Project Management
Prof. Raghunandan Sengupta
Department Of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No # 08
Lecture No # 38
Graphical Evaluation and Review Technique (GERT) IV

Welcome back my dear students and this is the thirty eighth class and we are continuing the discussion of GERT and how it is different from PERT and what are the implications for the exclusive or? Implication for inclusive or? And the networks i know that iam basically going little bit slow trying to discuss many things but the reason is that GERT or Q GERT are interesting topics.

Very rarely used in the man project management concept but there are huge amount of implications that if you are able to implement that to the maximum possible extent with all the implications all the actual concept brought into the practical phase GERT really gives you a very good picture. How it can be solved in a practical way? Considering all the concept of probabilistic network to the maximum possible.

Probabilistic means again iam repeating hence the probability treated to the time and probability related to whether that particular art or whether that particular arc or whether that particular route will be taken. And also if you remember in GERT we also mention time and again the looping concept. So as we were discussing in the last class when we just finished that lecture which is the thirty seventh one about the basic and network concepts. The second point is

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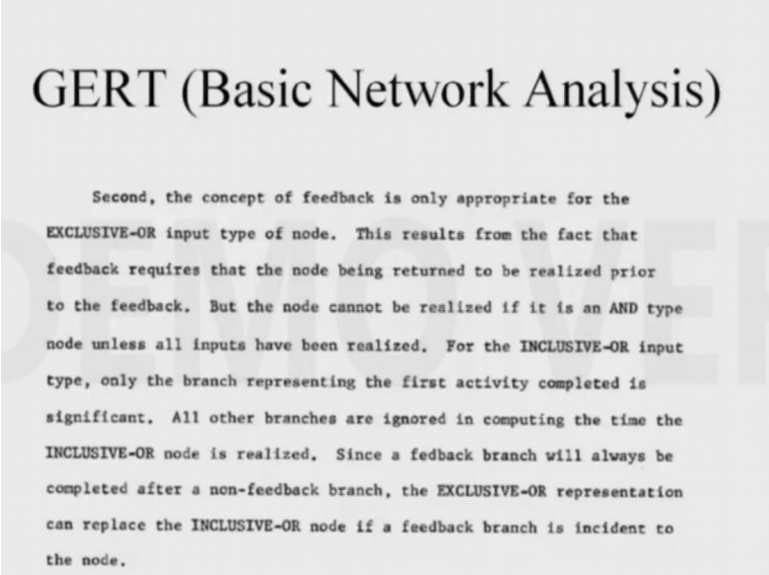

The concept of feedback is only appropriate for the exclusive or input type of node. So the first one if you remember was where it could basically replace a complicated network with a equal lent simple network which i did not mention and tried to basically bring a simile where you are able to replace a decision tree with this equal lent certain equal lent value. So that was may not be very derived but that should definitely give you a essence that how the equallence concept iam trying to bringing into the picture for the GERT concept.

So continuing with the second definition second explanation so this results which i just read this results from the fact that feedback requires that the loop node being returned to the realized prior to the feedback. So obviously looping would be there in order to give the feedback to it is initial stage once again before it starts again but the node cannot be realised if it is an AND type the reason being that if it is basically an exclusive or which means the loop would be coming back time and again.

So as that the looping is taken care off. For the AND one it has to be the if you remember the maximum one or the realization of all of them but if one of them fails the whole process would not work. So in this case the exclusive or would be in a position to take care of the loopings as that the reality or practicality can be brought into the picture. For the inclusive or. So for what we mention was for the exclusive or for the inclusive or input type only the branch.

Representing the first activity completed is significant all the other branches are ignored in computing the time the inclusive or is realized since a feedback branch will always be

completed after a non feedback branch the exclusive or representation can replace the inclusive or if a feedback branch is incident to the node so what actually mean is that when you considering the exclusive or and the inclusive or. If you go back to the truth tables like zero zero.

And how zero zero in one case can lead to an output or the truth statement in the example which i discussed in the example we did talk about during the class so that would basically make a sense in the statement which is just mentioned here. So which basically means that the feedback will always be completed on a after a non feedback loop is there hence the exclusive or representation can be replaced the inclusive or depending on or the whole sequence of events that the activities are scheduled.

