Project Management
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Module No # 5
Lecture No # 21
Concept of CRITICAL PATH METHOD (CPM) and Introduction to PERT

Welcome back my friends my dear students so this is the twenty first class we have just very briefly considered that what is the concept of end to start, end to finish, finish to start and finish to finish concepts based on which activities can be scheduled point one. Point number I mentioned that and did highlight quite a number of times initially in the beginning and also in the last twentieth lecture that the concept of PERT and CPM are only based on end to start.

That means once basically the job A and then only job B can start so number of days duration is given there and also the important fact is that no looping is allowed. So we will consider that in very simple example so as such we were going briefly through the concept of cost so it will be expanded later on in a more detailed problem. So we must remember here that we are considering the end to start criteria is just a repetition to me to much more clear to you.

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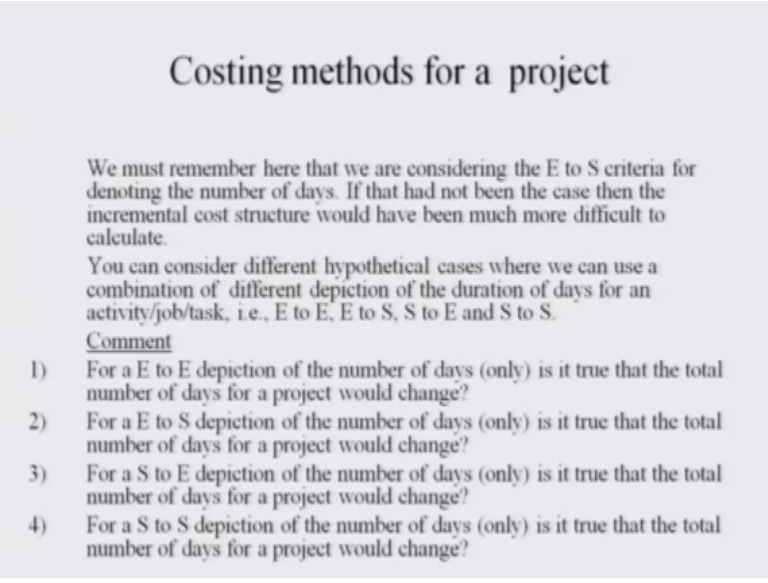

For denoting the number of days between end to start and what is the deviation between two jobs. If that has not been in that case then the incremental cost structure would have been much

more different to find out considering with just simple example which you considered in the end of twentieth lecture. We can consider different hypothetical cases where we can use the combinations of different depiction of duration of the jobs.

For an activity in a job or a or a set of work task and the concept as I mentioned it can be end to end, end to start, start to end, start to start. For end to end depiction number of days it is one two the total number of days for the project would change would not change to answer that. For end to start, start to end, end to start the concept would similarly mean implemented such that we can understand that the way of calculating the slide, the way of calculating the path. What we are going to do is only specific to end to start concept.

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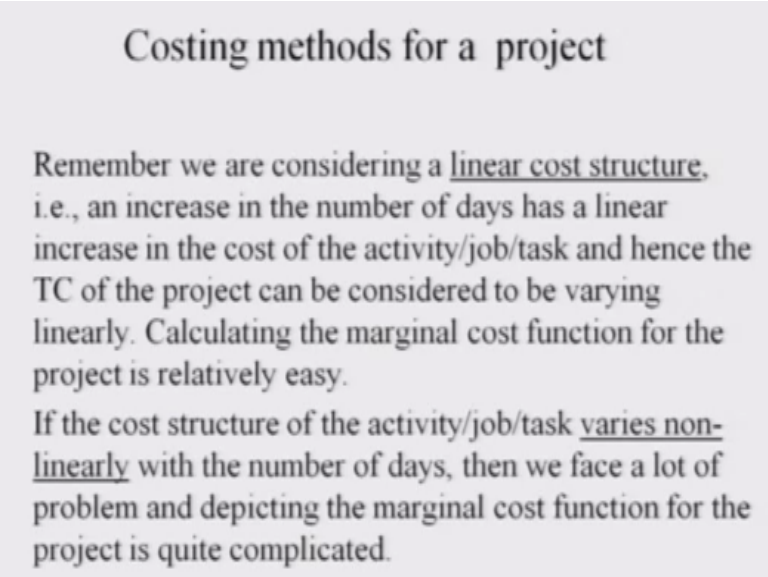

Remember here again as I mentioned when we are doing this very simple problem of the cost structure which was four forty in total. So would consider linear cost structure that is increase in number of days as a linear is in the total cost structure. There is not marginal rate are happening at the fixed rate. So the total cost of the project can be considered to be very leniently calculating the marginal cost function of the project is relatively easy if linear cost functions are there.

