Total Quality Management - I
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Lecture - 17
The OC curve and Sampling Plans

A very good morning, good evening, good afternoon to all my dear friends; I am Raghunandan Sengupta from the IME department; IIT, Kanpur and this is the TQM course 1 and I did mention in my last lecture. So, we will have the TQM 2 also.

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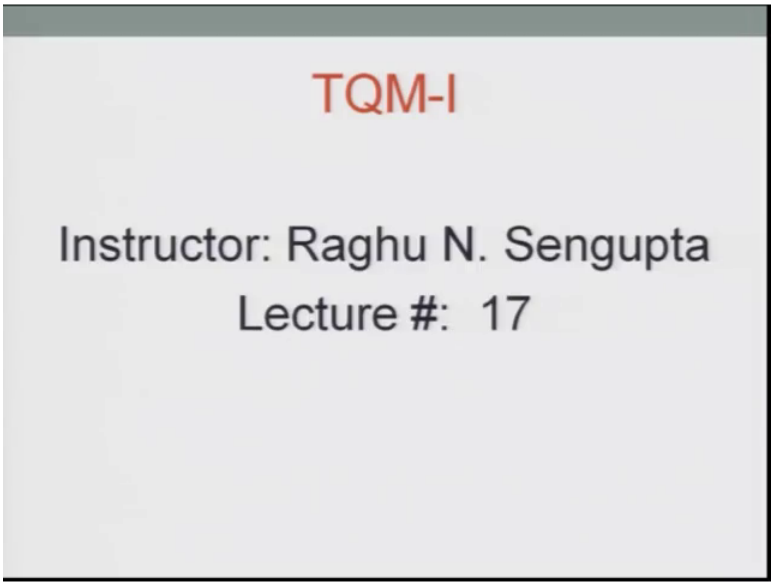

So, this is the 17th lecture which would be the second lecture in the in the fourth week because as you remember; there are 5 lectures per week. So, we would all already when is the 15th one, you have already completed your 3 weeks. So, continuing with the OC curves, the concept of AQL, concept of LTPD, concept of capital N small n c and how the curves would change and how the shape at the characteristic of the curve curves will change depending on capital N small n c and also that is distributions we did understand few of the fundamental things.

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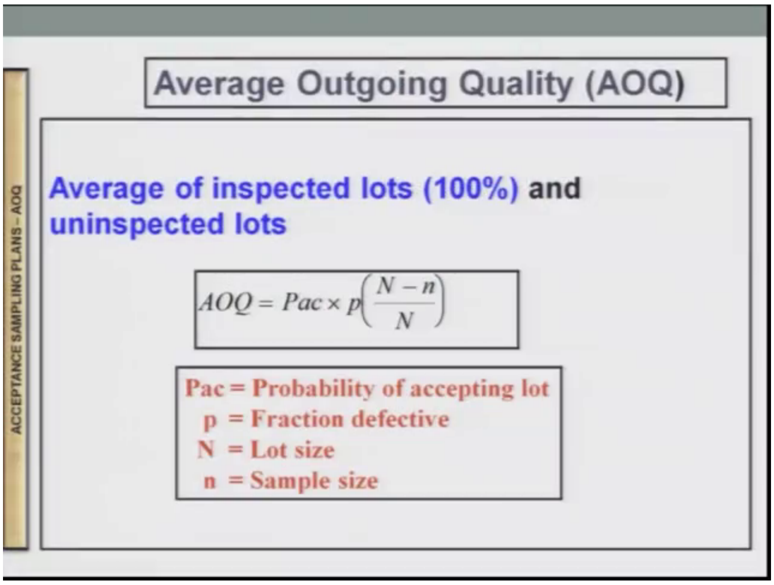

So, what we are interested to study now is basically average outgoing quality which is a AOQ; so, that the average number of of lots depending on the percentage and unexpected lots would be. So, now, in the lot you have technically the population is not infinite which is capital N. So, when you are when you are trying to find of the average outgoing quality. So, consider you have picked up small n. So, what we would have in front of us is basically we have a so-called lot which is of size one is n small n and one is basically capital N my small small n.

So, the number of defects if we are depending on small n would also have a consequence on the number of so called defects and accept acceptance on the other lot which is capital N my small n. So, we want to find out basically the multiplication of these following factors which are listen to me carefully, is the probability of accepting a lot. So, you either accept or reject the lot, would basically be dictated by P suffix a or a c and the fraction of the defects depending on what is their overall all probability which is which is there would be given by the value of P and the lot size which I mentioned the total size is capital N and the sample size is basically small n.

So, if you are basically considering these values. So, it would be capital N minus small n divided by n would give you the relative frequency that should be multiplied by p which is basically fraction defective and; obviously, that would also be multiplied by the factor of P suffix ac which is the probability of accepting the lot based on which you can find out the value of AOQ which is the average outgoing quality.

Now average outgoing quality limit so; obviously, these values would keep increasing. But obviously, they would we have a particular optimum value, some star value of AOQ star after which it should basically decrease. But that increase and decrease would be dictated by as if we remember by fourth actually in in set of informations. One is P suffix ac, one is p as I mentioned, the other would be the basically the lot size and one would with sample size. So, if you basically are able to plot the actual A or Q of any set of informations it would basic will be a distribution as shown in this figure.

