Project Management
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Module No # 07
Lecture No # 31
Detailed Explanation on Crashing of Jobs

Welcome back my dear students, hope everybody is fine, this is the thirty first lecture. So each lecture as you know and am repeating that, but sorry for that is for thirty minutes, so we will try to be start off from where we left. So I am Raghunandan Sengupta from IME department, IIT Kanpur, India. So now we will be discussing about the concept of crashing of jobs and in some way the concept of leveling of resources.

So if you remember in the last class, in general there are all standalone lectures which is happening, so I am trying to finish off each small topics or sub topics or special topics in one lecture series of thirty minutes. But this concept is intertwined with the thirtieth or the twenty ninetieth fag end of the twenty ninetieth lecture a thirtieth lecture in total and the thirty first would be considering the concept of resource leveling

And how resources are important in order to basically crash the jobs so if you remember, PERT and CPM generally consider time as important factor based on which we want to find all the critical path. What are the activities on there and what is the variances, what is the expected time and the important things. So further continuation of the concept we consider how the costs are also considered.

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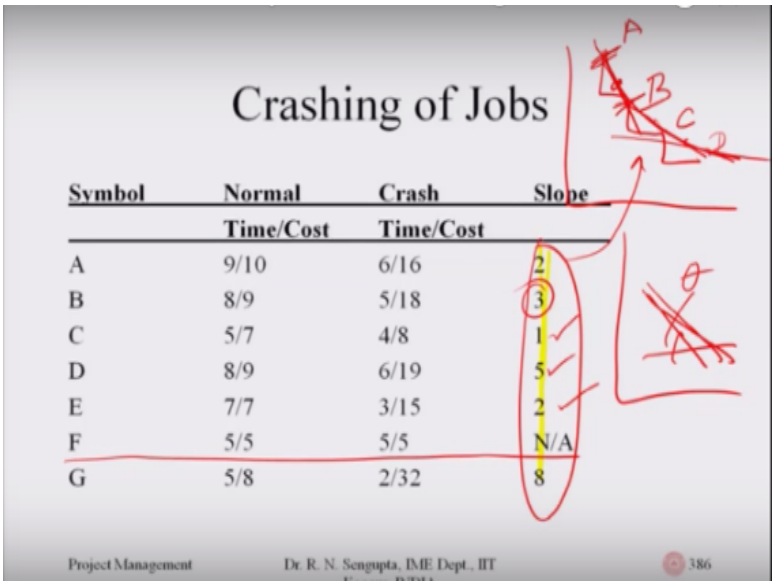

So this slide is basically the data related to the normal time and the cost which is in the second column for jobs A till G, then the crash time that means if you reduce the time so obviously there is an extra cost incurred so that is all the information for jobs A to G is given in column three and correspondingly the slopes are given, my apologies in the last class there was some error in the slide so I again start off from where you left. So if you consider job A the cost differences is 16 - 10 which is 6 and duration is 9 - 6 is 3.

So if your 6 divided by 3 it becomes 2, so the slope which is the marginal cost increase for one day direction of the time for only activity A is given by the cost of two rupees, two dollars, two yens or two liras, lira is not there like but just I wanted to mention this, so that is two units. So correspondingly if I go to job B it is 18 – 9 is 9, divided by 8 – 5 which is 3. So 9 divided by 3 is 3 which is there the number so let me highlight it.

So if the number three which is there which is the slope of that activity cost versus reduction per day for activity B on a standalone basis, so if you consider C, it is 8 – 7 is 1 divided by 5 - 4 is 1 by 1 is 1. So this is fine if I go to the next one, I am going slowly plus please bear with me the D activity is 90 – 9 is10 divided by 8 – 6 is 2 it is 5. Then if I go for the activity E it is 15 – 7 is 8 divided by 7 – 3 is 4, 8 by 4 is 2.

For the F1 note down it is here carefully so you cannot reduce the number of jobs because it is due to some reason or other consider this you want to get a CNC machine and the cost for utilizing the CNC machine as being supplied by the vendor A is only five and the person cannot try to basically try to be either extend the usage or try to give you some extra benefits because his clientele are also in line apart from you so you are you who is trying to basically do this crashing of the jobs is one of the clients for the vendors.