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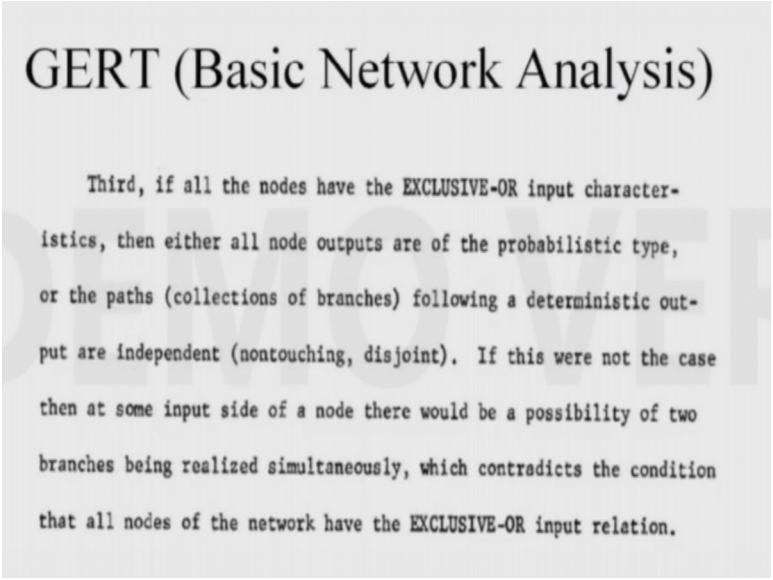

The third important is that if all the nodes have exclusive or input characteristic. So inputs iam only consider are the ex or or the exclusive or then wither all node outputs are probabilistic time or the paths that is the collection of branches following a deterministic output are independent that means they are not touching and the disjoint. So this will become clear as we proceed and we solve trying to discuss problems in details.

If this were not the case then some input side of the node there would be possibility of two branches being realized. Simultaneously which may not at all possible in the case of a GERT network that means practically impossible situations can come up if we do not consider the third point which we are just discussing and this would contradict the condition that all nodes of the network from EXCLUSIVE OR input.

So what it means that it would consider the exclusive or input characteristics based on that if you proceed and do not consider those assumptions which were just mentioned. So the end result which we get would basically contradict the truth statement of the exclusive work based on which you have proceed. So obviously it would mean that condition based on which we are trying to achieve the exclusive work would definitely be right. So it is basically you are trying to prove it to the negation concept.

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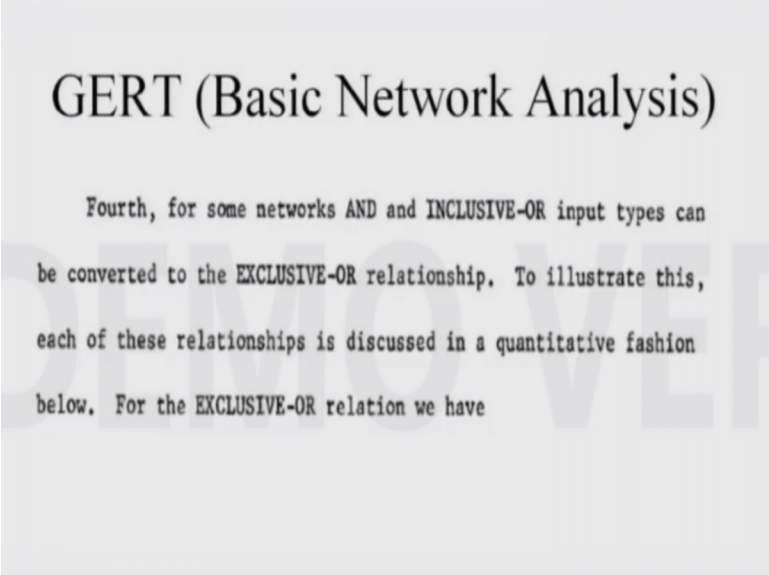

The forth characteristic is forth one states that for some network AND and INCLUSIVE OR. So we are always coming back to the EXCLUSIVE OR INCLUSIVE OR AND statement time and again the input and the output concepts. So the AND and INCLUSIVE OR input types can be converted to the exclusive or in the relationship depending on how you frame the overall logic statement to illustrate this each of the relationship is discussed in a quantitative fashion for the exclusive or relation.