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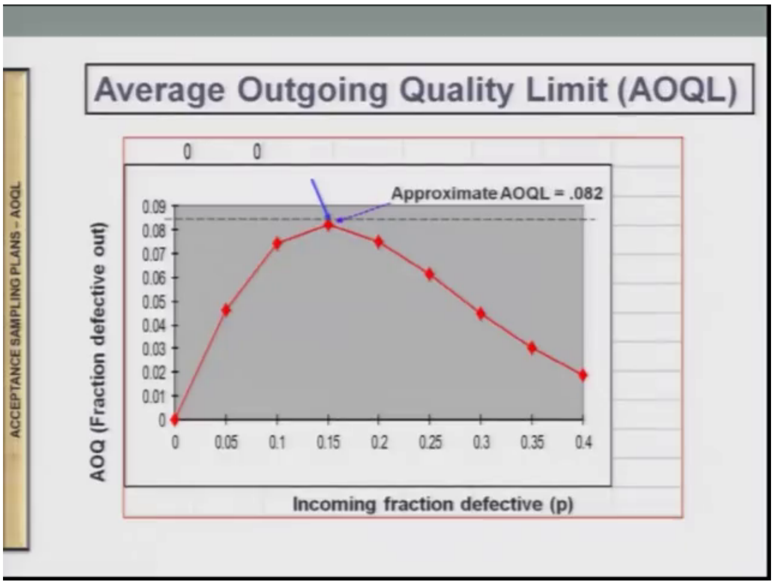

So, let me highlight it. So, this is the distribution which you have. Technically this is a smooth line if you have number of points is is infinite and there would be some point at the highest level would which would be given by the average approximate AOQL value which is the highest value depending on which you can find on the maximum level of AOQ. So obviously, that will change from example to example. It will change depending on what about distribution you have, it will change on the value of capital N, it will change in the value of small n and it will change in the value of P suffix ac, it will change on the value of p and all these things.

So, in this example as the incoming fraction defectives fees are measured along the X axis and the fraction defective out which is the e of value which imagining along the Y axis. Obviously, they would be a peak after which it will try to start decreasing and that the optimum value. We have to find out in order to understand that; what is the threshold value based on which we can take a decision for that particular lot for that particular distribution for that particular small n value for that particular capital N value.

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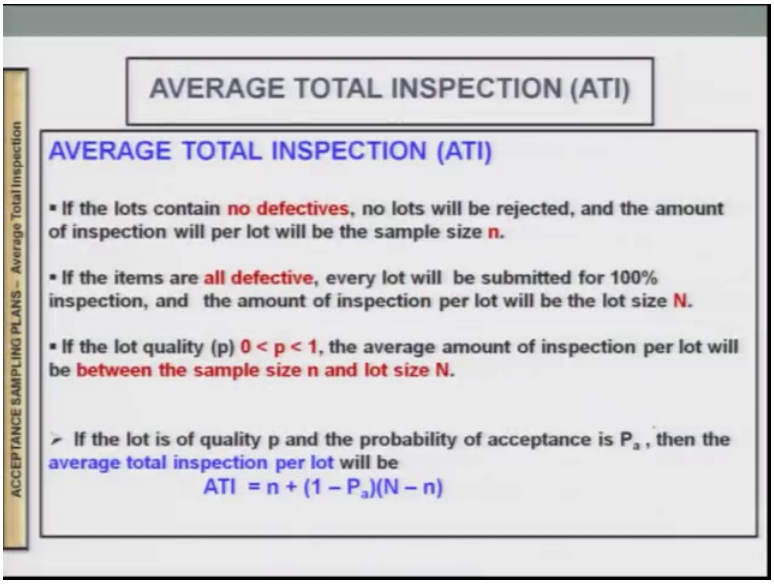

So, average totally in inspection obviously we will have some cost. So, I will come to the cost very briefly later on. So, average total inspection which is ATI. So, if the lots contains no defects no lots will be rejected and the amount of inspection would be per lot which will be given by the cost of inspection each item multiplied by the overall sample size. So, it will be as I mentioning, I will, I will read it again and the amount of inspection will per lot be the samples. I have n multiplied by the overall cost which is their per item.

If items are defective every lot will be submitted for hundreds per inspection and the amount of inspection for lot would be the lot size capital N. Because now you are basically actually inspecting each and every item. So, in the initial case you have the so called obviously lot in front of you which is capital N, from there you pick up small n and you make a decision. But now if you have to do in inspection for each and every lot so obviously, the not total cost would be the cost of inspection multiplied by the number of the of items which are there in the lot because as you are doing 100 percent inspection.

If the lot quality the average amount inspection per lot will be basically be there between the sample size n lot size which is obviously true because if you pick up a sample size and if you want to do full inspection or in one case you want to do some intermediate steps so the number inspection would definitely be in between a value of small n and capital N. If the lot is a if of quality p and the probability of acceptance is given by p a then the average totaling inspection per lot would be given simply by this. So, what you do is that you picked up a lot, so that enters a cost and also you will basically be doing and total cost structure for the values which are two things we should remember, is that what is the amount of such objects which are there which are being acceptable rejected.

So, those would be what so if you pick up a small end the rest left is capital n minus 1. So obviously there would be an overall probability of cost based on the fact that those capital N minus small n are being left out. So, that would technically be multiplied by 1 minus pa because pa we know is basically the acceptance probability of a lot we have depending on what is the overall numbers of the values of capital n small n and c.

Now the (Refer Time: 07:24) average total inspection.

