to the fact, it will be  $d_2$  into  $\sigma$  is the central line because  $\sigma$  is now known it did not be replaced with the value of  $R$ .

So, it would be  $d_2 \sigma$  and the upper and the lower (Refer Time: 12:02) values depending on the case that the overall coverage is about 99; would be basically  $3$  into  $d$  the; so, this  $d_3$  values will depend on the dispersion which you have based on the values of the total coverage is which you have. But let me tell you before and; obviously, we are considering that the distributions based on the fact of  $\sigma$ 's also we tend to consider the central limited theorem will be true.

But actually the distribution of the sample variance would technically be in the long run if you understand the basic statistics and going to the depth would be chi square distribution; which we would use that later, but not now. For the R charts, the upper control limit would  $d_2 \sigma + 3$ ;  $d_3$  into  $\sigma$ . So, this  $d_1$ ,  $d_2$ ,  $d_3$ ;  $a$ ,  $b$  and all these things can be found on from the table. So, I am not going to go into details for that.

Similarly the lower control limit would  $b$  of equal length below the central line. So, it would be  $d_2 \sigma$  which is the central line minus the value  $3$ ;  $d_3$  into  $\sigma$ . So, this three again have the same concept which we are discuss just few minutes back when we are discussing the  $\bar{x}$  charts. So, defining the constraints at  $d_1$  and  $d_2$  according; so, these are the capital  $d_1$ ,  $d_2$  which are technically used from the tables; we can find out the upper control, the lower control and the central line for the R charts.

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The slide is titled "Interpreting the charts" in a red font. It contains a list of four bullet points:

- In interpreting patterns on the chart, we must first determine whether or not the R chart is in control
  - You must eliminate assignable causes from R chart first
  - Never attempt to interpret the x chart when the R chart indicates an out-of-control condition
- Typical patterns may be seen and interpreted from the charts

So, now giving a decent background; I know the time was limited; I tried to give the detail qualitative field, but; obviously, I would request urge and really want the students to read the book that will give them lot of idea about total quality management.

But one thing the book goes very slowly; I am referring to the book time and again because I found in that to be very interesting book; it goes very slowly builds up the concept slowly, but is a huge amount of information; there huge amount of concepts you can be clear if you read that book slowly.

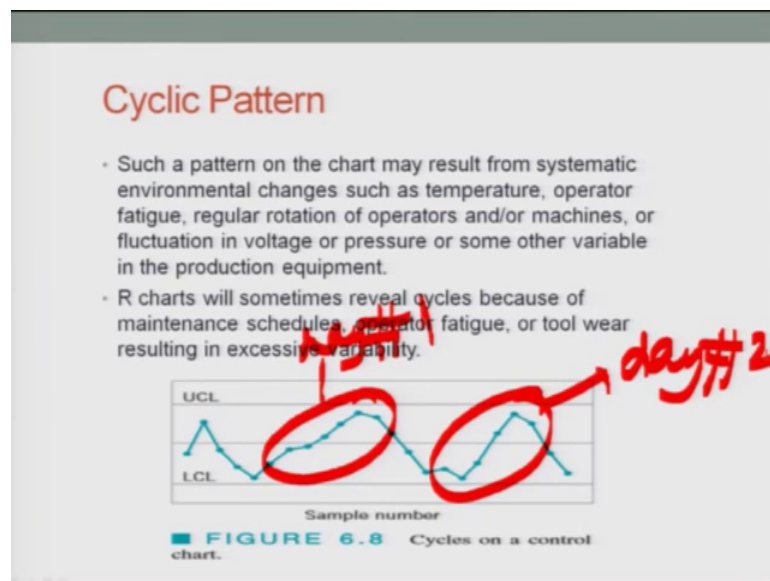
So, interpreting the charts is the next stage of our concern. So, interpreting the patterns of the charts; we first determine whether or not the R chart is in control. So; obviously, if it is not control there is some variability because R is the range which give us the information on the variability. You must eliminate assignable causes from the R charts first; never attempts to basically interpret the x charts when the R charts x bar charts when the R chats it indicates out of control.