We have this diagram that means the conversion of exclusive or inclusive or and depending on how you are able to basically convert the truth statement will be considered.

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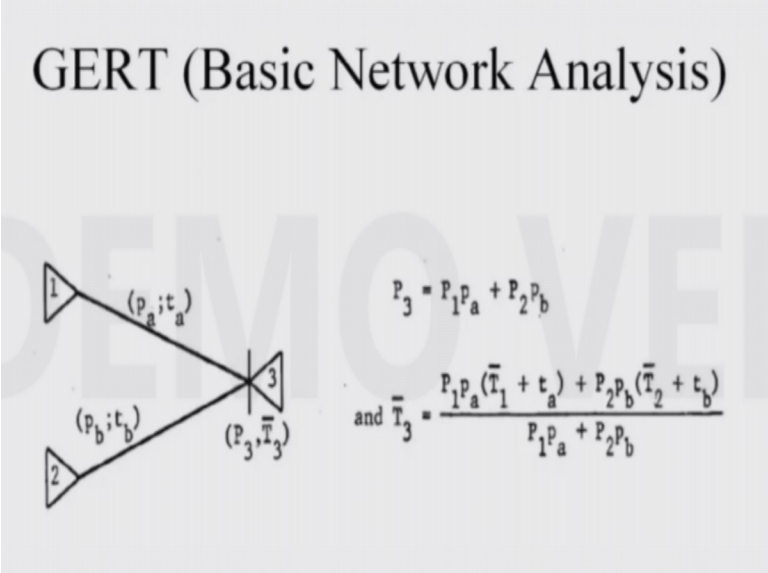

So there you have basically a network. So if you consider a networks the probability P suffix A and T suffix A .Probability P suffix B and T suffix B are given and if you remember these triangle which you have one and two number and the triangle is the mirror image of the triangles which you have on the right hand side with the vertical line it will definitely give in formations of what they are all the combination of six.

We have done exclusive or inclusive or the deterministic one and one corresponding to AND and the probilistic one for the input and the output. So if i consider the overall probability and time which is most important for me why because if you remember the main characteristic based on which we started discussing the GERT was basically mean to at which one was basically time.

One was probability because probability and time would be the while structure that what would be the time taken and what would the probability at those paths would be achieved so for this one when you are considering the overall probability it would basically be a simple multiplication of the concepts of P one into Pa plus P two into Pb which will be meet the probability P three for that particular node and the time would basically be a collective time based on the overall weighted average.

So what are the weights average which iam considering it would be so T one and T two bars are the average times. So the average basically is denoted by bar T three bar T one bar T two bars so and so and this concept of small T suffix A small T suffix B small T suffix C

corresponding to whatever different type of nodes and imports and edges will have so they would basically will note the time .

So based on that we can find out the value of T bar three bar is the average and based on that we will proceed .So now if it is expanded made into three nodes four nodes five nodes or say for example exclusive or being more complicated with many inclusive or and nodes. So they can be done accordingly. So what are these?

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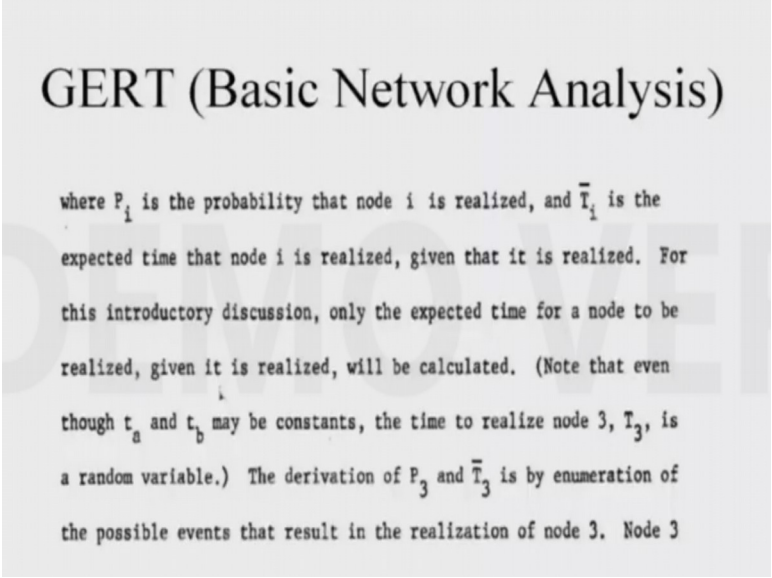

So now i will basically explain once the diagram has been put in two in front of the candidates in the last slide. So these are where P capital P so remember this is the capital P where capital P suffix I is the probability that the node I is realized point one capital T suffix I with the bar is the expected time that node I ids realized. So node one, two, three, four whatever it is it would basically have the corresponding probability and the corresponding given by the suffix.