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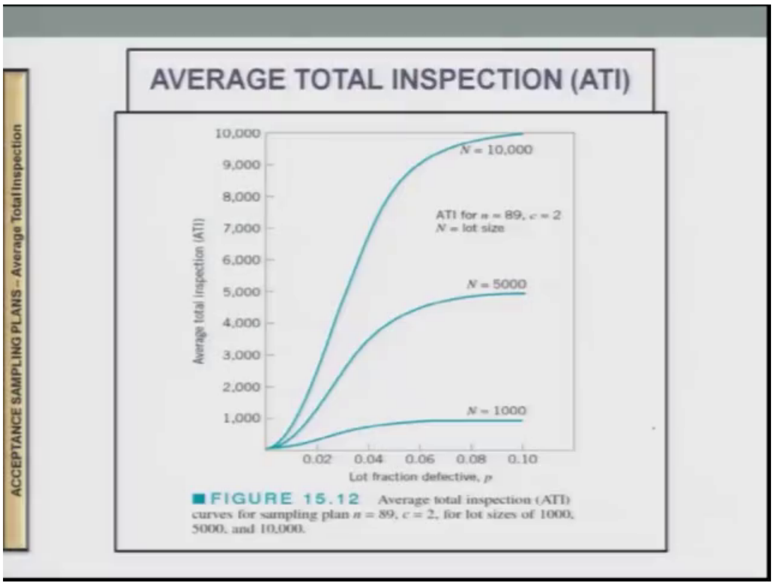

So just will I thought I will mention you the examples which would be there for either the problem or you and when you are trying to read the book though would be quite exhaustive from the point of the book. The book covers a huge amount of concept and absolutely very nicely done, but the examples which we will consider for this TQM would be based on the slides only and they would be a little bit more conceptual without going to the nitty-gritty that how those problems have been solved from the from the textbook sort of point of view.

So, the average total inspection ATI basically been measured along the Y axis and the lot fraction defective as we have been doing for the OC curves are being measured along with the X axis. And as the capital N value changes; obviously, small n changes c changes. So, keeping small n and c fixed you will basically have different curves as shown in the plot.

Now, the question is that as capital N is changing, small n is changing and c is changing; obviously, we will have different scenarios enacted in front of you. So, that you have to be careful in order to understand, how the total, the average total inspection or the ATI would basically look like depending on the change of capital N, change of small n and change of c.

So, this figure 15.12 gives you the average total inspection for the curves giving the fact, given the fact that small n is 89, c is to for lot size is depending on how you have taken them. So, the lot sizes in 1 case is 5000, another case is 1000, another case is basically 10000.

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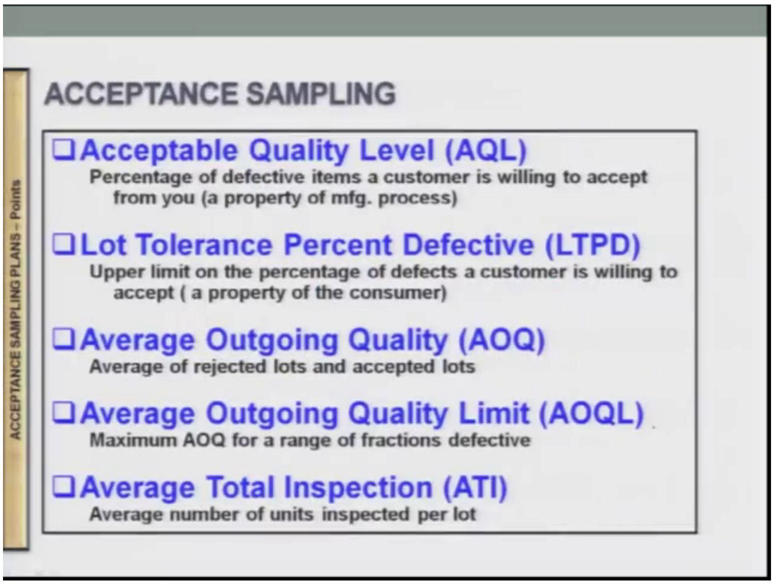

Acceptance sampling basically the important points is the AQL level is the percentage of defective items a customer is willing to accept from you, a property of manufacturing process which is basically a sort of error. So, as he as he as I mentioned these are basically alpha and beta errors which you also encounter in hypothesis testing. Lot torrents percent defective which is the LTPD is the upper limit on the percentage of defector customers willing to accept which is basically also would have a consequence or or the information based on the alpha and beta value.

The average outgoing quality AQ AOQ is the average of rejected lots and accepted lots. And similarly, you have basically AOQ and ATI which gives you the maximum AOQ value is the peak happens for a range of fraction defectives as mentioned and the average number of units inspected for a lot would be given by the concept of ATI.

Now, you have to basically design a sampling plan and we will very simply consider it the single sampling plan only. We won't be going to the complications out double sampling plan, sequential sampling plan and all these things

So, operationally 3 values need to be determined before a sampling plan can be implemented.

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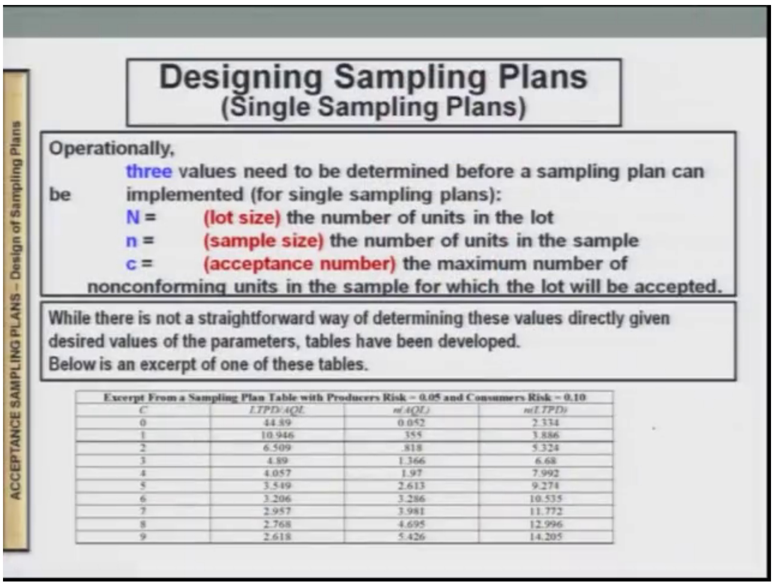