So obviously it means that no such request can be entertained, when entertained so hence the cost structure is as given in the second last row, which is F1. So there is no concept of slope here and for the last one is 32 – 8 dived by 5 – 2 it is 8. So now all this cost which I am also circling again and which I will again highlight so they mean that these are the linear costs, so linear costs even though everything is clear to the students.

I am sure they are very intelligent and have understood the concept in clearly as we explained so the slopes are these, so the rate of the change is fixed so tan of theta is fixed. So obviously in case if they are not fixed is nonlinear, I did mention using a small diagram that how you can basically linearize them. So the cost factor is like this so I use a step function so as that, this portion which is marked by A and B has to be replaced by a straight line.

So this straight line is basically tangent, so this slope would basically replace that part of the curve, so similarly I have another tangent starting from B, that tangent would basically replace the portion of the curve which is B and C, so if I go to C and D again I have to draw a tangent so it is not drawn very clearly and very nicely but still I am sure people would understand that, so if go to C and D, tangent at this point would be the best way how you replace the curve C and D.

So all the slopes corresponding to AB, BC, CD would replace the corresponding cost structure. So activity A consider activity A has a nonlinear cost as shown here, in that case for each one day reduction the cost would basically increase accordingly so it will be minimum for deduction from D to C increase by a certain amount as it basically scratched from C to B again it will increase to the maximum value as it is scratched from B to A.

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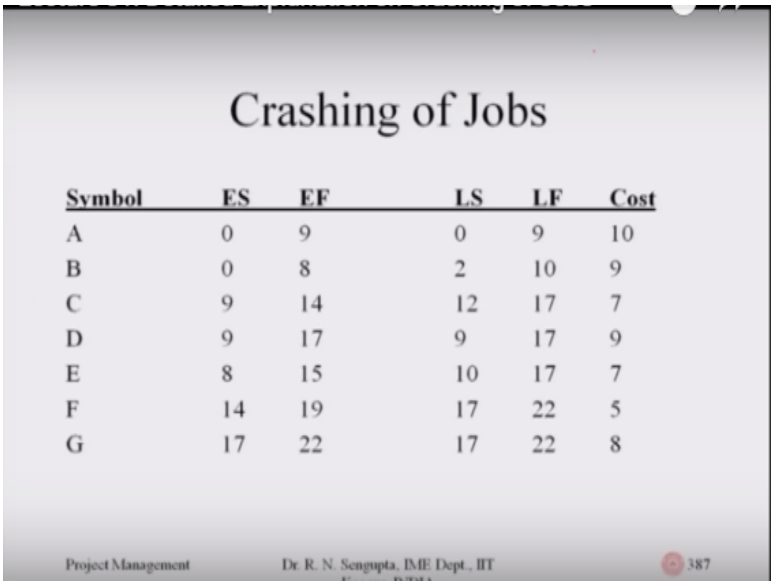

So the corresponding values can be found out if it is given, so correspondingly you can have for B activity, for C activity, for D activity and all the activities which are given. Now consider the data is given so data is given can be again from two point of view points for repetition please bear with me it was the critical path method the concept was used, the fixed cost would be time cost my apologies for the time would be given.

If the time is given so obviously there is no concept of variance but consider the PERT concept is used, you have A and B corresponding to the fact this is the optimist time, pessimist time, M is the most likely time from that you find out expected time as if I know the expected time for each and every job you will try to find out the variance also for each and every job, always remembering the fact that the variances can be added for the whole critical path.

Because we assume the activities and jobs are independent to each other, now here I would like to expand this concept of independence even though it did make sense from the calculation point of view, so what I mean by independence is consider you have one resource, and the resource is for this example consider that is very skilled laborer, a welder or say for example a person who does the drawing very well or person who does the actual the concept of trying to basically utilize the engineering concept he or she is very well.

And when we are trying to basically utilize the resources of the welder for one job we considered that the resource allocation is such that utilizing at one end the resources would not affect the resources utilization for the other end even if there is resource constraint, that means the interdependence structure should be there which in the practical sense may not be true. Anyway leave aside that discussion.

Let us come back to the problem based on the fact that you have the expected time, the variances, the using the concept of forward pass, and using the concept of backward pass, so I write down in the second, third and fifth column the respective values of early start then early finish, as you can see in this slide. The let us start the later finish and corresponding to the cross structure or the one information which we had in the in the last slide.

