Total Quality Management - I
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Lecture – 16
The Operating Characteristic Curve

A very good morning; good afternoon; good evening, my dear friends; welcome back to the TQM which is total quality management course which under the NPTEL program and as you must know, I have got used to my name, I am Ragunandan Sengupta from the IME department, IIT, Kanpur. So, this is the 16th lecture technically which would be if you follow the principle of 5 lectures per week, it would be the starting of the fourth of week.

We as you remember, we were discussing about the producers risk and the consumer risk. Now to give to illustrate the background in more details I have done that, but I will still like to dwell on that fact much more is that whenever you are supplying any product. Whether you are giving services, trying to basically supply a motor, trying to supply a car, sell, some product whatever it is. They would be basically and obviously, there are some inspections to be done for these products the quality assurance criteria and those norms have to be met. They can be 2 level of risk or 2 different type of risk. So, this risk is basically from 2 different levels. Levels means 2 different players one is basically who is the supplier of that good or that product or the service. And, one is basically the person is buying it so; obviously, 1 risk or if risk, I am using the word to make it very simply some sort of loss or some sort of negative impact, it has depending on the outcome.

If it is a risk for one, obviously it will be on the other side it would be a positive one for the other player which is if it is risk for the vend for the supplier, it would be the positive 1 for the buyer and vice versa. So, one would be the consumer risk and one is the producer risk. Technically the words are what we mean by the concept of alpha and beta. If you go back to the basic concept of hypothesis testing, it will be you are making some product that has some problem, but still is being accepted. Which would be some positive part on your game, on your part, but it would be a definite to be a risk for the other person who is buying it.

Now, if the scenario is turned, in the sense good attempts are there, but they are being rejected, which means that you are facing a problem, you have a risk the other person has a positive part. So, basically this is the overall thing which we would be analyzing time, and again depending on the distribution depending on total sample size, depending on what is the total acceptance level and also depending on what is the total population size.

Now, technically population size should be infinite, but for the hyper geometric and asajj distributions where quality concepts are there, their capital N would also come into the picture. Obviously, capital N is very large with respect to small n, but still it will come into the picture, such that capital N small n and c are apart from alpha and beta which is the producer risk and the consumer is could dictate how the graph looks like.

So, to give you an example as per the concept of acceptor sampling; now, again I will again a step back. Acceptor sampling when you are saying that it means that you picking up some chunk and you are making a decision whether to accept or reject that pact you a lot; now, picking up a set of observations can happen in many ways. If you remember in the last lecture we did mention a word known as sequential analysis, or sequential sampling, or the double level of sampling and so on and so forth. So, technically means that you pick up set of observations, make a decision based on what the outcome is, either you proceed for the second step or try to take another set of observations or basically reject that actual sample or also accept the sample depending on the num c; that means the ratio of c by n would be the relative frequency or the actual probability of such defects in a small sample; whether that is exactly equal to some stipulate value, whether is more than that stipulate value, or whether that less than stats stipulate value based on these three outcomes you will take a decision.

So, these technicalities how you can make if and then concept like if the number of defects are greater than equal to some number of c if they are less than c, of they are equal to c; how you make that decision would basically depend on your level of competence, level of accuracy, the cost structure, the sample size, the population size, what type of distribution you are we are considering all this would come into the picture.