Excerpt From a Sampling Plan Table with Producers Risk = 0.05 and Consumers Risk = 0.10			
C	LTPD/AQL	n(AQL)	n(LTPD)
0	44.89	0.052	2.334
1	10.946	.355	3.886
2	6.509	.818	5.324
3	4.89	1.366	6.68
4	4.057	1.97	7.992
5	3.549	2.613	9.274
6	3.206	3.286	10.535
7	2.957	3.981	11.772
8	2.768	4.695	12.996
9	2.618	5.426	14.205

And is if you remember I have been k i I have kept mentioning the concept of capital n small n and c repeatedly. So, these as you see are turning out to be the important factor based on which you can find out and design the sampling plan. So, capital N is the lot size the number of units in the total lot which you want to actually inspect given time and money is is not an issue. Small n is the sample size the number of units is in the sample which you pick up from the lot and c is the accepted numbers for that particular lot depending on the value of n which you have.

So obviously the ratio or the probability of defects which you have from the overall lot would also have some implications what is the value of c for the sample which you are going to pick up to n to test that lot. While there is not a no straightforward way of determining these values directly given below are the desired values of the parameters and the tables have been developed accordingly.

So, basically gives you 4 columns in column 1 is basically the value of c if you remember the number of defects another value you have basically is the ratio of LTPD and of AQL. So, AQL so obviously, you can have 2 different columns also to find out the LTPD and a AQL values which are technically gives you some information about the alpha and beta or the producer risk and the consumer risk. Do you have n which technically is the a AQL value and you have basically the for those values of of n you have basically the LTPD value.

Now, only important thing to remember is basically on the second column you are trying to measure the ratio of the LTPD then a AQL value based on which you will take a decision accordingly. So, as c changes from 0 to 9, these values are LTPD to AQL or sing or on a single column which is the third one and the fourth one you have the AQL and the LTPD values as noted down.

Now, I have been mentioning if you remember the word alpha beta time and again. So, let us come back to this concept again.

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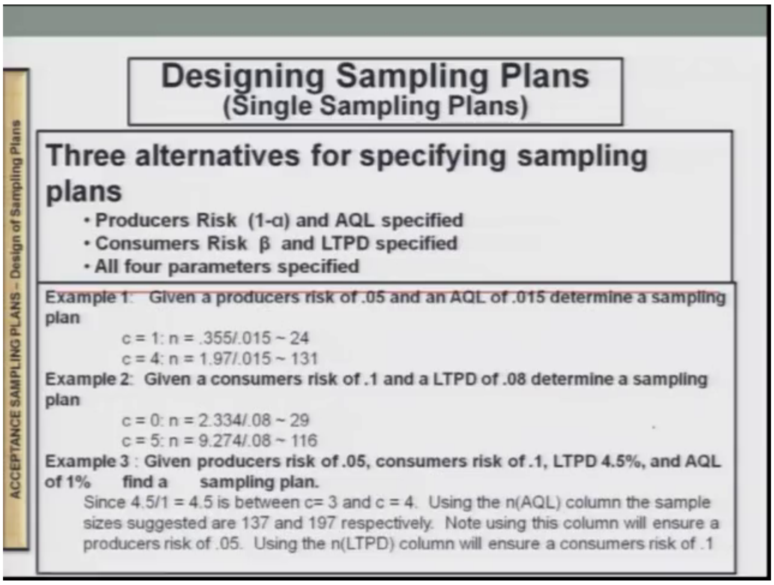

So, 3 alternatives for specifying the sample plans are the producers risk which is basically 1 minus alpha or the AQL specified values. The consumer risk which is beta which is the LTPD value and all the 4 parameters need to be specified, 4 parameters are if you remember capital N, small n c and d.

So, in this example given a producer’s risk of 0.05 and a AQL value of 0.1, 0.15 you want to find out or determine the sampling plan. So, the son though from the tables and the informations you will have c is 1, the n value would be calculated depending on the ratio of the values of AQL and LTPD which use already mentioned from there you go down the table find out the c value and then go horizontal to find out the values of AQL and the LTPD values from where you can find it out.

So, for that you find out the n value coming out to be one case about 24; that means, for the whole lot you will pick up a sample of 24 given that c value is 1. In case is c is 4 when you have to basically find out the value of n which comes out for the lot as 131. And for the second example given a consumer risk, so initially it was the producer risk. Now it is a consumer risk and the LTPD value of 0.08, we want to need to determine the sampling plan. Again you use the same formula find out the values of n as the ratios and those values comes out to be 29 and 116 depending on the value of c as 0 and 5.

In example c; given producer risk of 0.05 consumer’s risk of 0.1 and LTPD of 4.5 and the equivalent of 1 percent we want to find out what is the sampling plan. Again, we use the formulas which are given and you can find out the values of of n accordingly. So, here I am going a little bit fast, but trust me if you read if you go through the slides read the books the overall general plan of how you solve the problems would be, would be clear.

Another thing which you may be finding in the last 3 or 4 lectures I am going a little bit fast, because the reason is that total quality management is is a very vast subject very interesting subject. So, rather than basically spoon feeding you I would basically try to encourage you to read the topic, understand the slides, ask questions and try to understand the concept of quality control or total quality management or the quality control process tools or the mathematics based on which the overall philosophy of quality works.

Now, till now we have been considering the single sampling plan only. That means pick up only test that and basically accepts or reject. Now in the second stage before this equation sampling plan comes we have the double sampling plan.