For this introductory discussion only the expected time for the node can be realized when it is realized it will be calculated note that even now coming back to state even small T suffix a small T suffix B may be constant and the time realize node three. Which is capital T three is a random variable because this times which are given are on an average. Where we basically they would basically be this T bar three would be the average of the realized value for the random variable.

So let me try to basically simplify that T bar three consider or T bar whatever it is considering ever die when you play in the lead game of ludo. So the dies has six phases and consider is an unbiased die the six bases are marked one to six. So before you roll the die the phase which is coming out is totally unknown to you and it is a random variable and let us denote it by capital X as is the notion in probability sense.

Now once the die phase comes out and we note it down and know it. Then it becomes a realized value which now becomes fixed and it is given by the symbol or small X. Capital X is before you start which a random variable when it is realized it becomes a small X. So when you are trying to find out the average of X capital X and we denote it capital X bar it is technically the value of for this example.

Which i just discussed would be one plus two plus three plus four plus five plus six where is the actual sum of all the values which will come out when you roll the die divided by six because six is basically the number of such occurrences are there and if somebody wants to basically have a look at this picture differently it would be one into one by six plus two into one by six plus three into one by six plus four into one by six plus five into one by six plus six into one by six.

Where this one by six are the corresponding probabilities point one. Point number two is that considering that iam trying to basically expand my thought process and trying to bring to the fact that why T three can be or random variable the average value can also even though it is not mentioned here but average value can also be a random variable in the sense consider you have a box and the box has numbers chits marked one to five and if i tell you what is the average?

If i pick up one each of them chit ones note it down and keep it in the box that is iam doing a random sampling this is the concept iam just utilizing for probability for the first time. Do not be bothered too much about the words which iam using try to understand the problem or significance of the problem with respect to what we are discussing. So consider that chit has an average value which is one plus two plus three plus four plus five divided by five because the probability or getting any one chit is always one by five now on the other hand.
Consider i change my set up of the experiment i pick up three chits at each time with replacement. So what it would be one instead if i pick up three chits at one at a time with

replacement it may be possible in one scenario the first it is one second chit is also one the third chit is also one. So one is the average average one plus one plus one divided by three which is one now consider i repeat this experiment which is been done by the same person or different person in the same setup and he or she picks up three chits with replacement.

No problem every setting is same it may be possible and the other extreme the chits which come up out is five five five in that case the average value or that particular sample which we pick up iam using the word sample for the first time but you can check up the simple concept of probability that would be much easier for you to understand sample is basically a small set of the whole set of things which we have.

So in that case the average of sample is five plus five plus five divided by three which is five if i consider this number one which i just mentioned average and number five in the long run they seem to be average but they are random very well themselves because if we keep picking up the observations the average would change. So in the sense the average concepts which is written here we will consider them in to be deterministic but there can be sequences where are the outputs depending on what type of example which you have.

Where the realization has not be realized that means all the values that had not been realized to that extent where all the possibilities have been looked in two. So as that the average can be random point one but remember that the average is on the average in the long run shoot actually be the bar value which is deterministic and just to wrap up the discussion here the averages or the averages whatever you take in the long run should be exactly equal to the population. Population means the whole set of observation.