If the cost structure the activity job task varies non linearly with number of days then you face a lot of problem and depicting the marginal cost functional for the product it is quite difficult in order to conglomerate the cost structure.

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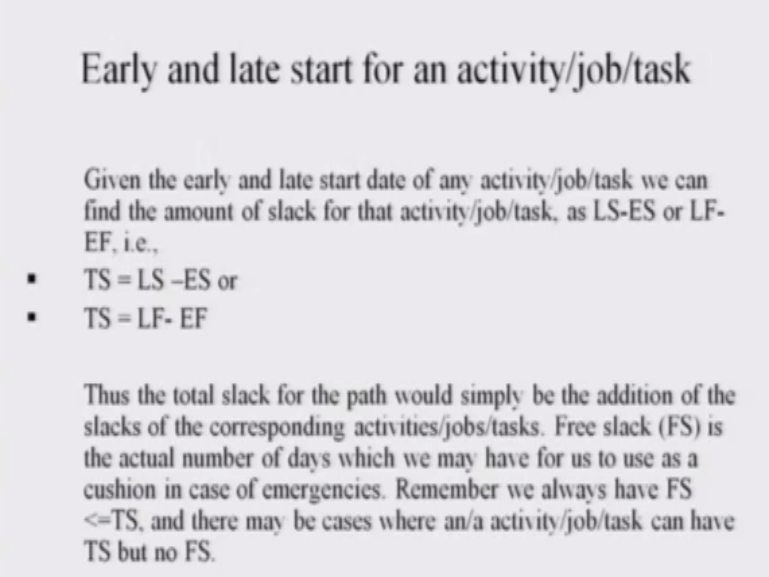

So given the early and late start dates on any activities considering end to start concept to prove which would be use for the calculation means we find the amount of slack for the activities job as given by this concept. So as it is given the total slack would be given by this concept by late start LS the late start minus early start or it will be given by late finish minus early finish. So these are the general formula for the end to start concept out of the so concept.

So thus the total slide for the part would simply with the addition of the slacks for the corresponding activities jobs so add them all, all of all the paths accordingly. And the free slacks which is FS is S is the slack not the start so remember that TS which is there by pointing my finger this one the two forty eight slide. So this is basically the total slack and this and the two forty eight slack which is also the FS which is there with the free slack is nothing to do with the start while the concept of E and end to start end to end was different concept based on which we do the calculation.

So the free slack is the actual number of days which may have for us to use the cushion in case of emergencies. Remember we will always have the free slack been less than equal to the total slack and there may be cases where activity of job can have total slack but no free slack. So and what it means that if that is true and to what it means actually total slack free slack everything is zero that will also make sense as we proceed with the problem.

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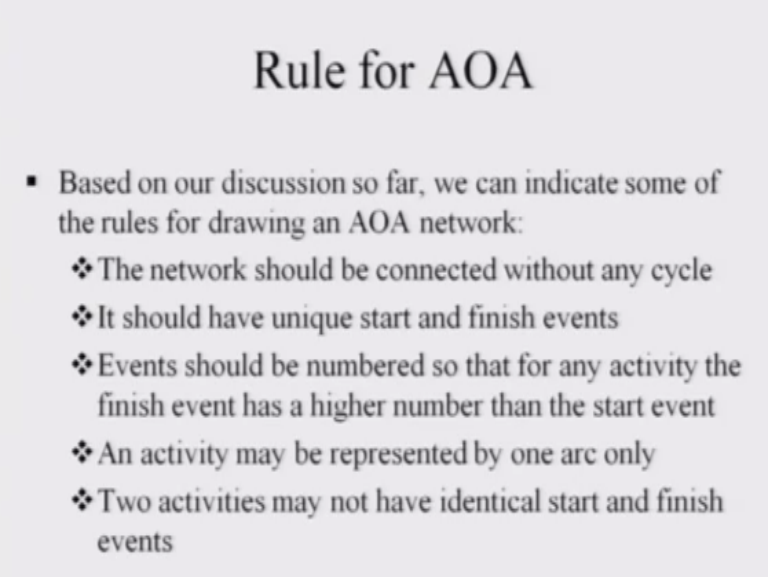

Based on a discussion if you have been having from the last part of the twentieth lecture have been initial few slides for the twenty first lecture. So we can indicate some of the rules for drawing the activities and arc concept. So it can be implemented for the activity on node concept also there is no change. So the network can be connected without any cycle it is just a cycle of the same thing if we consider.