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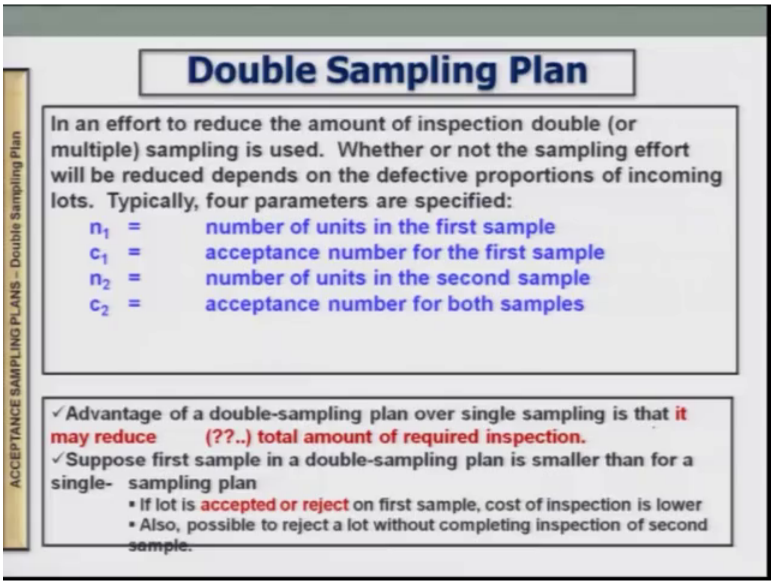

In an effort to reduce the amount of inspection or double or multiple whatever it is we basically do a sampling where we pick up on a set of observation for the first sample which is given as n 1 inspect it. If that is given a c 1 and if d 1 is less than c 1, we you basic and accept it, if d 1 is later the greater than c 1, you reject it

But there may be some instances that the values of d would be between some specified limit. So, so, so typically the four parameters based on which you will try to do the studies. Obviously, d 1 and d 2 are the number of defects which are happening in the first sample and in the second example depending on how you have picked up these observations from the lot capital N. n 1 is the number of units in the first sample and the acceptance number in the first sample is corresponding number is c 1, n 2 is the number of units in the second sample and the corresponding and acceptance number of of both the ones is given a c 2.

So obviously, the accepted number in the in the second lot would be c 2 minus c 1. So, the distinctions have been can be made accordingly where you can only specify the number of defects in each lot or you can basically see the number of defects or allowable in the second lot when you have already picked up the first lot would be c 2, where c 2 would be the sum of the number of defects. You are getting in the first first picking and the second picking if you do it in the third case if you have c three then c three would be the sum of all the defects which you are getting in stage 1, stage 2 and stage 3.

So, advantage is a double sampling plan over single sampling is that it may reduce which do not know, but in technically in equation sampling plan where do you go in different multi stages methodologies, the total amount of record for inspection is low which would basically have a positive effect on the total cost structure.

Suppose first sample in a double sampling plan is smaller than for the single sampling plan, so obviously it would mean that overall cost structure is being affected. But obviously, that may compromise on the overall total quality. So, if lot is accepted rejected on the first sample cost of inspection is lower also possible to reject a lot without completing the inspection for the second sampling may be there.

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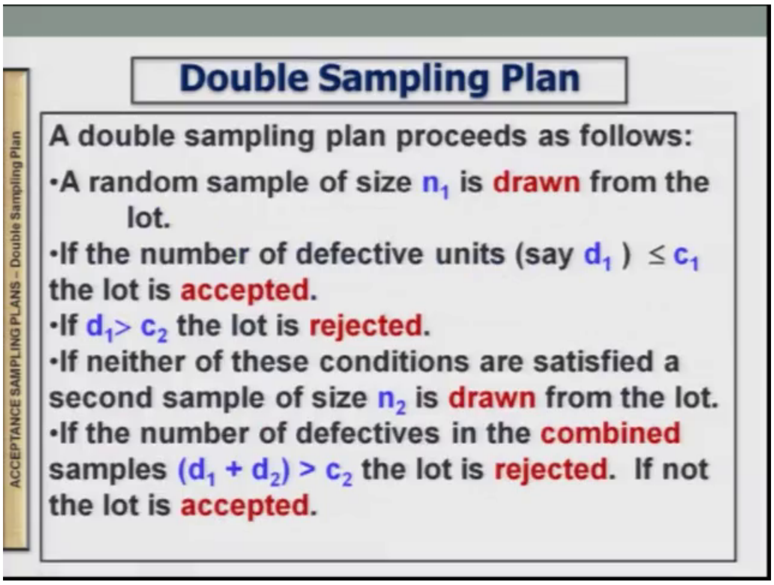

So, double sampling plan is basically what I mentioned in words comes out like this. A double sampling plan proceeds as follows. So, then the first step a random sample of size n 1 is drawn from the lot. If the number of defects which is d 1 for the first lot is less than equals to c 1, the knot is accepted in case c. For example, d 2 is greater than c 2 then obviously the lot is rejected, c 2 is basically the total sum of of defects which will have in the to picking we are doing.

Now, if neither of these conditions are satisfied; obviously, because c 2 is greater than c one. So, in case when we pick up d 1, d 1 values may be greater than c 1, but less than c 2. So, and then what do you do is neither of these conditions are satisfied a second sample of size n 2 is drawn from the lot. If the number of defects in the in the combined sample which is d 1 and d 2 is greater than c 2 it is rejected if not the lot is accepted. But in many of the cases it may be possible that the sum of d 1 and d 2 if they are less than c 2 then other some decisions has to be taken accordingly.