Which we have for example there were the chits one two three four five collectively so it would basically be equal to the actual population average. So continuing our discussion is that note that even though Ta, Tb may be constants the time realize for T three is the random variable and so can T three bar B but do not be too much bothered about that the derivation of P three and T three is by the enumeration of the possible events or different events or outcomes. You can have that result in the realization of node three. Thus

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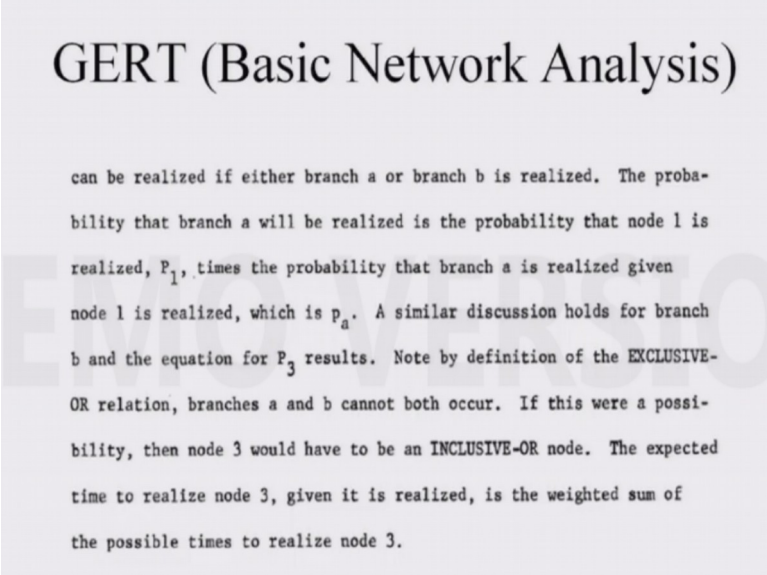

Node three can be realized if either branch A or branch B is realized. So whatever different type of inputs which you have which is in this diagram branch A with probability P suffix A time T suffix A branch B being probability P suffix B and time T suffix B is there. So these are two only two there can be more than two also that would only make things complicated on the calculation form but the concept remains the same.

So the probability that a branch will be realized is the probability that the node one is realized which capital P is suffix one. So this small P and capital P would now become clear to you one is probability along the edge and one is probability of the realization of the node edge means the arc. So node one is realized with capital P suffix one times the probability that branch A is realized which is Pa. So when iam trying to find out the probability it will be multiplication of the probabilities of capital P suffix one multiplied by small P suffix A.

Now if i have two branches it was what capital P. If i go back here capital P suffix Pa then capital P two into Pb and if there were more nodes the calculation would just be repeated accordingly. Where you basically bring the node probability? And the realization probabilities accordingly. So a similar discussion holds for branch B and equation P three P is capital P suffix they would follow note by definition of the EXCLUSIVE OR relationship.

Branch A and B cannot occur together which is basically as per the concept of EXCLUSIVE OR obviously if we change increase in bring into the picture or AND obviously the concept would remain the same the way of calculations will differ. So if this was possibility the node

three would have to be an INCLUSIVE OR node and based on that we will basically do your calculations the expected time to realize node three given.

It is realized that means achieved is the weighted sum of the possible time to realize node three and the weighted sum which we have just saw and which i did not mention very fetidly let me go back here. So i mention the last equation was the average. So it is basically a weighted sum weighted sum what you are taking is the probability of this realization multiplied by that time.

So if you see the numerator you have times that is T one bar plus T suffix A which is the time realization of the node in the arc multiplied by the corresponding probability which is P one into Pa and the second realized values is t bar two plus tb multiplied by corresponding probability which is basically p two into Pb so that is the weighted probability values which you have.

So it would basically be divided by the probability which you have here in front of us which is P one into Pa plus P two into Pb.

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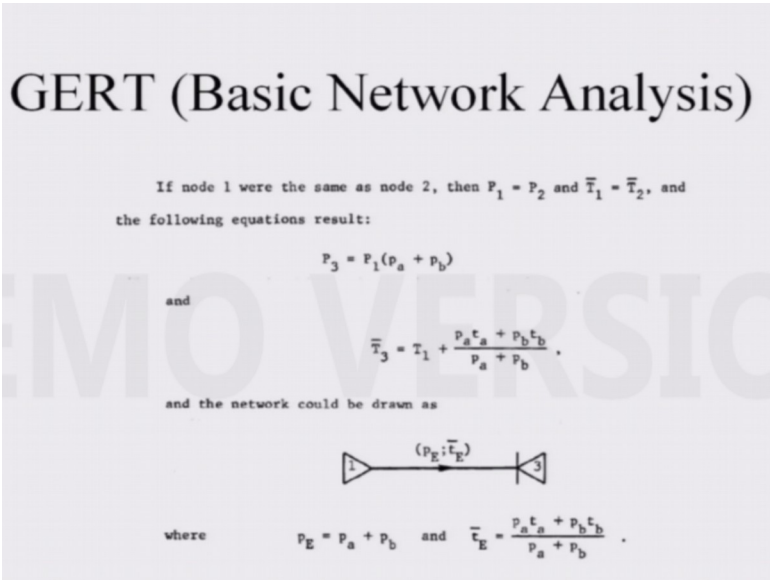