I doubt down the costs also, so the costs are given in the last column, so if you see job A it has got the early start of 0, job B has the early start of 0 similarly C, D, E, F, G have the early start of 9, 9, 8, 14, 17 corresponding to the late start which is basically the fourth column if you see in the slide for A to G, are the values which is given from 0 to 17, similarly the early finish and the late finish for A is 9 and 9 and corresponding to G.

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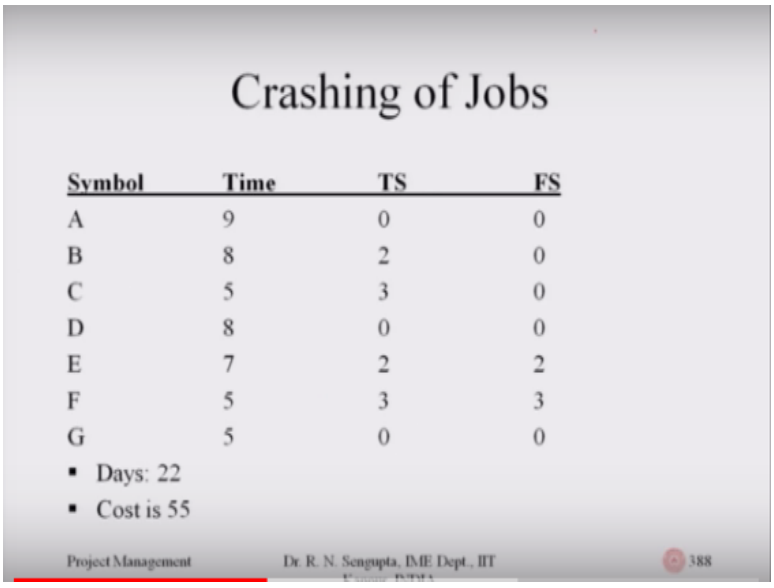

Which is the last column if you see the values where I am pointing my finger, this is twenty-two which is early finish, and 22 which is the early late finish. Now based on the calculation how you

do that if you see remember the formula for finding out the total slack the pre slack, so I do we do that and then we again draw the table but with the necessary information as stated what are those, the first column is again the jobs A to G.

Then in the second column you have the time was basically the time taken on the normal scale, for each and every job, starting from A has a time of nine tilt he last job activity G has a time of frame of five the total slacks are given in the third column, the free slacks are given in the fourth column. So as you know the realization between total slack and free slack is known to us and based on also the fact that in the critical path what should be the value of the free slack.

What should be the values of the total slack, we are able to find out what is the critical path, now of you consider the critical path the number of days I am not going to come to immediately to the path as such but only concentrating on the number of days for the critical path and the number of cost which is incurred, total concept of cost. So if you consider the total number of days so now first time I will basically in this whole series of lecture starting from the first to the 31.

I generally did not go back to the last slide, so considering they were moving in a sequence and people could understand them, I purposefully did not go back because that would basically break the sink or the flow of the lecture, but with we due permission of the students I would like to go back to the slide as that it is easy for me to explain this concept of the crashing. So if I go back to the last slide, so it gives me the early start early finish, late start and late finish.

The cost and if I have the normal so the time normal times are 9 + 8 is 17, 17+5 is basically the total value which would not match with the answer because I am going to come to that so what is important is 9+8+5+8+7+5+5 so that is total duration would be taken for everyday activities collectively but that would not be the case for the critical path which is obvious but when we come to the total cost remember that cost would now be not the critical path only for the other jobs also which are there in the sequence.

So that is why I wanted to mention that, so based on that the total number of days is twenty two so the critical paths if you can find out they would be immediately very apparent from the total

slack and a free slack which is there, so if you see their total slack and the free slack for A as shown in the slide A is total slack three slightly 0 if you see D total slack free slack is 0 if you see G the total slack and free slack is 0.