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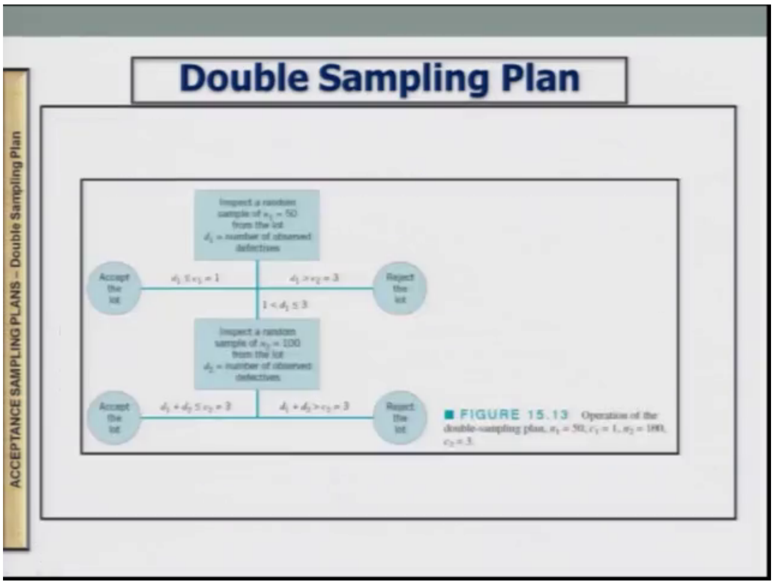

So, as given in this figure 15, 15.13 again taken from Montgomery is inspected random sample of size n 1 from the lot is d 1 is the number of defects and have some value of c and 1 and c 2 already specified. So, if you lick look into this these curves or the flow process the operation on the double sampling plan is for this case given as n 1 is 50, c 1 is 2. That means, out of this n if you find out n 1 50 as defects as 1 and if they are less than that you accept that if they are more than that and in the second in the in the single sampling you would have rejected that.

But now you are not doing because you are going for the double sampling plan. Then in the second lot do you pick up n 2 observations from the capital n and now the total number of defects is 3; that means, there is a probability of getting to, not the probability the overall number of defects in the second lot is now 2. Now if with this information we see the flow process it basically works it works in this way you pick up on a set of observation which is n 1 from the lot and try to count the number of defects. So, if the number of defects is less than c 1 which is 1, you immediately accept that lot.

Now, if it is greater than 3 you reject that lot. But they may be instances where d 1 is basically between c 1 and c 2 which is basically between 1 and 3. So, this is what is mentioned at that level where I am pointing my fingers or let me make it more explicit, where I am basically surfing this. So, if d 1 between c 1 and 3; so, you basically inspector the second law of the second sample of 100 observations from that lot if d 2 is the number of defects which have been observed and then what you do is that find out the sum of d 1 and d 2. If d 1 and d 2 is greater than c 1, c 2 you basically reject that lot if it is less than equal to 3, basically except that lot.

So, in this sequence you go from can go from stage 1 to stage 2, stage 2 to stage 3. Say for example, you have the triple sampling plan. And there you basically have the sample size n 1, n 2 and n 3, d 1, d 2, d 3 of the number of defects you will get in in the first picking, in the second picking and the third picking. And consider c 1, c 2, c 3 are the numbers which means the c 1 is the number of defects which is allowable in first picking, c 2 is the total number of defects which is allowed by the sum of c 1. And the, and the second stage pickings which you are getting and similarly in that case you will have c 3 which will basically mean the total number of defects combined which is c two plus the number of defects we are going to get in the third sample would be equal to c 3. So obviously, your sequence of movements out of the flow process would be a tempered accordingly or made accordingly.

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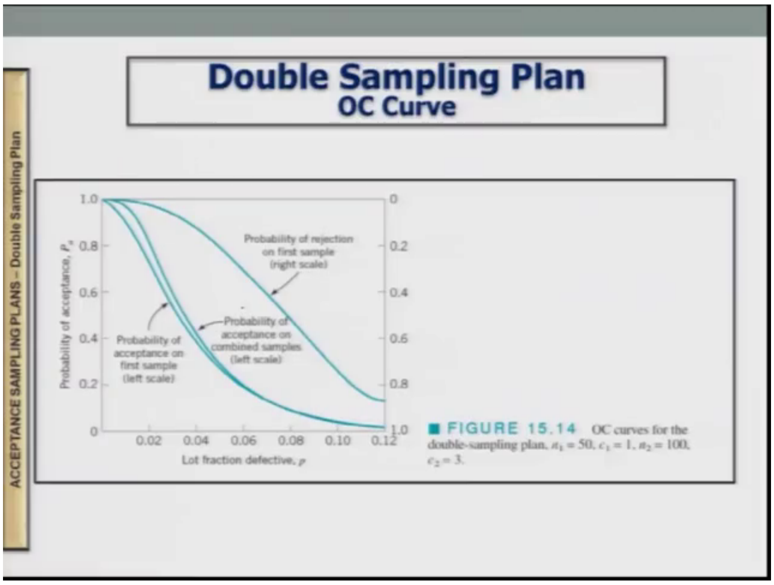

So, the double sampling plan or operating characteristic curve, the double, the triple sampling OC curve should also be dictated according to the values of capital N, small n 1, small n 2, c 1 and c 2 for the case of double sampling. In case if its triple sampling the OC curves would be dictated by this this values, which are small n 1, small n 2, small n

3, capital N then the other values which will dictate OC curves, how they look like shape and size would be basically c 1, c 2 and c 3.

So, again we are measuring the probability of acceptance pa along the Y axis lot fraction defect along the X axis and the probabilities of acceptance depending on on the level of these values which I just mentioned. So, capital N different combinations of n, n i, n i is means I can be 1 for single sampling and 2 for double sampling can be 1, 2, 3 for triple sampling.

So, the probabilities of rejecting of the first sample is given on the on the right-hand scale. So, they are given on the Y axis which where I am pointing my palm. So, then the probability of acceptance of the combined cases are given in the left scale and based on the combinations you proceed to find out that how the lot size inspections will be done based on the algorithm you have which you have already discussed.