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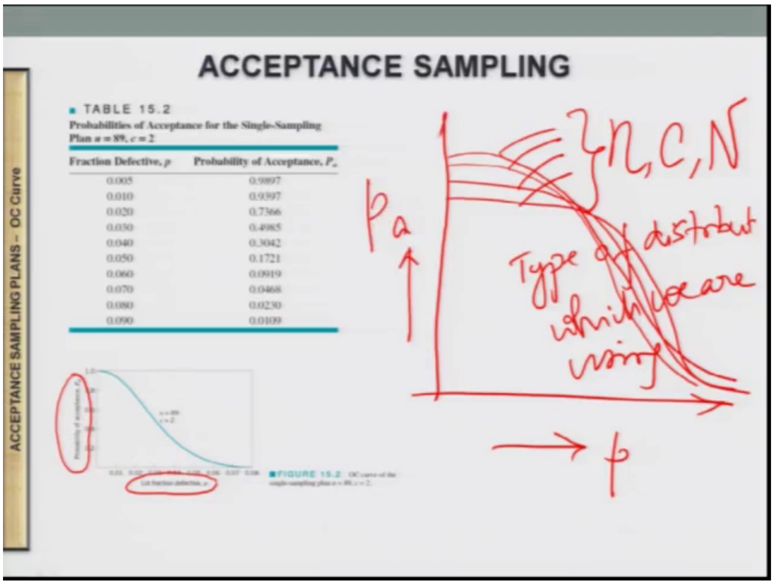

Now, if you have the probabilities of acceptance for the single sampling plan as per tables 15.2 so, let me mention this table has been taken from the Montague Murray book, which you are trying to follow for our TQM 1 course as well it will be followed for our TQM 2 course which will be floated later on.

So, probabilities of acceptance for the single sampling based on the fact that N small n is 89 and c which is the total number of or defects which you can have is 2. So, the overall fractions of defectives are given in the first column and the probability of acceptance now if you remember when you are drawing the m and the AQL curve. So, you will basically have something measured along the x axis and something measured along the y axis. So, those informations as already stated one would be basically with the fraction or defect which is given by p, and what another would be the probability of acceptance which will be given by p suffix a.

As the fractional defects are given the probability acceptance are given so; obviously, it means that as the fraction increases then the probability of acceptance decreases; obviously, that would be true. Because more defects you find less which would be the chance of accepting that particular set of observations of sample.

If we come to the graph which is being shown in this slide, this obviously the graph looks simple and; obviously, you will encounter such graphs later on also, what I want to mention is like this. Now, if you remember let me make the color. If, you pay attention to the y axis you are measuring basically the probability of acceptance which is p suffix a, if you are paying attention to x axis is the lot fraction defective. So, what you actually do is that depending on small n and c, you will have a particular graph. In case is a hypothetical thing which I am trying to draw, in case see for example, you have this y axis as P a x axis as p and you have different n senses. 7:36

So; obviously, your graph sum would be like this, some maybe I am just drawing it in order to make you understand depending on the distribution and capital N small n and see it will change, some of the graphs may be like this. Some of the graphs may be like this and so on and so forth. So, all these graphs which you see would basically be depending on n, on C and also on capital N 0.1 and it will also depend on the type of distribution which we are using. So, this would measure P and this would measure P suffix.

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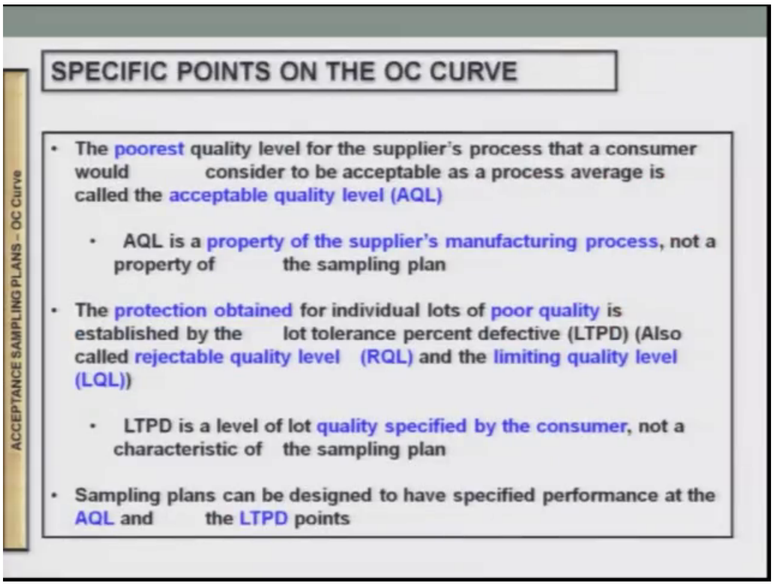