So now continue our discussion if node one were the same as node two then obviously P one that is capital P one suffix P one is will be equal to capital P suffix two and hence the average T one bar and T two bar would be the same. So in that case the following equation would result where you can find out p which is capital P suffix three you can find out capital P bar suffix three and the network can then be drawn as shown here .

Where you will basically have one connecting three the nodes and with an arc or path where the corresponding probability and time would now be replaced by the corresponding probability and time which you have just calculated .So what is P suffix E is sum of Pa plus Pb and the corresponding tie would be given as just calculated. So in this case sorry i will just went very fast into the next slide but i wanted to mention.

If you see that last diagram it gives you nodes are been connected by the arch corresponding probabilities are given and just a simple calculation to find out the average value and the sum of the probabilities. Now consider next the analogical relationship. So we will basically try to go through the logic relationship and try to highlight how the average value can be calculated now the and calculation will be based on the fact that the probabilities and the time would be given accordingly so if you have the values as given.

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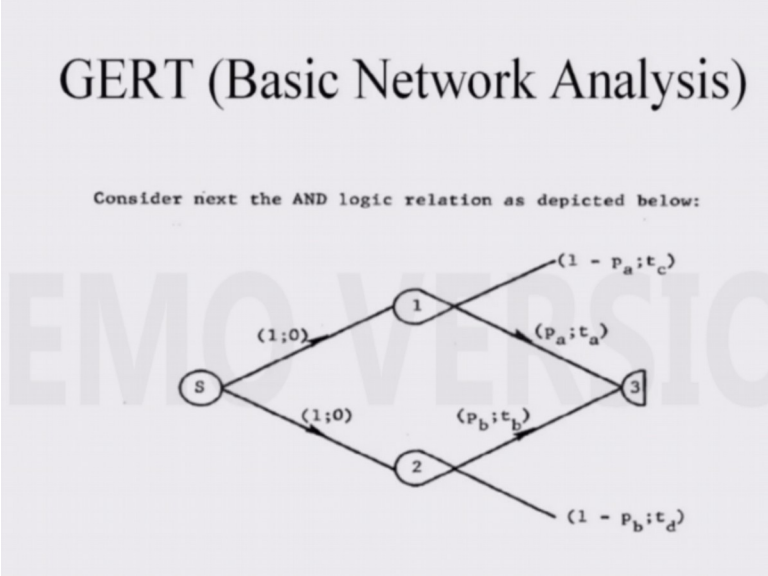

So consider the very simple decision tree the diagram were exactly in the decision tree for the concept what you are trying to utilize in the decision tree whether you take that path whether you do not take the path if you remember the example of drilling. Whether you will drill do some specific test or you will come of with the product in the market that opened one whatever it is or you want to do some pre marketing of that try to understand whether it really sense or exactly whatever it is given here as we are considering.

So you have one comma zero for the node which from S to one and it is also one comma zero from S to two so this one and zero are the corresponding values which we have already

discussed for the probability and the corresponding to time. Now the S you proceed later on from node one and two depending on the concept is in AND logical relationship you will find out now.

You will basically have the time and probability come into the picture but remembering the fact the probability of sum of all the probabilities of all the paths would exactly add up to one as they should be as per the normal nomenclature and by the basic concept. So if i consider that and lay attention to the so called two paths or edges which are joining three iam just mentioning three as a number and people should be able to see the node which is there.

So it has a probability of P suffix A comma T suffix A which is the probability in a time and them corresponding to the fact that three has been joined by two the corresponding probabilities are P suffix B and T suffix B and if you see the alternative node arcs which are going from one and two separately obviously one would be one minus probability for the path which is joining one and three and for the other path which is the bottom most one which is going out from two and not joining three.