It should be unique start and finish event should be there events should be number so that there is any activity if finish event as a higher number than the start. If you start from the left which is the start right would be the finish they basically go in in a branch or a tree concept from the start to the finish. An activity may be represented by only one arc only because it is a arc one if it is node one to be correspondingly done.

To activities may not be identical start and finish event so if it is in the activities of arc concept the last bullet are also to be applicable in in from the from the activity on node concept. So this again what we consider I am just highlighting with different diagram the end to end, end to start, start to end, start to start.

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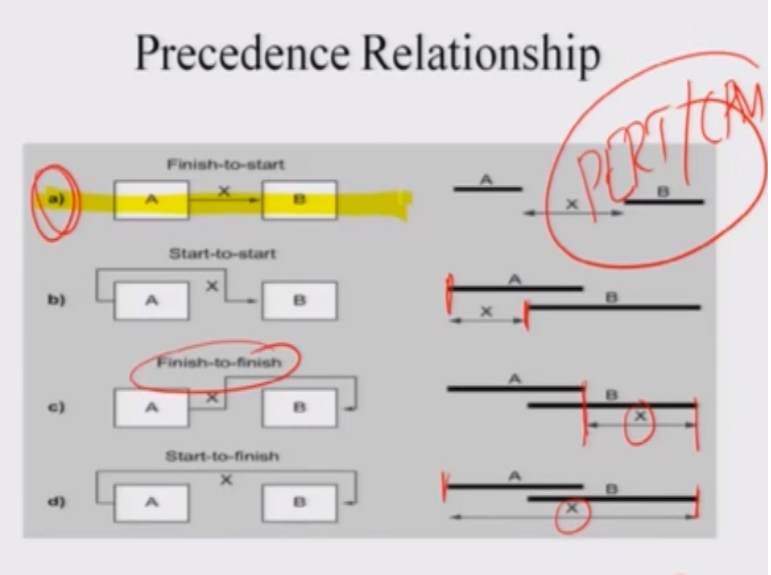

So this is again given diagram A, B, C, D or the same implication this is end and finish words are interchanging so please bare with me. Finish to start X is the number of days so if I basically go into the right side diagram which means there is number of days between A ending and B starting. Start to start means X is the number of days which means between the number of days where A starts and B starts the X is the actual number of days similarly if I go to finish to finish or end to end.

So it means that one once A ends and B ends the number of days is given as it is implied so let me at least in the diagram try to make it much clear. So this is the end and this is the end so this X is basically the finish to finish or end to end. Similarly if I go back to point B start to start so this is starting of A and starting of B X is there. If I go to finish to finish start to finish so where A starting and where B is ending.

So this is X is defined so arch is PERT and CPM we only consider point A that means we can highlight it so we will only consider this for our discussion. Even though the slacks and critical path can also be done for ABCD.

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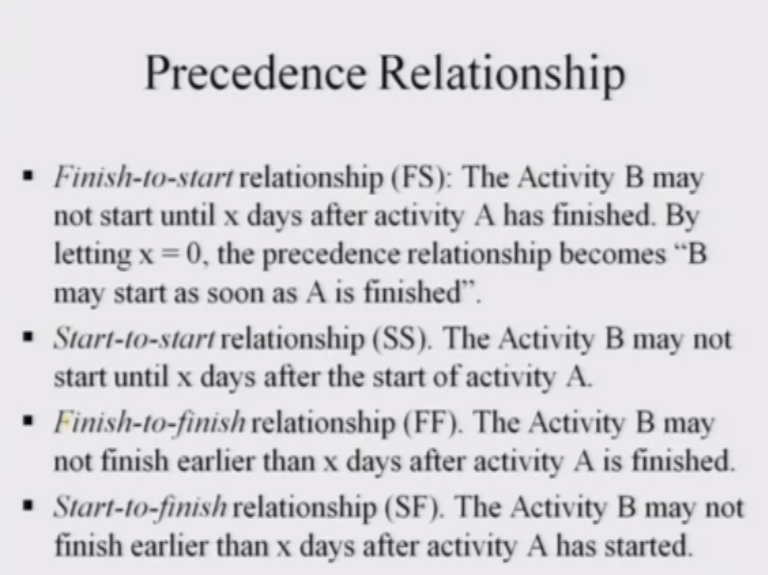

So finish to start relationship the activity B am just explaining time again in order to make understand more clearly in the qualitative sense. The activity may not start X days after activity A as finished but letting X = 0. So the pessimist relation is B may start as soon as A is finished. So these are important like X = 0 what implication it has for all the three different concepts. Start two start is basically S SS the activity B may not start until X days after start of activity A and if X is zero obviously it will have implication.