So, the specific points for the in the operating characteristics curve, you will basically have again the LTPD and AQL levels, which technically has some informations of the alpha and beta. So, the OC curves would have the characteristics which are in general true are as follows. The poorest quality level for surprise process that a consumer or a customer would consider to be acceptable as a process average is called the accepted quality level or the AQL level. AQL is a property of the surprise manufacturing process not a property of the sampling point plan. So, generally if the sampling plan is say for example, some capital N some capital M small n and some C so; obviously, it would depend so that would basically keep the curve fixed. But where you basically go to accept or reject a particular lot would depend on what is your manufacturing process as such.

If you finally, visualize the curve remains fixed, but where you basically cut on the horizontal axis and the vertical axis of the values of P and P a would depend on how the manufacturing process is going on. Now going to the second point the protection obtained for individual lots of poor quality is established by the lot tolerance percentage defective, which is LTPD also known as reject able quality level which is RQL or the limiting 1 quality level.

So obviously, it would mean that AQL and LTPD are some concepts which generally go hand in hand that is true, but whether one increases and other decreases vice versa, that has to be studied and if you have understand and the overall concept of the producers risk on the consumer risk, it will come out immediately it will be to be true. That as if you remember that as I started the class and when I was showing the 16 slide and when I began the class, I did mention, the risk or loss for one person is basically the positive gain for the other person. So obviously, that will give in the information, that how does the concept of AQL and LTPD behave which will give you some good idea.

So, LTPD is a level of lot quality specified by the consumer not a characteristic of the sampling plan. Sampling plans can be designed to have specific performance and the AQL and the LTPD points depending on what the manufacturing process capability is.

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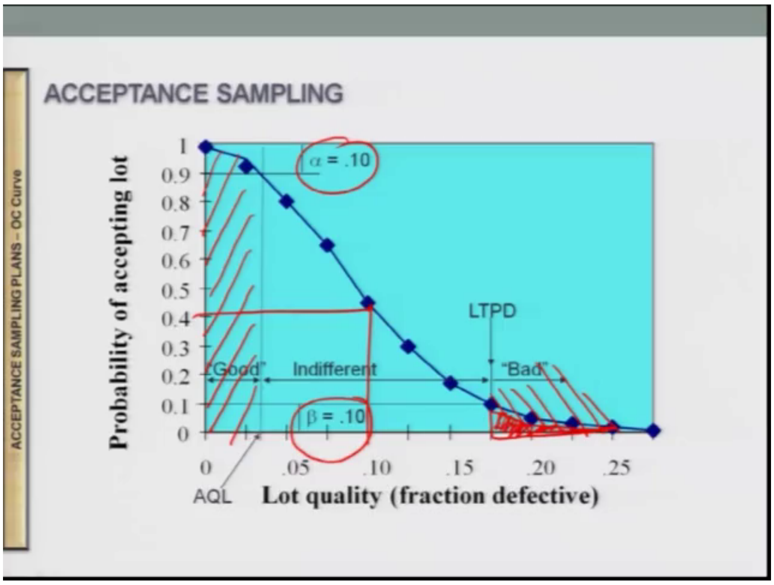

Now, with this graph it will be much easier for me to explain which I have been talking about time and again at the beginning of the class as well as the fag end of around over 10 15 minutes when we were completing the 15th lecture.

Now, if you consider again the x axis and y axis they are same. So, you have basically probability of acceptance lot which is given along the y axis. The lot quality was which is the fraction defect, is given along the x axis and if you consider the graph given. So consider the graph given as specified depending on some value of capital N some value of small n some value of c, and consider the graph is given as it is.

Now, let us consider the details of LTPD and AQL values in more detail. So, let me change the color to say for example, red**.** So, it would be easier, now depending on the manufacturing capability, you basically specified some value of alpha and depending on the other end of the it depending on the consumer is can produce a risk you have also specified some beta value.