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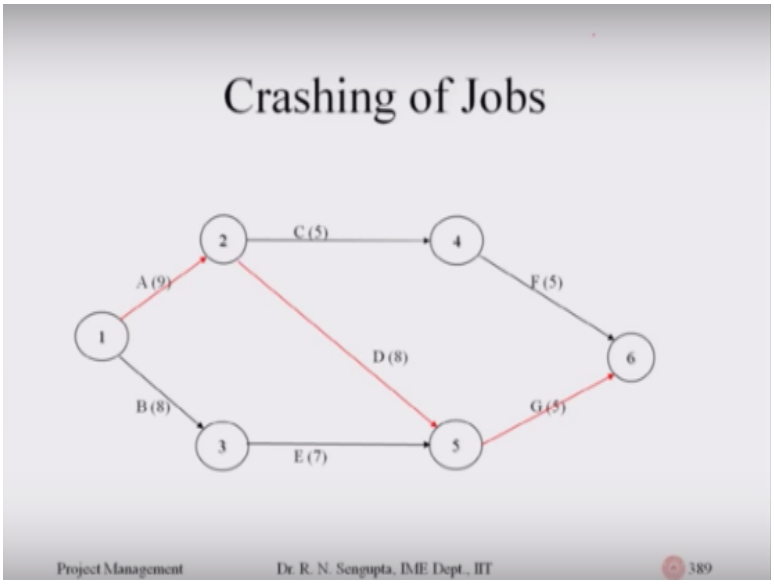

So that should give you a hint the critical path what it is and the total cost is the addition of all the costs for all the activities beside the critical path. So now it has been replicated in the diagram format, this is the activity on node concept so now if you remember the activity on arc concept my apologizes on a quod concept so the jobs which are there on the critical path considering the total slack and the free slack they are A, B, D, G.

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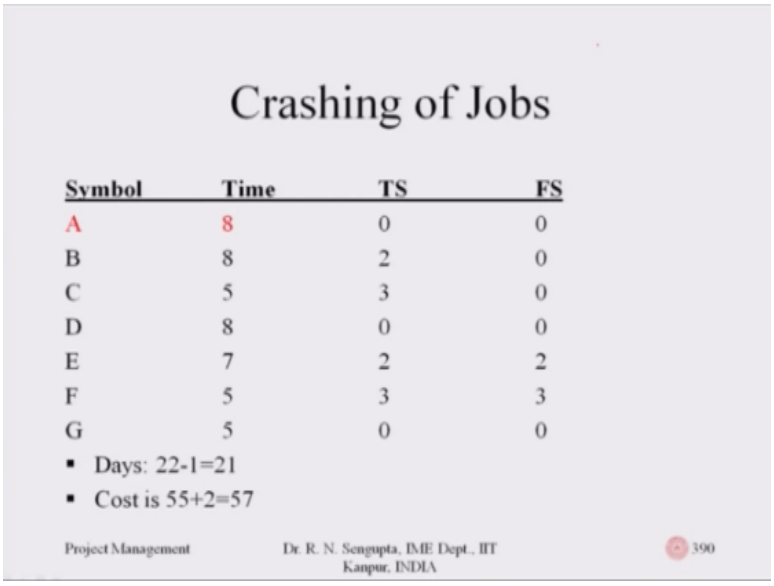

So the total time required if you remember in the last slide was 22 so how do we get 22. 9+8 is 17 + 5 is 22, but if I want to find out the total cost would be for activity A, activity C, activity B, activity D, activity E, activity G, activity F all combined together that was the total cost which was coming. Now let us crash the jobs, so crash the jobs in such a way that we will try to see that how it can be crashed in such a way without exceeding some set of cost which is not given here but trying to understand.

That as I decrease the number of days what is the overall effect on the job, and the time duration so for my first highlighted activity A, the time initially if you remember again with due permission and apologies to the students I will go back to the last slide. So here A where I am hovering my pen is 9, so 9 is now going to become 8. So A becomes the number of days technically was basically twenty two so critical path has decreased by one.

Because still it is critical, this job this path which is A, D, G is critical now it is 8 it is not 9, 8+8 is 16 + 5 is 22. So if you see the number of days is twenty one now immediately as what is the increase in the cost so now if you go to the cost structure of A, so again go back to the cost structure so if you see the first row, cost structure is given by the slope of two that means per one day decrease in activity duration for activity A your total cost increases by two units.

So if it is two units so let us go back total cost was 55 as you see in the slide, actually when there was no crashing now obviously it will increase from 57 to 55 I am sorry for that it will increase from 55 to 57. So as noted down here so if you consider one day reduction in A decreases their time duration from twenty two to twenty one and increases the cost from fifty five to fifty seven.