Similarly for finish to finish then activity B may not finish earlier than next day after activity A is finished. So if X is zero obviously it will have a implication if at all it can be made zero so in many of the situations you will find out whether for finish to start, start to start, finish to finish and start to finish X may not be zero or cannot be taken as zero. Start to finish relationship activity B may not finish earlier than X days after activity A has started.

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Precedence Relationship

- In order to perform a forward scheduling we have to "normalize" the FS, FF and SF time relationships to a SS relationship. Correspondingly, to perform a backward scheduling we "normalize" the time relationships FS, SS and SF to a FF relationship.

In order perform for the forward scheduling of the forward bias task method and the backward pass method which I mentioned in the twentieth lecture. We have to normalize the finish to start, finish to finish, start to finish relationship to the start to relationship. So they should be normal normalize should be done such there is not confusion how this that implemented. Corresponding to perform a backwardly we normalize the time relationship between finish to start, start to start, start to finish and finish to finish relationship.

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Rules for calculating Precedence Relationship

Type	Forward	Backward
FS	d_A+x	d_B+x
SS	x	d_B+x-d_A
FF	$d_A+x- d_B$	x
SF	$x- d_B$	$x-d_A$

So how we do that I will simply now the rules for precedence relationship for all the four concepts so I will only consider the one as a problem and highlight that people can utilize the same concepts using the normalization concept which is placed in front of you in this slide

number two five three. So for finish to start if the total number of duration of days of A is given as D suffix A duration for B is given as D suffix B. Then for the forward concept it is DA + X is basically the overall way when you calculate the forward pass method and the backward pass method this will use the formulas.

How will you use the formulas am going to come to that within few minutes so bare with me for pay attention here. It is basically if it is hence similarly for the backward method with DB + X at it will calculate accordingly. So in the forward method you go from left to right and do the calculations accordingly and for the backward method we will come for the right to left so and we do the calculations accordingly.

For start to start if the forward method will consider X as the number of days which is there between job A and B and for the backward method it will be the submission of DB which D suffix B is that duration for B + X which is number of days – D A which is total number of days for A.

It may not immediate sense but once I solve the problem you will understand for forward to finish to finish method you will basically have as you going from left to the right it will be now DA + X means how much you have proceeded on to the right and then – BD to find out the forward points of starting and finishing of the jobs. And the backward method you will basically consider X the delay which the number of days of gaps which is there between A and B in proceed accordingly from the right to the left.

Finally to the last one is start to finish it will be number of days gap minus DB which is D suffix B which is number of days required for job B and you will continue doing it for job A, B, C, D accordingly from left to right. And if you going from backward method it will X – DA, D suffix A is the duration for job A so how you use that we will consider now.

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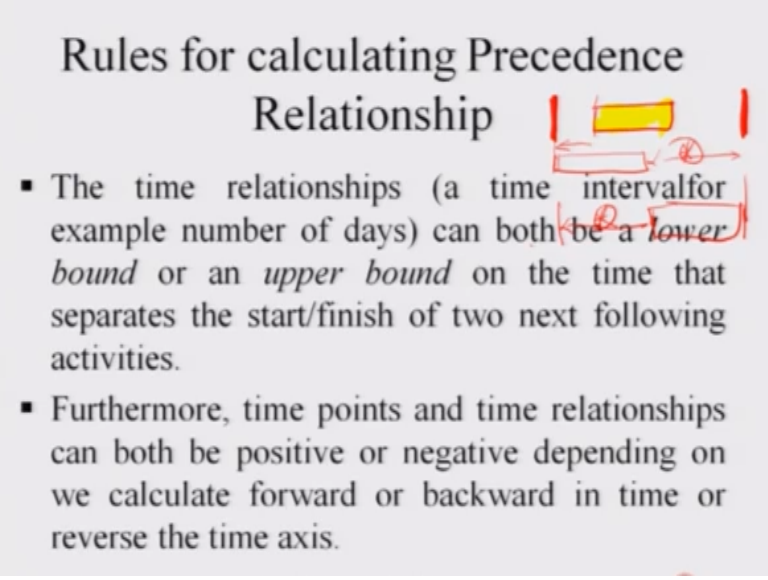

The time relationship time interval for example number of days which there in between two jobs can both be the lower bound and the upper bound based on which we do the calculation. So what we are trying to utilize is the forward method and the backward method is trying to find out what is the earliest is you can start up the job what is the latest you can finish the job. Because based on that if the earliest start and the latest start of a particular job say for example is the second day and the eighth day.