So operator, OC curves for the double samplings; for one example is given where you measure pa again also along the Y axis, p along the X axis.

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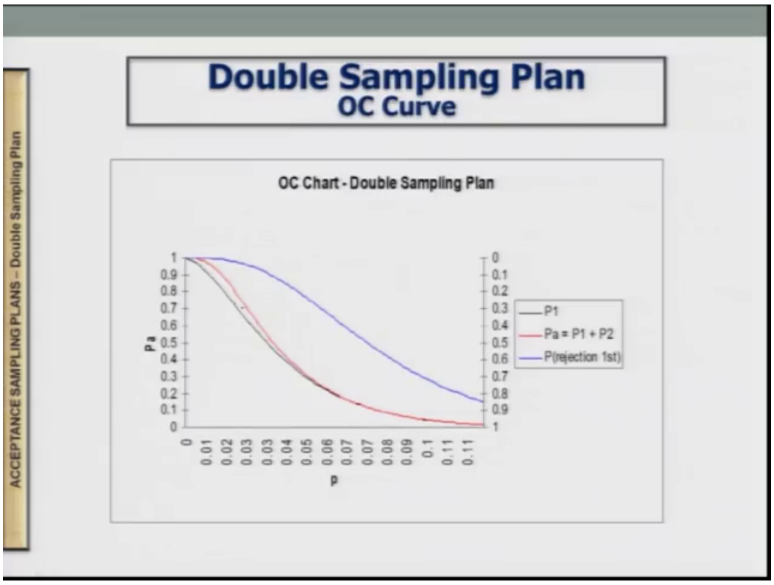

And then the first stage of the second stage probabilities are given on the on the right-hand side. So, in in one case you have basically the OC curves when you are doing you have done the first level then another case would be the combination of the first level. So, based on that you will have different values of producer risk and consumer risk and you can proceed accordingly to find out what is the total cost of defects which are being accepted or the number of good items which are being rejected.

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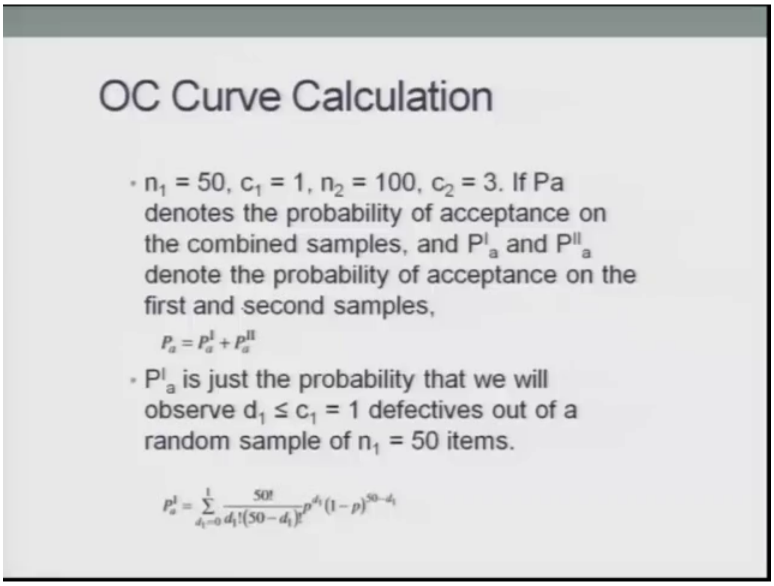

So, OC curve calculations depending on the binomial distribution. I am mentioning the word binomial distribution because either you accept or reject that particular lot which is there in front of you. So, n 1 values is 50, c 1 as you remember is 1, n 2 is 100, n 3 is, c 2 is 3. So, if p a p suffix a denote the probability of acceptance on the combined scale and p stage 1 suffix a and p stage 2 suffix a denote the probability of acceptance on the first and the second sample, obviously it will mean the probability is basically sum of p suffix 1suffix suffix a plus p suffix 2 suffix a; so, based on that you basically find out.

So, I should be word using the word prefix of 1 and 2 rather than suffix, sorry for that. So, P’s prefix 1 a suffix a is just the probability that you will observe that d 1 is less than equal to c 1 which is d. That means, because in the first order when you pick up your already specified that c 1 should be the number of defects. But if the d 1 is less than c 1; obviously, we will accept that lot.

So, out of a sample observations of 50 which is given by n 1 and based on that you basically find out the probabilities sum of the probabilities so. So, now, the some of the properties d 1 can be either 0 or 1. So, if c 1 is basically 2, then d 1 values can be this 0, 1, 2. So obviously, you have to add them accordingly though then in in the first lot again is a binomial case because you had basically n as 50 which is the total sample size and then and the number of defects and non-defects would basically be either d which is the effective one and the non-defective one would be 50 minus d 1 because 50 is the total sample size which is have in front of you.

So, that would be multiplied that would basically be the values based on which you will multiply the properties of p 1 and 1 of p and 1 minus p which is the level of acceptance or rejection which you already have now.