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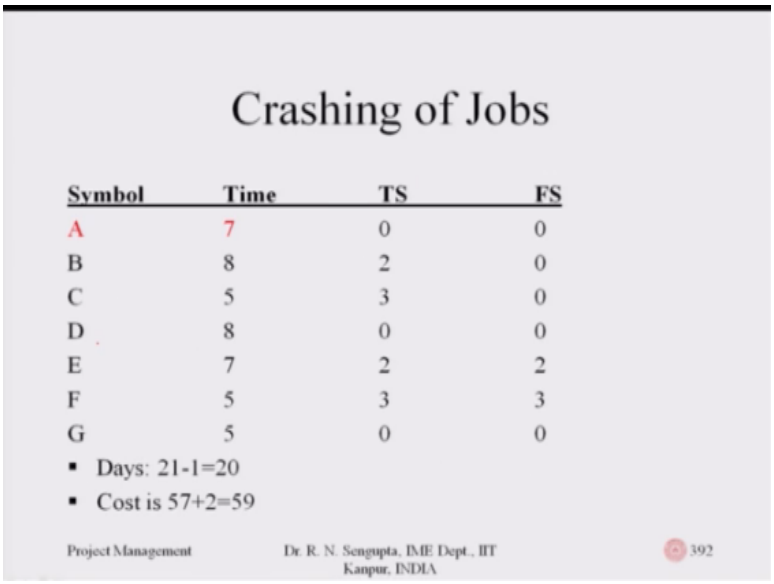

So now I will ask whether it is possible to reduce this further, so let us go step by step, so this is the situation of the network diagram here everything remains same, but only concentrate the highlighted red color which is A was A duration was 9 now it is 8 so based on that I have drawn. The next instance of the activity on arc diagram for this problem. Let us further reduce A, it could have been done for the others, but let us go one sequence of steps that I reduce A.

A now reduces from 8 to 7, so now the overall duration of the critical path which is A, D, G, is basically 7+8 is 15 +5 is 20. So as rightly pointed out 21 decreases by one units to 20. Now on the other end will also see that what effects does it have on the cost, so cost had increased from 55 + 2 that which makes is 57. Now 57 will again increase by two units because this is the linear cost marginal rates if you remember I had mentioned that.

So now it will increase from 57 to 59 as pointed out in this table so I in this slide and then I will again draw the actual network diagram, so here again the network diagram everything remains the same only the A number of duration which was eight earlier has now reduced to 7. So let us again pause and see yes this is a footnote which I wanted to add well you may be thinking that this work has been going on I am trying to deduce A by one unit.

That is fine, but does it bring any other activities into the critical path let us see it was twenty one as of now by a deduction on the number of days of 1 unit in each step, if I want to find it out

whether there is aa reduction in any in A or change in the critical path to other set of activities, the answer is no, why? If I basically consider the sequence of activities ACF what is that? 7+5 is 12 + 5 is 19 so nineteen is definitely less than 7+8+5 so hence it is still not critical.

Which is not critical, let me repeat it that set of activities A, C, F is not still now it is not critical, just look at the other set of activities 8+5 is 7+5 is now equal to the same value as 7+8+5 which means now if you see and as rightly pointed out in this diagram there are two sets of critical paths and both have been marked as red color so the first set of critical path is 1 to 2 which is A 2 to 5 which is D and 5 to 6 which is G so the total sum is 7+8 is 15 + 5 is 20.

The other critical path is 1 to 3 which is B, 3 to 5 which is E, and 5 to 6 which is G, so hence again add up 8 + 7 is 15 + 5 is 20. So that means now rather than having only one critical path now you have two different critical paths. So now let us pause here one minute, in case I will show that in detail calculation later on but let me explain, now if there are two critical paths, if you reduce G by one day so it basically affect both the critical spots at the same time.

So in that case if I want to find out that one day reduction in the critical path which is actually happening from 20 to 19 what is the actual critical paths now the one is A, D, G and B, E, G duration is 7+8+4 and here it is 8+7+4 which is in both these cases it is 8+7+ is 15+4 is 19 now you will ask the question that does any other path become critical now? So let us answer this question because there is only other set of activities path is 7+5 is 12 + 5 is 19.

So till now what we have is that 8+7 which was 15+4 is 19 which means that the moment we reduce from 20 to 19 the rest set of activities which were still not critical now becomes critical that is point number one, point number two is that you will ask what is the actual cost increase so the cost increase has increased from 57, 55 to 57 to 59, but if I reduce the job G by one day.