And if the total duration of the job is four days that means there would be some leverage between which you can change the overall job. So let me use without the values use the diagram to explain that. Say for example so this is this one is the earliest job can start and this is the latest job can end. Now consider the job duration is like this this block is what it denotes the total duration. Now if I consider the job duration and let me high light it for better understanding it means the starting of the job which is here can be shifted here.

So if this whole jobs comes there trying to use the two scale it means the starting comes here this end point comes here. So this means the total amount such (()) (14:04) which you have and on the similar lines if this ends coincides with this point. So consider this and this is same and this starting this starting then again you will basically have leverage. So this considering that it has been drawn accurately the number of days the slack or the cushion which you have here which is star and the star which is here which is the cushion would be of the same duration.

The time relationship or time interval can be both a lower bound and upper bound the time that separates the start and finish of two next following activities based on which you do the calculation. Furthermore time points and time relationship can both be positive or negative depending on we calculate the forward pass and the backward pass.

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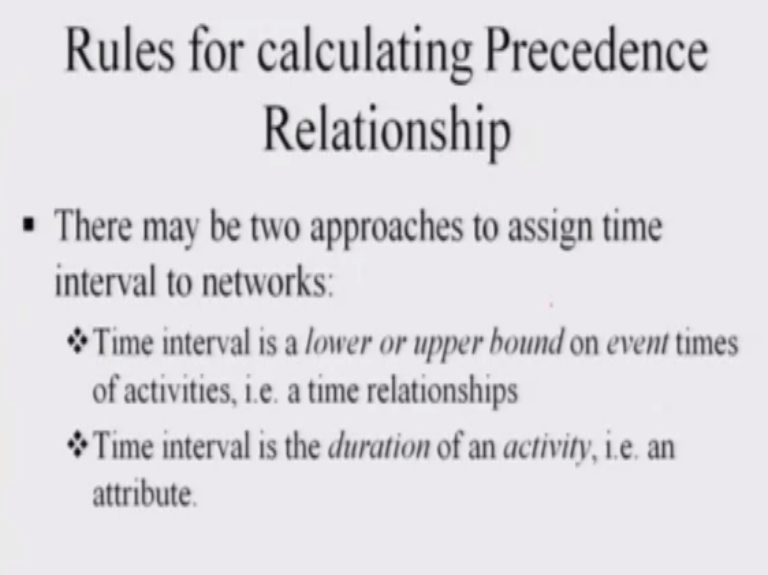

There are two approaches to assign time intervals to networks one is basically time intervals lower bound and upper bound considering the activities or even times are activities there is a time relationship is given. And time interval can be also consider from the duration so rather than the start rather and the finish the duration.

The duration is NX so rather than only considering when it starts and when it finish and it only consider X or rather than only consider the X I will consider that when it starts or when it ends or when it ends or when it starts depending on which way you proceed from the forward which is basically from left to right or the backward which is right to left.

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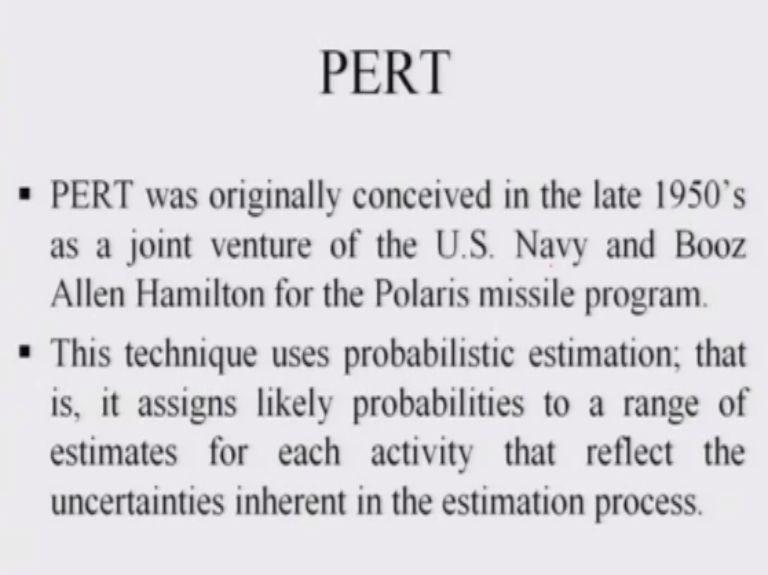