Now, what you want to do is that, if you are between these 2 ranges whether below or above would depend on which side of the story you are looking at, you will basically accept or reject that lot and; obviously, that would have a positive or a negative consequence for the consumer and the producer. If it is basically lost for me it is again for you. So, in this region where I am now hashing, these are all goods, good things, goods needs, not the product good means in the characteristics sense is a good one and all the things which are there on the right are basically bad. So, technically I would consider only this I did draw the hashed lines above that also, but technically this is the area, which is bad.

Now, what you technically do is that depending on the n c and capital N value, you basically have some lot quality fraction and defect or the probability acceptance whatever is given, you go vertically up find out the point and find out say for example, what is the level of the probability of accepting that particular lot depending on the lot quality fraction of defects which are there. If we considered on alpha and beta again I am mentioning, they are the risk from the producer and consumer which technically means that if products are falling in this range on to the left or in this range see for example, on to the right. So, there may be chances that a good quality product is being rejected or a bad quality product is being accepted. So, that has to be basically understood in such a sequence, that alpha and beta value would basically dictate to what level good and bad items would definitely be rejected or accepted, depending on from which side of the story you are looking at.

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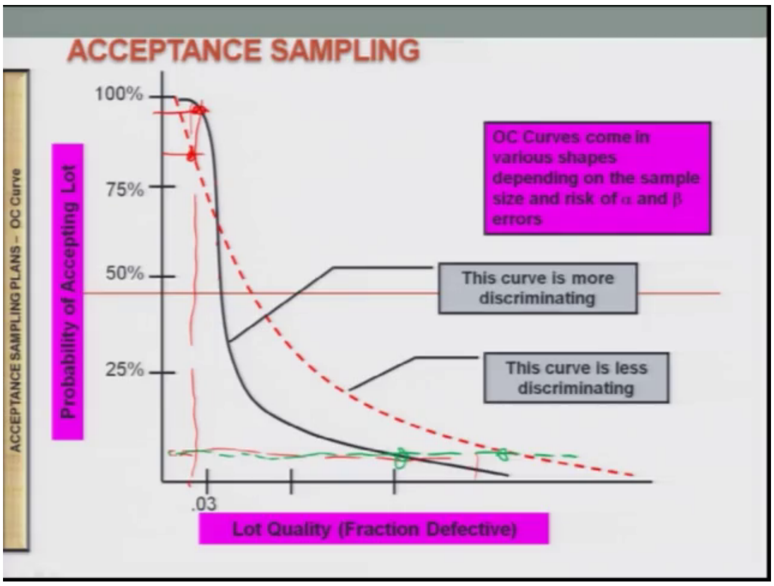

So, to explain it further again the same graph, but basically the actual example which is being given is not with the data, but I am only trying to specify the overall coverage areas of the different portions of the graph. As per the same norm we are measuring probability of acceptance lot along the y axis, we have measuring the lot quality fraction defective now along the x axis and the curve which is shown here, which is the black 1 where I am hovering my pen now, I am not drawing it, is basically the quality characteristic curve. And another curve is which is the red 1 which is dotted is shown like this. So,15.19; there are 2 theoretical graphs which have been drawn so, actually they would depend on the values of capital N small n c and the distributions which you have.

So, it mentioned that OC curves come in various shapes, depending on the sample size and the level of risk of alpha and beta, alpha and beta which have been talking about, so, in this case if you see the black 1. So, there would be some if you draw and the alpha value is somewhere here, beta value somewhere here, if you draw the vertical. So, the vertical point basically cuts these 2 graphs at, 2 different places the black 1 which is more discriminating is basically being cut by the vertical line, at a certain point where the probability of accepting that lot is totally different from the case, if where the actual vertical line basically intersects or cuts the red line.