It would be probability would be one minus the corresponding probability which is one minus Pb where Pb is the probability joining two and three and the corresponding times would be given accordingly node three will only be realized if both

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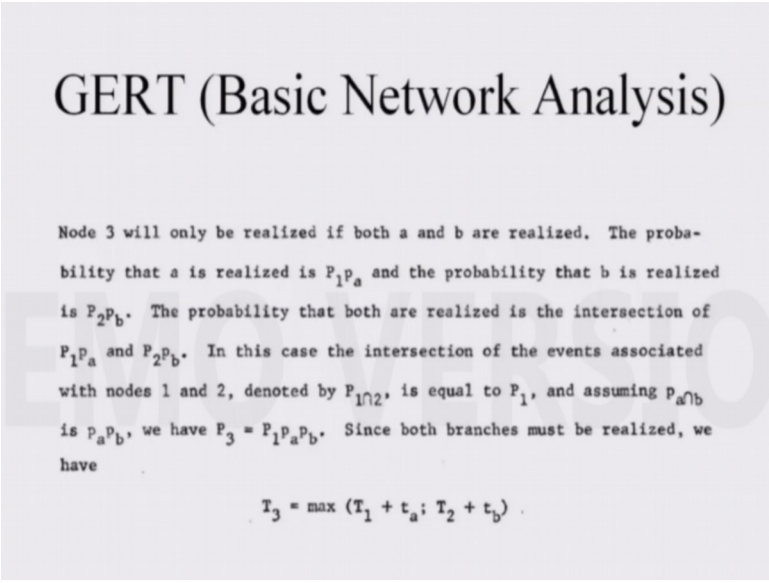

A and B are realized because corresponding to the fact that the AND network is in force our logic is in force. The probability that is realized would again using the same concept would

be capital P one into small Pa suffix A is those two values multiply and the probability there will be B is realized will be similarly the same way capital p suffix two into small P suffix B multiplied together the probability that both are realized in the intersection of the concept that both of them are realized because AND network.

So if you go back i do not want to mention that iam sure that there would not be required if you consider the concept of EXCLUSIVE OR, INCLUSIVE OR, AND network it would have some implications due to the fact that the main diagrams could be utilized in order to explain this concept or logical operations .So in this case continuing the discussion in this case the intersection of the events associated with nodes one and two are denoted by P one is intersection two that means both of them are being realized accordingly.

And it will be equal to P one and similarly when you are doing it for Pa intersection B it will be basically probability Pa into B or according to the fact that they are happening. So as that the probability should be multiplied and again if i want to find out P three it would be multiplication of the first term which is basically p one multiplied by the multiplication of those terms of Pa and Pb because now it is basically the AND network .

Similarly if you remember the AND network and i have discussed in this class when i have gone to the characteristic of the nodes it was basically maximum time. So if i come to T three it is basically the maximum time which occurring and we will take that value which is more and what all those values Ta plus T one plus Ta as the first time part towards that is the node which is joining as i mentioned the node three one of those edges and the second one is T two capital T suffix two plus Tb and based on that you will take the maximum time.

If there are three you will take the maximum of this three and corresponding to the fact that how you will do the calculations. So probabilities it will be again the same thing in this case i have used where iam pointing my finger. If you note down this P one into Pa into Pb corresponding to the fact that if you have three it would be the second part Pa Pb Pc and correspondingly P one will be calculated accordingly.

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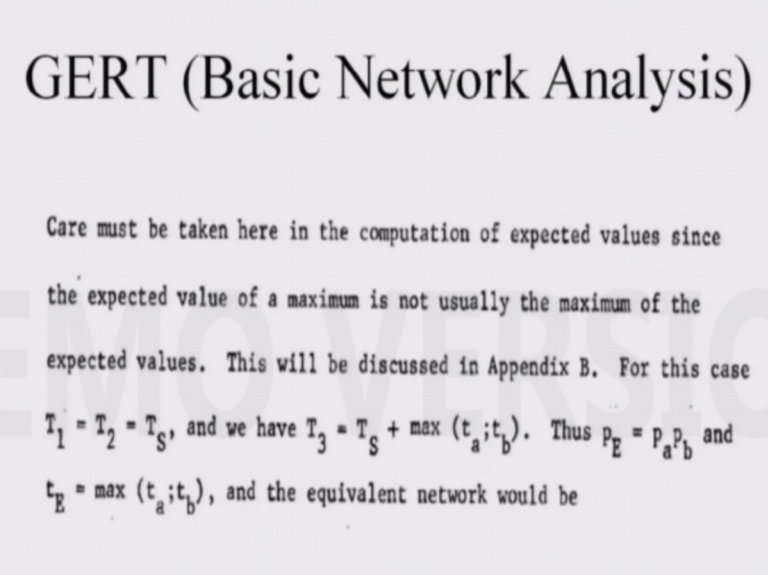