So PERT was originally conceived in nineteen fifties as a joint venture of U.S.Navy and Booz Allen Hamilton for the Polaris missile rocket work. This technic as probabilistic estimation that it assigns likely probability to a range estimates for each activity that reflects the uncertainty inherent in estimation process.

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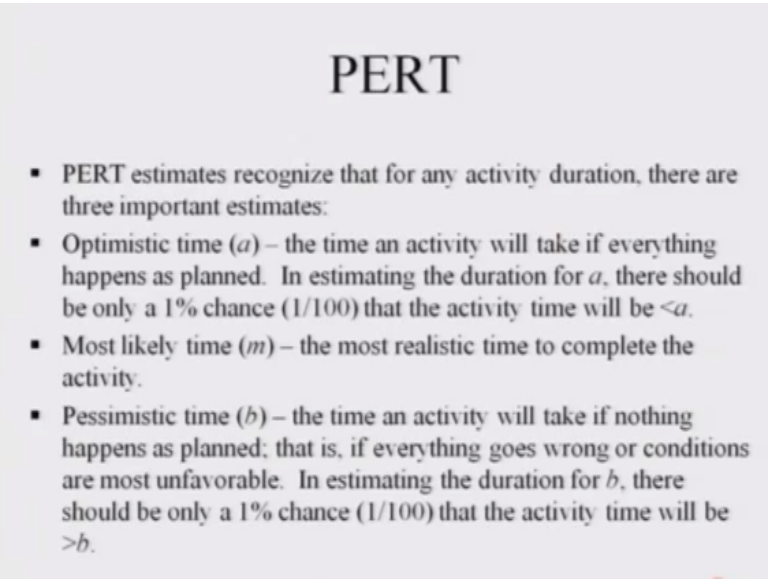

PERT estimation recognizes estimates recognize that any activity duration there are three important estimates similarly remember T suffix P, T suffix O, T suffix M. So one is the optimistic time one is the most likely time, one is the pessimistic time and based on the proved with the calculation we already seen it basically optimistic plus pessimistic plus four times from

mean time. So all this things divided by six and I mention that what is the probability one sixth, one sixth and four sixth.

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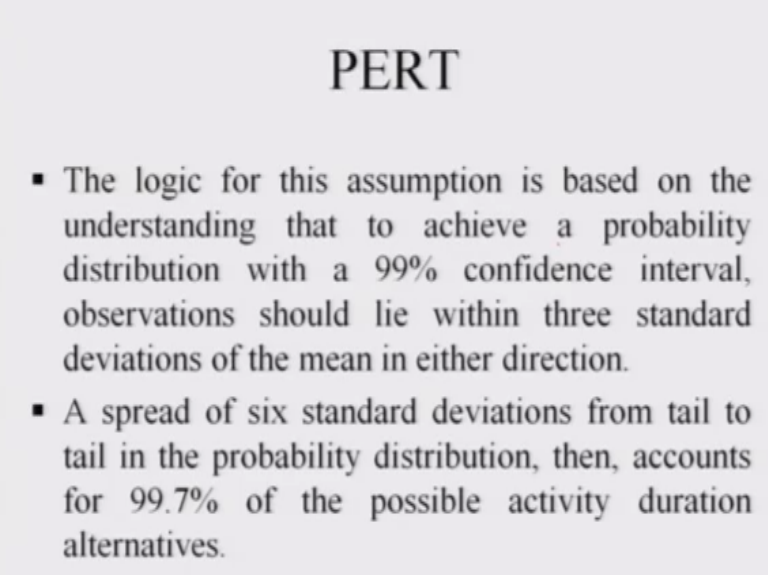

Now logic for this assumption is based on the understanding that to achieve a probability distribution with ninety percent confidence interval observation should lie within three standard deviation of the mean value. So considering is not normal distribution it was normal distribution then three sigma into the right sigma on to the left considering the normal distribution would basically cover about ninety nine point nine seven percentage of the overall area but that as it is not normal not cemetery.