Because what is important if you remember; I did this discuss that the x bar charts are in somewhere trying to give you some information's about the movement of the sample means. So, the more number of sample means which you have; in a sample if n is increasing it is basically tending to give you some average characteristics which need not would not give you much information's about the variability.

But when you come to the R charts; which basically give you the variability the dispersion per sample in each sample because you need to find out the difference between the maximum and the minimum; that would give you much better information's whether these amount of variability. So; that means, the first set of information which we want about the process or how the mechanical process or the manufacturing process or whatever this charts are being used for that information would be coming out from technically from the R charts.

So, that is why it is mentions and never attempts to interpret the x bar charts; if the R charts are out of control. So first try to understand the R charts and then go into the next stage of trying to understand the x bar charts. So, typical pattern may be seen and interpreted accordingly from the charts.

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So, we will very briefly discuss some cyclic patterns and some patterns which are evident from the conglomeration of the information; which we get from the x bar chart and R charts.

Now, cyclic patterns are basically some cyclicity; there is a cycle which is going on cycle means the we see a pattern and how this pattern of cyclicity can be studied. So, it says that such a pattern chart may result from systematic environment changes like in the summer is temperature is high; then the winter the temperatures where low; again the

cyclicality of the summer comes and the temperature increases and decreases so; obviously, it will come repeat in a particular cycle

Or these is a operator fatigue so, in the morning in the operator it is absolutely peak of its performance his or her performance and in the evening or after lunch; the performance of that particular person decreases. So, obviously, that may happen each day or there may be regular rotation operations operators. Like say for example, in the first shift you use in operator who is very skilled and the second shift you use an operator who is not does skilled.

So, that and if that that pattern of labor utilization is same so; obviously, that cyclicality of the pattern would be very evident from the process control charts. Or say for example, in the morning, you are using the same operators, but you are using two different machines; in the morning half you are using a bad machine and that means, old machine I will not use the word bad, but in the second half depending on whatever production planning which you have done; you are using a new machine.

So, now; obviously, there would be a cyclicality of the patterns which would be coming out and very evidence from a process control charts. Or fluctuation in the voltages are there say for the example in the night; when the operators are working, the load of the electricity is not much from the residential areas; obviously, there would not be much fluctuation in the voltage.

But in the morning; if you are working your factory is working; there is a huge amount of load which is coming out from the residential areas. Because say for example, you work in an area; where people are using in the mornings the different type of cooking machines; some geyser are being utilized for heating up the water, the electrical ovens are being used or say for example, different mixies are being used. So; obviously, there is huge load on the electricity that would have an effect on the usage of electricity for the machines which you are trying to utilize for manufacturing the product.

So, I am giving very simple examples which should make sense to you. Or say for example, you are using water to as a coolant or you are using water to manufacture something for cutting tool at high pressure and consider in the morning the water pressure is low because all the residential areas are using the water and in the night the pressure is low because the water usage by other parties are less.