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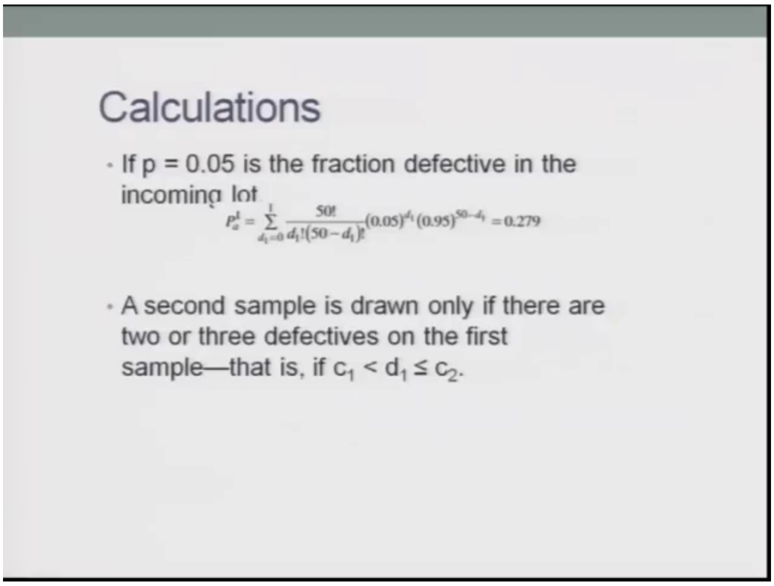

In case if a small p is given as 0.05 which is 5 percent ag and that is given with the fraction or defective in the un incoming lot, then you obviously put those values and find out the value comes out to be about 27.8 which is 28 percentage.

A second sample is drawn again if d 1 is between c 1 and c 1 and c 2 then obviously, you will like you will basically accept it. In case if say for example, d 1 plus d 2 is more than c 2, you will reject it. So, a second sample is drawn only if there are 2 or 3 defects on the first sample and that depending on how the example has been formulated. Based on the values of small n 1, small n 1, capital N, small c 1, small c 2, small d 1 and d 2 would basically dictate that what the values are.

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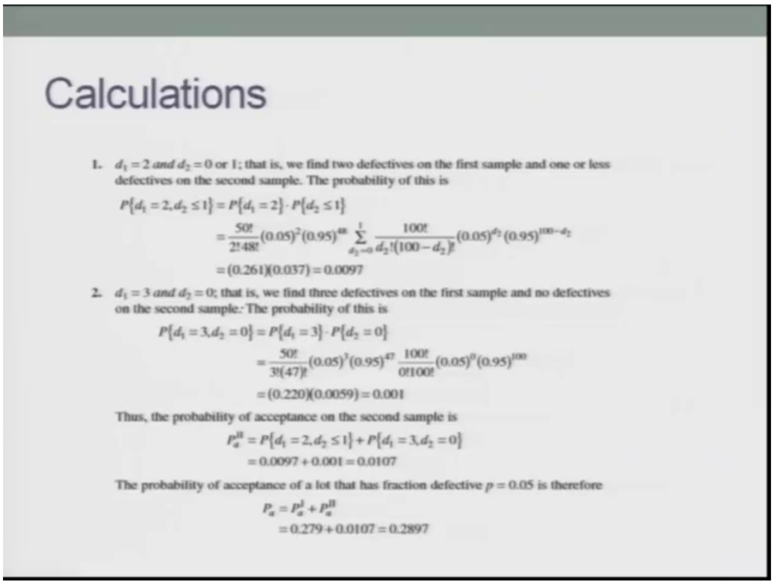

So, calculations would be done accordingly. I will just go through the steps in general. I will again illustrate that later on and then and in the class following that. So, d 1 is given by 2 and d 2 is given can d 2 can be either be 0 and 1. So, if d 1 is 2; obviously, you to it would mean that in the total combination is basically c 2 which is already fixed. So, which means that d 2 in the next d 2 which is in the number of defects in the next lot would be 0 or 1 because 0 means this value of d 1 and d 2 is less than equal to 3. So, if this base is fine we will accept that or basically put to consider that as a as a good lot in case see for example, d 2 is 1 still 2 plus 1 is 3 which is less than equal to the value of of c 2 which we have and we will basically accept that particular lot. But in case say for example, d 2 is 2 and more than the sum of d 1 and d 2 is more than c 2, obviously we will reject that.

So, with that concept let us proceed. So, d 1 is 2 and d 2 is 0 or 1 that is we find 2 defects in the first sample and one or less in this defect in the second sample then the probability would be found accordingly. So, this would be a multiplication probability because they are independent of each other. So, what we find in the first lot would basically definitely have an consequence on the decision, but the calculations would be independent because the probability of one lot doesn't affect the probability of the second not and vice versa.

So, you find out the probabilities. So, the probabilities in one case would be that you have already 50-50 remember is n 1 based on that you find out because d 2, d 1 is specified at 2. So, that number is fixed. So, one this number is fixed this calculation of this is not a summation is basically a fixed value because this 50 is the total size n n 1, 2 is the value of d 1 which you have and based on the probabilities of the values which is already given as 0.05 and 1 minus 0.05 which is 95 percent. We found out these values in the second stage we sum up. So, sum of summing up are done for what values they are done for d 2 is equal to 0 and d d 2 is equal to 1. So, if we see the limits in this notation. So, this d 2 basically changes from d to 1 as specified and from that you find out the probability comes out to be about 0.97 percent.

In case d 2 1 is 3 and d 2 is zero; obviously, in that case because you have already specified a c 2 as 3. So, in that case if you have again you find out the probabilities multiply them because they are independent once you find out the multiplication those values it comes out to be 0.01 which is about one person about 0.01 yes

Thus, the probability is now in the in the second stage would be in this case a combination in the first case we have a combined combination of see for example, d 1 s 2and an d to d 2 is less than equal to 1. So, and on the other case you will basically have d 1 s 3 and other case. Obviously, d 2 would be 0 because you are already picked up all the bad items accordingly. So, if you calculate the values the probability comes out to be about point 0.29 first value as in the probability sense in the problem in the percentage sense it would be above 29 percent.

So, I will go through these problems one ag once again and the problem set would be considered accordingly. So, have some patients go through the book I am sure will understand the problem set and the concept are the very huge, but once you start reading you will definitely get a hang of the picture accordingly and we are open to suggestions open to queries and please get in touch with us for whatever queries you may have have a nice day.

Thank you very much.