Hence this calculation not utilized in order to consider ninety nine percent coverage of total area. A spread of six standard deviation of six sigma from tail to tail the probability distribution that account for about ninety nine point seven percentage for possible activity duration alternatives.

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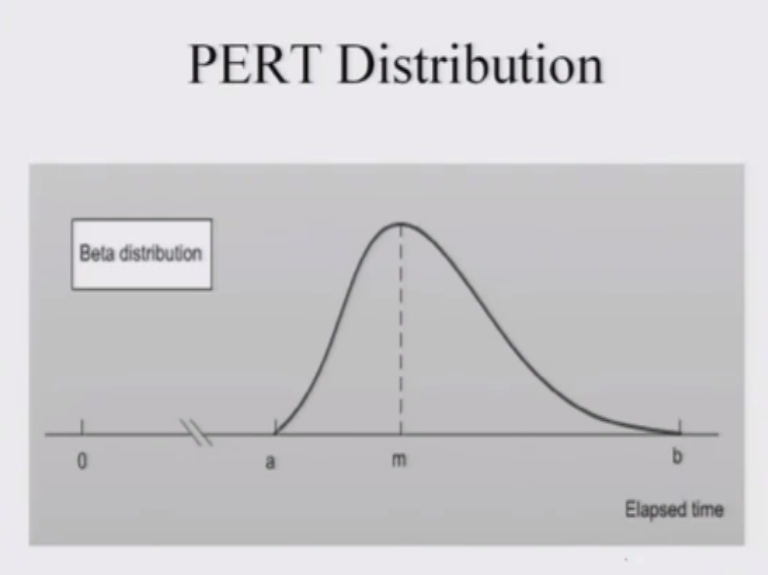

So this is the beta distribution which you have been taking about and if we look the distribution in front of you is not normal. So technically if it is not normal am trying my level best to draw it would be like this. So hope I would be able to draw but I just erase that so this is the beta distribution. This value is basically the time estimate which is as A, B or O, P depending upon whether you want to denote by optimistic time or pessimist time and M is the mean time based on which you build your calculation.

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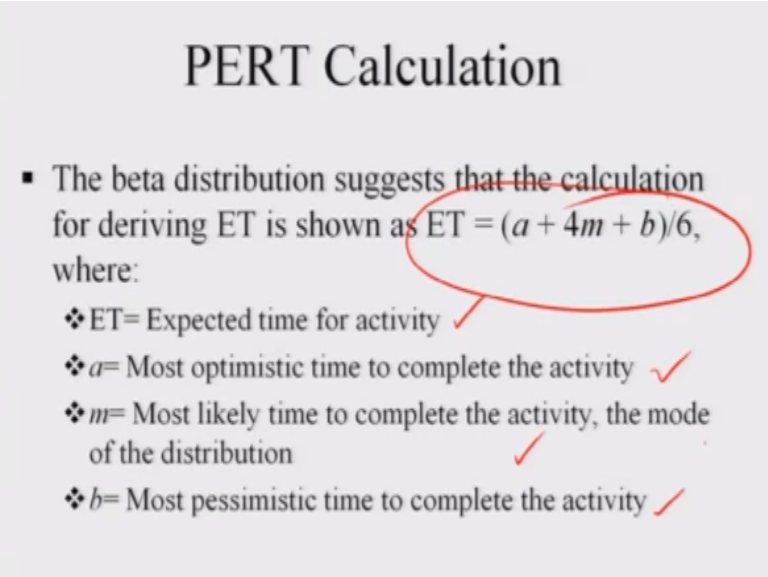

So again the estimated time is given by this formula the expected time is the optimistic time M is the most likely time B is the pessimistic time and will use this for the calculation as well as for the average value varies.