So I will basically go back to the sequence of the job reduction which is happening what is the margin rate so let me go back there so for G the reduction one day detection cost increased by 8 so which means that the total cost which is now in the new one is fifty nine will now increase by

59 +8 which would basically become 67. So this I am trying, I just wanted to explain so let us proceed with the problem.

So now here that was one sequence now let us see the other sequence ok let me before coming to this diagram later on let me explain it with a different set of tables so there are different instances based on which you can reduce the overall critical path the costs in will increase but you will need to find out which is the minimal cost, you basically draw different sequences and take a decision psychotically.

So now consider you are trying to reduce A and E at the same time, A basically now reduces in such a way that it becomes from 7 to 6 and E basically which was 7 now reduces to 6. So now let us go back what was A and what was E was earlier. A was 7 and E was 7, so now A has reduced from 7 to 6 and E has reduced from 7 to 6. Now if it is reduced by 1unit so let us see what is happening so the total duration which is happening and the reduction of the number of days which is happening would now be.

It was earlier 20 now it is basically reducing by one unit and becoming from 20 to 19, point one, point number two the cost was 59 and that had an increase from 59 if we only consider other the activity G if you remember that has increased from 59 + 8 which was 67 but now we are taking a different approach. A cost is increasing on two fronts one for A and one for E.

We will try to answer that what is the marginal rate of the increase of the total cost both an account for A and E all of them or both of them reducing by one day each, so here the marginal rates for A if you remember was two, marginal rate for E activity was two so hence the cost increase would be 59 + 2 + 2 which 63 so you try to answer this question that trying to reduce the cost of A and E separately will it be beneficial with respect to the concept trying to reduce the cost of G which was for G.

You incurred an extra cost of eight units for one day reduction but here in trying to reduce A and E separately you incur a total cost increase of 2+2 which is four units, which means that it is much more viable much more logical to increase the cost by only four units by concentrating on

A and E separately rather than only concerning on G which would basically increase your total cost by eight units. Here I give you the diagram so this is basically now A is reduced from 7 to 6, B is reduced from 7 to 6, D remains as 8, G remains as 5.

C and F remain as 5 and 5. So this is the actual network diagram for the total reduced network depending on the one units of reduction of the total number of jobs which has happened at the last page for A and E separately. Now let us come to the increase in the cost and the reduction of the days if we consider only G. So now A to F are not being affected, G which was initially 5 is being reduced from 5 to 4.

The moment it is reduced from 5 to 4 the total number of days for the that path, so now that by the word that path I mean there are two paths which is which are being affected by G what are those that is A, D, G and B, E, G so if you see the total reduction is happening from 19 to 18 by one day and the increase in the cost would now happen by eight units. So this I have just changed it here this is eight unit as you can see in this slide so it has increased from 59 + 2 + 2 which is 63.

Sixty three would now increase by 63 + 8 which would become basically seventy one, so now with the reduction number of the number of days on different accounts for different activities the total cost as of now is 71. So this is the diagram which you have here A, B, E, D, C, F are not being marked by red, only G is being marked by red because we showed that if that what is the overall reduction in the number of days happening from 5 to 4.

So now again I concentrate on G in the same line, G is decreased in the number of days from 4 to 3, again the increase in the what would be what it was basically say seventy one, so 71 would now increase from 71 to + 8 to basically 79 an the number of days deduction was already 18 it is being reduced from 18 by one day. It is become 17 so here is the set of jobs you have so again the overall cost structure, overall network diagram is shown as given.

Lastly we reduce the number of days from G from 3 to 2, the cost increased which would be now here it would be 8 it was 71 + 8 which was 79 + 8 which is eighty seven, so this value would

become 87, which is increased from 55 and the reduction of the days would be given corresponding to the case when G is decreased by one day. So now this is the total network and if you see now all of them have become critical.

Because the number of days is if you count out any of them is 6+5, 11 + 5 is 16, 6+8 is 14, 15 and 16 here also and if you see B, E and G is 8 +6 is 14 + 2 is 16. So all of them have a duration which is 60, so the total duration has been cashed from as mentioned from 22 to 60 with the increasing cost as mentioned here. So with this I will close this lecture and continue in the thirty second about the concept of more crashing and RZ and QZ concept. Thank you and have a nice day.