Similarly, when you are going onto the right hand side, and trying to find out the values of defects based on the value of beta so obviously, the intersection or the horizontal red line which I am, now drawn let me change the color it will be much better for us to discriminate. So, where they intersect the black 1 and the red 1 the level of lot quality fraction and defect and the corresponding probability acceptance would definitely be much more discriminating for the black 1, because there are levels of beta and alpha even if they are fixed, or even if they are different. They would be higher propensity of accepting bad lots or they would be higher propensity of rejecting good lots.

So obviously, the graphs would should would make, you available that whether they are too discriminating or less discriminating, as for the general concept. Now the OC curves calculations have to be done, but technically in any statistical process control book. We will have the details of the calculation of the OC curves given based on the values of capital N small n and seen, but will try to basically be kept that and try to understand how they can be utilized.

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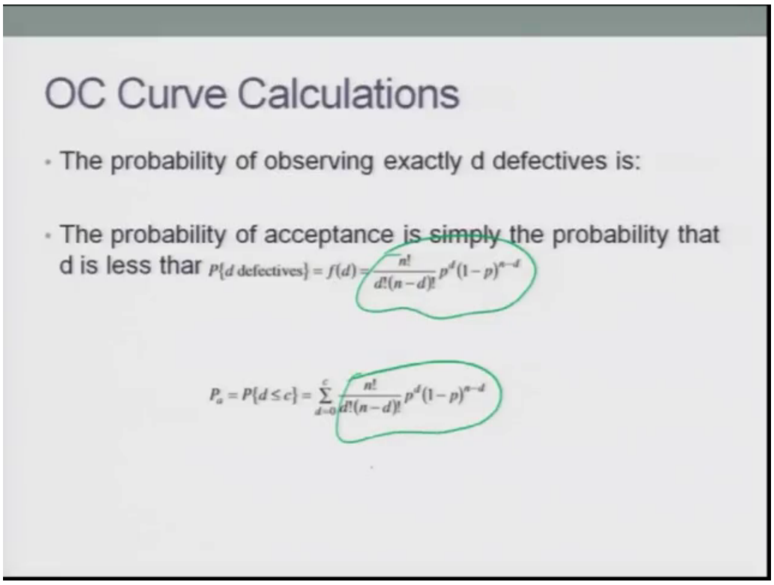

The OC curve calculations in this case is the probability of observing exactly d defects are s to be mentioned. So, then the probability of accepting is simply the probability that d is less than some probability of defects which is given. So, what I need to find out; if you pay attention to this, the that probability distribution function, which is the PMF because PMF would be the case for the discrete case, so there would be 2 case 2 slots 1 is defective lots and 1 is non defective lots. So, the total lot we are picking up is n so; obviously, the total number of defective and non defective should be such, that it will give you a picture that in what different combinations you have those lots. So, 1 would be deep factorial d, 1 would be n minus factorial d.

Because you are basically dividing to do 2 lots, so d that d can be any 1 out of those lots which you are picking up and obviously, the probabilities are picking up considering the picking of the lot or, good or, tipping up the observations as good as P, then obviously, 1 minus P would be the probability of picking up the bad item, or vice versa depending on; however, we here basically explained the overall problem.

So, P can be good or P can be bad. If P is good then 1 minus P would be bad or vice versa depending as I said, that how the framework of the problem has been done. In this case if I want to find out the number of defects d being less than equal to c or greater than c and depending on that, whether we will basically accept or reject. If d is less than c, obviously, if you have made the plan you will accept that particular lot, if d is greater than c you will basically reject that lot. In this case what we need to find out is the total probability of the sum of the probabilities. So, depending on the distribution function which is there in front of you will find out all the values of these. So, these are the number of defects. The defects are 1 and if c are basically 3. You will find out the probability for all the number of defects which are happening once.