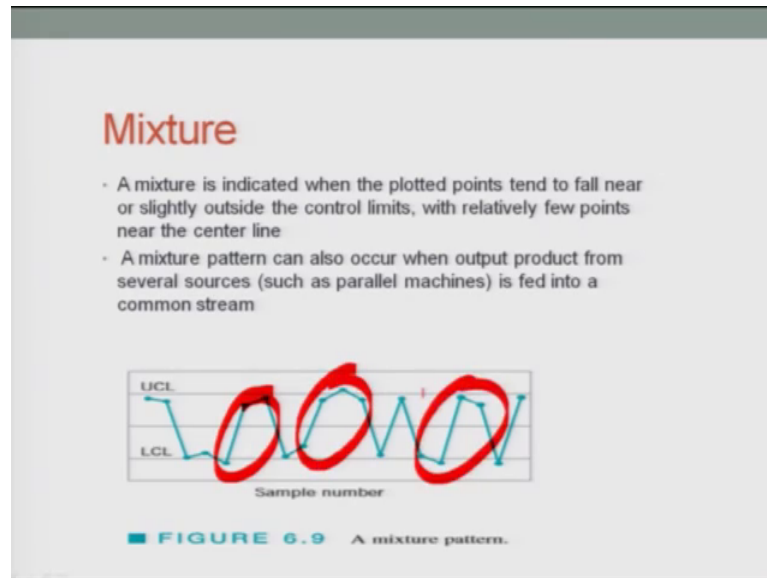
So, they can be some examples like this which I mentioned; electricity usage, water usage and operator being changed, machine being changed. Even if the same operator is there fatigue being there in the afternoon fatigue not being there in the morning; maybe the fatigue in the evening would be the highest level because there the overall efficiency of the worker is keeps decreasing as the total working pattern of the operator comes to an end; on a 8 hour shift.

So, R charts will sometimes reveal cycles because of maintenance schedule; maintenance schedule maybe say let us say for example, the machine is not old it is relatively new, but it needs a lot of maintenance. So, if the moment is maintained all the different parts have replaced, they have been oiled, they have been checked then the variability would be much less.

But as there is a wear and tear depreciation mid machine is being utilized day in and day out so; obviously, there would be some change in the variability on onto the higher side. As I mentioned operator fatigue is there; tool wear who can be one use like in maintenance; when you check the machine the you change the tool, but as in tool wear and tear is there; obviously, you still have a negative effect on the machine ability of the products which are doing utilized and the efficiency may decrease off the tool which would not give you the right product.

So, if you see the patterns; they would be and try to basically use the red color. So, this cyclicity would be repeated depending on say for example, this is day 1 from morning to evening; this is say for example, day 2 from morning to evening. So, you see in the morning due to some reason the upper control limit and the lower control limit even if they are same it is the cyclicity of how the readings are being done and it will give you lot of information about how the process capability is.

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They can be mixture of patterns; a mixture is indicated when the plotted points tend to fall near or slightly outside the control limit and basically it repeats this mix and match of different type of effects which comes out or they falls slightly outside the control limit with really little a few points near the central line. So, there is a dispersion, there is weird movement of the points from the central line.

But basically they are being clattered in and around the upper control or the lower control and is give some signal that they some different effects which can be considered as a mixture of different type of effects which are happening. A mixture pattern can also occur when output product from several sources suggest; parallel machines is fed into common stream.

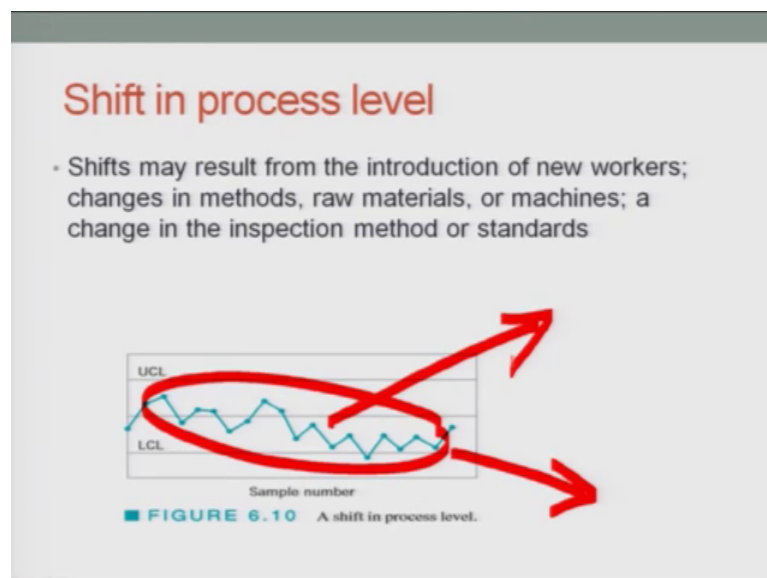
So, there are different type of process have being done for a single product on different same type of machine; some of them are old, some of them are new, some of them are being operated by new workers, some of them are being operated by old worker, for some the wear and tear off the machine due to some maintenance problem is there, for some of them the maintenance problem is not there because they are been maintained by the operator himself or herself on regular basis.