Care must be taken here in the computation of unexpected values since the expected value of a maximum is not usually the maximum the expected value. So this is just for information and be careful about that so that this will be discussed in later on but i do not think that is very much necessary for this course because it is not an exclusive detailed discussion of word GERT but is more cursory discussion.

How you can basically build up a network is in the GERT concept? For this case you will basically have T one is equal to T two is equal to Ts we are taking constant value T and we will have basically T three is equal to Ts and basically maximum of the two other values which is Ta and Tb you are taking based on that the probability would also be calculated which is Pb Pa into Pb and then the maximum value would be given.

As Te would be equal to max of Ta and Tb and you will basically do the network accordingly now .Before i go to the next slide i will try to basically summarize what we are doing is trying to basically go into the integrities of the AND the EXCLUSOIVE OR the INCLUSIVE OR and try to basically see how the calculations can be subsumed and brought into the forefront for any network such that combination of with any edges with any number of arcs can be done very practically.

Considering both time as well as the realization of the networks of the arcs are probabilistic and obviously you will understand the logic of AND and OR should be taken into the consideration when you solve the problem. So coming back to this diagram

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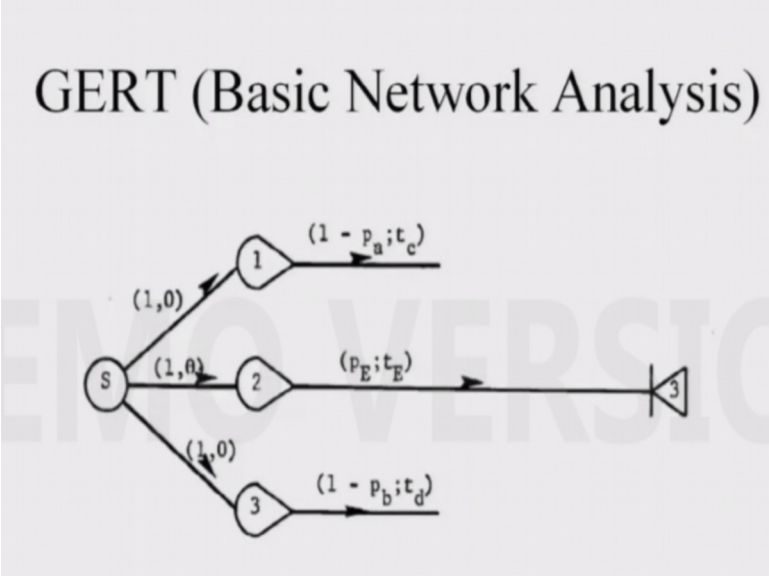

So what you have is S is leading to one. S is leading to two. S is leading to three. So one, two, three have their implications if you consider the diagram and the probabilities are given one minus Pa and for three or going out from three it is one minus Pb and the corresponding times are given in the first one is Tc and the third one is Td but if you consider the node two it is reading to an the combination of the node which is a triangle with a vertical line on the left. So i would leave that for your explanation to get it clear.

But it is corresponding probability is Pe and time is Te which we have just considered as given .So in case say for example it was not one ,two, three but it consider it was one two on the top then three then four five. So obviously one two would have the corresponding probabilities and time and similarly four five would have the corresponding probabilities and time and the value if three which is now the middle one in the thought of example we are discussing would have the corresponding values calculated.

So with this i will close the thirty eighth lecture and try to wrap up GERT and seem some concept of GERT and do a very simple problem from the point of view of GERT one in the last lecture in the fortieth one and try to basically summarize in the fortieth one in few minutes. What we have covered? And what we can expect in general form by the students? And try to basically do this course. Have a nice day and thank you very much for your attention.