If defect is 2, 2 in number so, obviously you will basically except they are not whether d is 1 or 2 so obviously, we will sum up all the probabilities for the values of d, as 1 or 2 in case if for example, if c is 5 and that number of defects is four so; obviously, it would mean that you find out the probabilities for all values of d, when d is 0 when d is 1, when d is 2, when d is 3 and d is 4 because then in that on all these instances, you are accepting that not say hence you have to find out the probability depending on the sum of all the different type of scenarios, which are there in front of you. For example, if the lord fraction defective is given as P is 0.1, then the lord fraction are not defective would be 1 minus P. Now, if n is given as 89 c is 2 then; obviously, if d is 0, you will accept d is 1, you will accept and d is 2 also you will accept, but in the case if d is more than two, or you will reject that.

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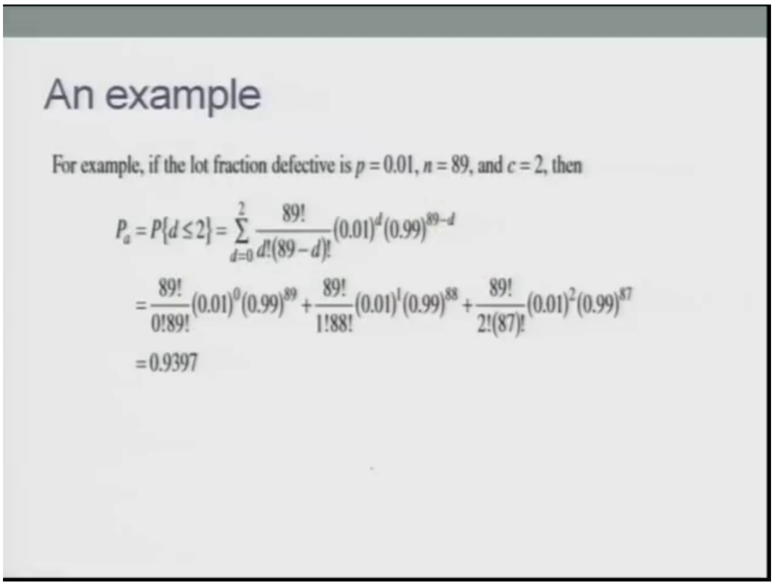

So, if you are trying to find out the probability P a. It will be given by the summation on all the properties that value comes out to be about 93 for 97 which means if the P a value which is the overall concept of how you will accept that lot, would be given that out of a lot of fun of a size of hundred, or doing that experiment 100 number of times they would be a propensity that you accept it about 94 number of times, 93.97 is almost 94 that is why, I said 94 number of times.

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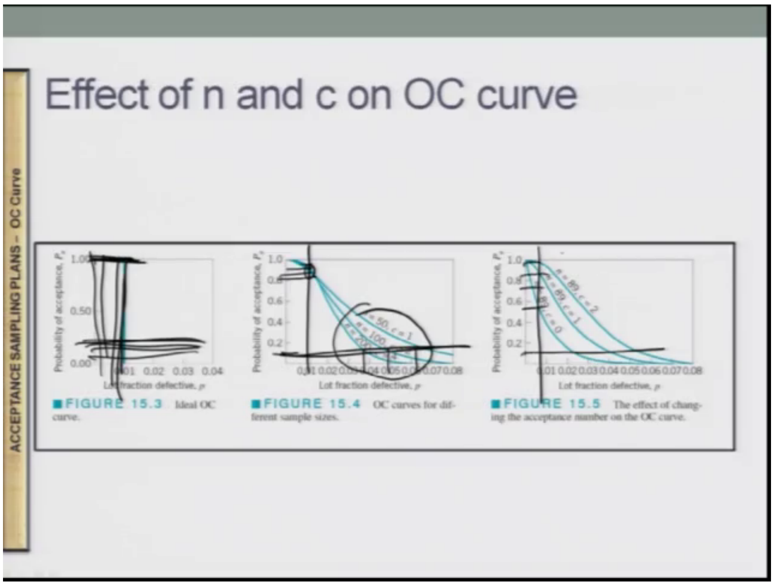

Now, as I mentioned that the operating cursive curve depends on capital N small n see the type of distribution so on and so, forth. So, here in this figures which are figure 15.3, 15.4 and 15.5 which all have been taken from the book by Montgomery you will understand that the how the overall discussion changes.