So; obviously, these comes those comes into the picture; you will see the mixture pattern would be repeating in such a way; that you will find out that the effect of those

assignable causes which can be controlled are slowly revealing their phase through the charts which can be either the x bar charts and the R charts.

And you remember again I am repeating; variability concept or trying to understand the variability dispersion would be a more important than trying to find out how the x bar chart behaves. Because variability is an essence of production process is an essence of any process with you took take because they are random. But if randomness basics exceeds the limit; it gives you some signal that there is there are some problems in the overall system for which corrective action should be taken.

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There can be shift in the process level that; that means, due to some process capability being changed or due to say for example, the products are coming out much good because the raw materials input has changed, because you are trying to get some different vendor to get the raw materials or if the coolant has been changed.

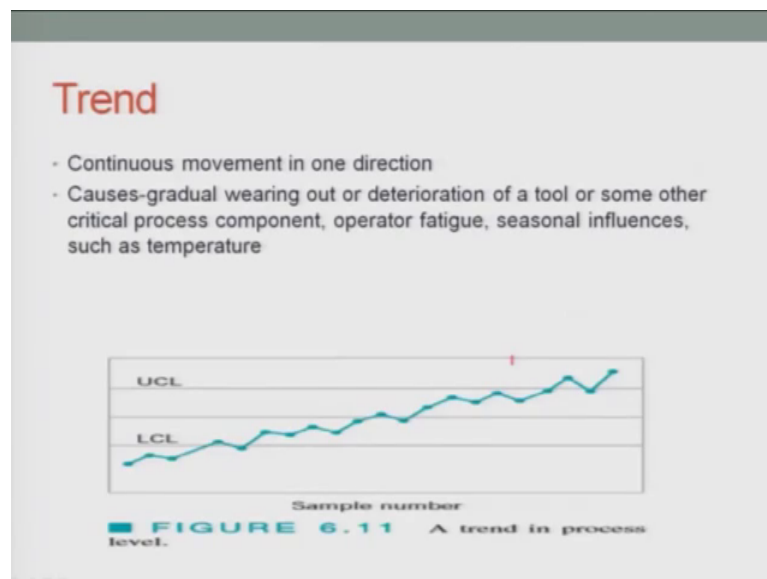
Because that as you keep using the coolant; obviously, there would be different sites and types of factors which may be affect the coolant, the temperature may be affecting the coolant or say for example, granular particles which or the residue particles when you do either grinding or some using the lathe machines all these things enter the coolant and you are not able to take those external particles from there.

So; obviously, the coolant effect on to the machine ability may have an have positive and negative affect; whether the train may be on the higher side like the they are moving towards the upper control limit or they are moving towards the lower control limit. So, shifts may result from introduction on new workers may change due to method change, may be due to the raw material change that the machine or machines have been changed.

So, change in the inspection methods or standards can also apply like say for example, a new inspection pattern on inspection idea has been used or utilized or the inspector himself or herself has different concept of how you wants to take the readings or rather than taking 9 'o' clock in the morning you are taking at say for example, 9.15 in the morning.

So; obviously, there those patterns would have an effect and if we see the shift can either go on to the downward trend weather positive or negative will understand later on or it can go into the upper trend.

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So, you have to basically understand in which direction is moving and what is the overall effect of this type of shift in the process level.