If you consider the ideal operating characteristic curve, it would basically be a vertical line. So, vertical line which I mean is basically depending on the so, the vertical line gives you the case that what is the value of alpha and beta, that can be found out. So, if alpha and beta are given from here you will know that the lot size acceptance and lot side defects dependent alpha and values, and beta value would be accordingly.

So; obviously, it would mean that wherever the values of verticals are, they were always cut their intersect, but on the horizontal line at a particular point. So, this probability of acceptance of dejection, space given the same again if you are going in the in the horizontal direction they will basically give you the value or depending on what the beta value is, it will give a you will understand. And accept and depend and rejection would be at the same level.

Now, when we are considering the operating characteristic curve for difference sample size; so, now, what we are doing is that, we have limited or made fixed the sample size small n and c are changing as they keep changing as I drawn they are about 3 or 4 slides away. You if you measure P a along y axis P along x axis. So, these values of n and c would give you different well types of curves. If you want to basically find out given a beta value what is the level of c P and P a we will again draw the horizontal and basically, find out the inter intersection point based on which you will find out p. And if you hope, if you are interested more from with a given value of alpha, you will basically go vertical up where they come here because here, they are too much concentrated so, you cannot basically see them, but they would be different levels of P a values which will give you the probability of acceptance. And based on which you can take a decision.

Now, if you basically understand that for a same sample size we are changing the c value, which is the level of accepting. So, c is 10, c is 5, c is 4 again the graphs changes, but the graphs changes characteristic change, the graphs is totally different with respect to the case when you changing both NMC. So, again in this case, if you find out beta and alpha accordingly draw the horizontal, draw the vertical, they give you different level of points, at which the probability of acceptance would be dictated depending on the value of n and different values of c which we have considered for the third graph.

So, again I am mentioning for the 2nd graph which is there we are considering different values of n, different values of c here, any small n in the third graph you are considering kept small n to be fixed, but we are changing the value of c; obviously, it will also happen that if you keep c fixed, and change the value of n you will have different curves also, but the general characteristics of the OC curves would remain the same.

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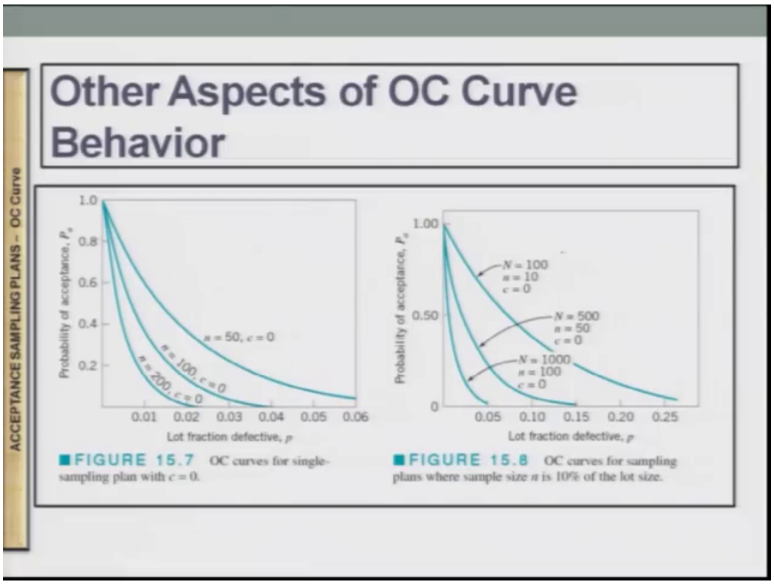

So, other aspects of OC curve behaviors it would be OC curves for single sampling plan which c is equal to 0 that means, you are only sampling one. So, those curves which are given in front of you for different values of n as 1500 and 200 with different values of c are given again you are measuring P a along the y axis and P along the x axis.

Now, let us come to the other type of graphs all are the OC curves, but here what is important to note there is that in many of the examples, which I did mention though fleetingly that the main dictating parameters, based on which the OC curves would be drawn are capital N small n and small c. Till now we have only discussed the sample size small n and the number of defects which are there in this on the sample and based on the number of defects, which is now in actually is d, whether d is less than equal to c or greater than c you accept or reject that particular lot, it may so, happen that the population from where you picking up may not be infinite. Technically if there is infinite; obviously, the problem solved so, but if they are not infinite obviously, that actual value of capital N should also come into the picture, in order to dictate or give you the information. That what are the values of P a, and P which will give you some information about the overall quality concept based on which you are trying to do the sampling plan.

So, lots fraction defective P then probability acceptance P a and all measured along the respective axis which are x and y. And in this curve we see that depending on our fixed value of c our changing value are small n and a changing value a capital N you will basically have different values of OC curve, and again those OC curves can be utilized to a study, the overall sampling plan. Which is there to just to mention these capital N, can also be changed small n, can also be changed, and c can c value can also be changed accordingly.

Now you want to basically concentrate on the concept of rectifying the inspection plan based on whether things are right or wrong.

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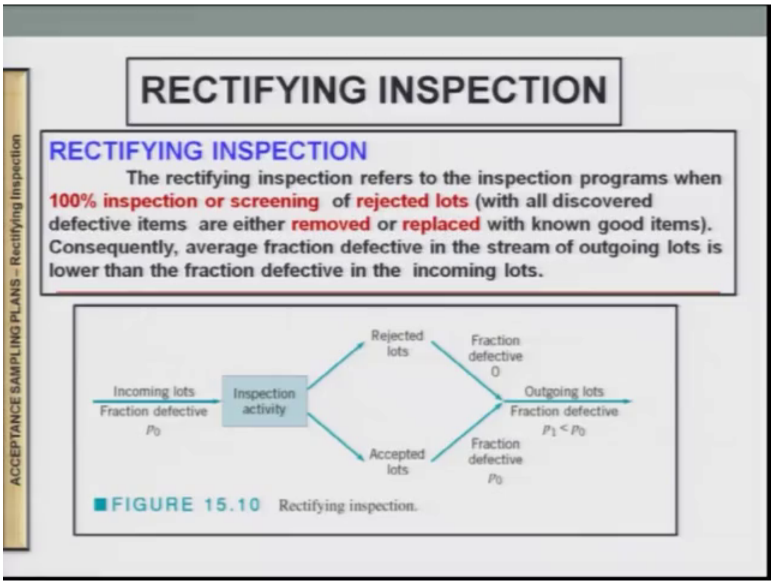

So, the rectifying inspection refers to inspection programs when hundred foreign inspections are screening of the rejected lots, or which are all have been discovered those bad items have been discovered, are either removed or replaced with known good items. Consequently average fraction defect in the stream of outgoing lots is lower than the fraction defective in the common lots because those lots which are bad, I basically replace them, but; obviously, they would be some other some observations also which are bad which I have not been able to replace. So, if that is the case when the not after the replacement of the bad items which have found out, are replaced when they go into the final inspection in the mean along the process; obviously the overall P a value in P value would change because, now the total quantum of such lots is less.

So, what did happens is the incoming lots fraction defect is given as P 0 an inspection takes place if rejected lots are there; obviously, you will basically try to replace them in accepted lots are there, the fraction defect remains continues to be P a and the fraction defective if the rejects and lots are there what you do is replace them, and continue along them. So, outgoing lots fraction defects p 1 would be less than a P 0 and again if you follow this policy, again a different set of such clothes would be there, where you have a P 2 value.

So, P 2 value would definitely be less than P 1, but what is important to note is that all this inspection entails a cost whether it is actually worth trying to spend that amount of money in order to overcome the defect, that would be the main question which will try to answer in the practical situation. So, with this we will I will end the 16th lecture, and start with the 17th lecture continuing the concept of AQN OC curves and so on so, forth have a nice day.

Thank you very much.