There may be a trend also; that means, continuous movement in one direction maybe there. So, and this may basically give you some information's that there is a use your wear and tear of the machines; the tool being utilized very heavily the coolant is heating

up or the machine is heating up or the temperature is increasing and the summer is coming or the temperature is decreasing and the winter is coming.

So, I will continue reading the second bullet point which means the causes of gradual wear and tear of these machines are there; which will basically give you. There is a depreciation of the tool or some other critical process of the component there may be an as I said that operator fatigue may be there seasonal influences like temperature humidity would be there.

And this would basically give you an information that in which trend; the overall process is moving; whether it is positive or negative you have to understand it later on, but these patterns would give you that information that there is some change which is happening in the overall process control. There can be stratifications also; so, which means that tendency of the points to cluster artificial around the central line.

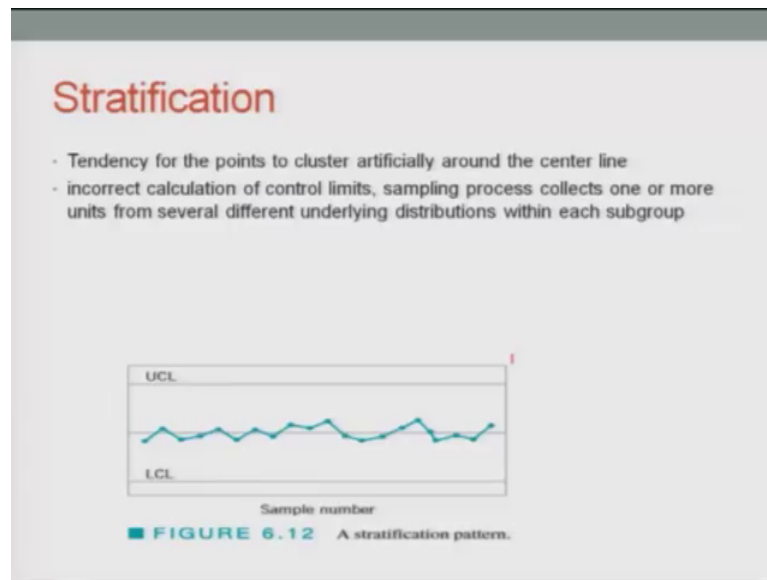
So, incorrect calculation the control limits may be a problem; there maybe sampling process collects one or more units from several different underlying distributions within each subgroup may be important; that means, say for example, due to some reason you were planning to take the observations at 9'o clock; 10 of them then again at 10'o clock 10 of them, 11'o clock; 10 of them and you should continue.

But say for example, due to some of the reason at 9'o clock you are not able to take the observations what the operator may do; which is definitely not right here she would take about 20 observations from 10'o clock in the morning and try to be see will compensate for not taking the set of reading of ten in nine 9'o clock. So; obviously, it mean that due to some reason that the 10'o clock pattern; on the 10'o clock work might change for 10'o clock usage of the machine or the coolant or the temperature was definitely not right.

So; obviously, all the set of so, called variability which could be control are in those 20 reading which are taking at 10'o clock in the morning. So; obviously, that is not right, but that will come out in the stratification and if the overall reading set of observations are very good so; obviously, those set of readings 20 number would be cluttered in and around the mean; if they are not they would be dispersed.

So; obviously, that would give you a negative signal that there are something wrong. So, obvious you have to study it accordingly.

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So, if you see the graph here again I am using the red color; the stratification would be in and around the central values; obviously, these can be stratified in and around the upper values also, but that would be studied separately. So, this will give you then information that how the patterns or is the cyclicity or stratification and so and so, forth would give you some information that how the process control capabilities from the point of the charts are going on.

And you can basically study them in a very simple manner and with a lot of good judgment and common sense if you are aware of the whole process; pass very critical information's about where the process is going, in which direction whether it is right or wrong. So, with this I will close this lecture and continuity with the 25th lecture later on. Have a nice day.

Thank you very much.