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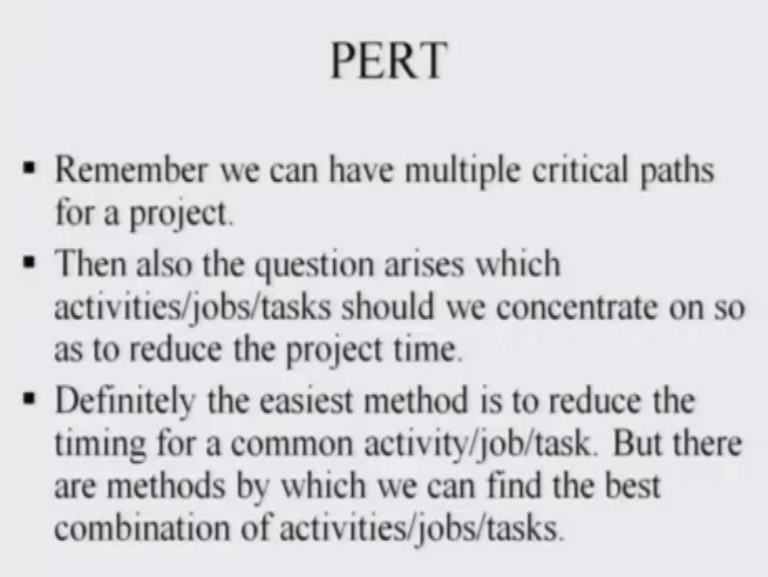

Remember we can have multiple critical paths that is important note and multiple critical path means that means there are many activities which are very important. So any crashing any over estimation of the time duration for any other activities would have a huge amount of consequence both from the time perspective as well as for the scheduling perspective.

So then also the question arises if there are critical many critical paths so which activities job task should be concentrate on so we reduce the overall cost to the maximum possible extent. So it may be happened that there are two critical paths. But we may only concentrate on one of the critical path because the overall saving of trying to crash the job would be higher in critical path not for the other. So depending on what is the cost structure what is the number of duration and such information which is there.

So definitely the easiest method to reduce the timing for the common activities job pass would be which is common to both the critical path. So if it is not common obviously we have to choose the pass in such a way that if you say for example. If you reduce one job A arbitrarily consider in critical first path then it may possible that you have consider job D and E in critical path method path two in order to have the same consequence for reducing the overall cost for the overall project.

But incase if it is avoided that this best so what we need to concentrate is that whether there are activities on jobs in basically have the consequence for both the critical paths consequence there are two critical paths. So that would basically help a person to solve the problem in much easier way. So but there are method by which we can find the best combination of activities of the jobs and the task which are there in front of us.

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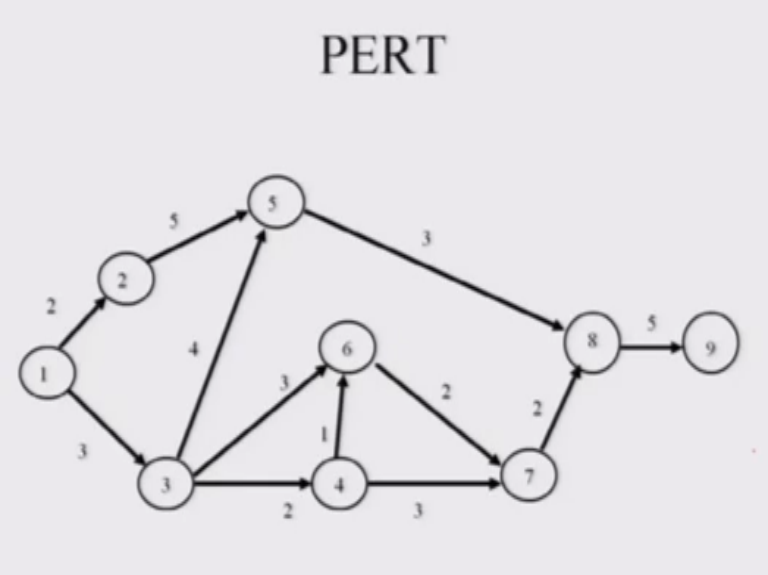

So consider diagram here so there are activities which starting on the whole project is starting at one and then it goes from one to two and one to three then three goes to three two five, five goes to two two five. So am not drawn table in order to under the say what is the precedence diagram concept which is there which job or a activities follows what. So then five goes to eight, three goes to six, three goes to four, and so and hence so forth till the last end which is job number nine.

So the number which I have given like one to two is two, two to five is five, one to three is three three to five is four these are the duration which is consider. So now we have not considered CPM critical path method not the critical path concept as such we are considering the PERT. PERT we have the average time so what is important to note that in the critical the duration not given as it is while in the PERT method the optimist time and pessimist time would be given.

And we have to utilize the most likely time also use the formula which is optimist plus pessimist plus four times is likely sum them up divided by six. So that will be considered as the expected time for any activity and do the job. So that expected time which is just mentioned in the calculation for the PERT is these values which are return here or the the the PPT slide which is two sixty two in number.

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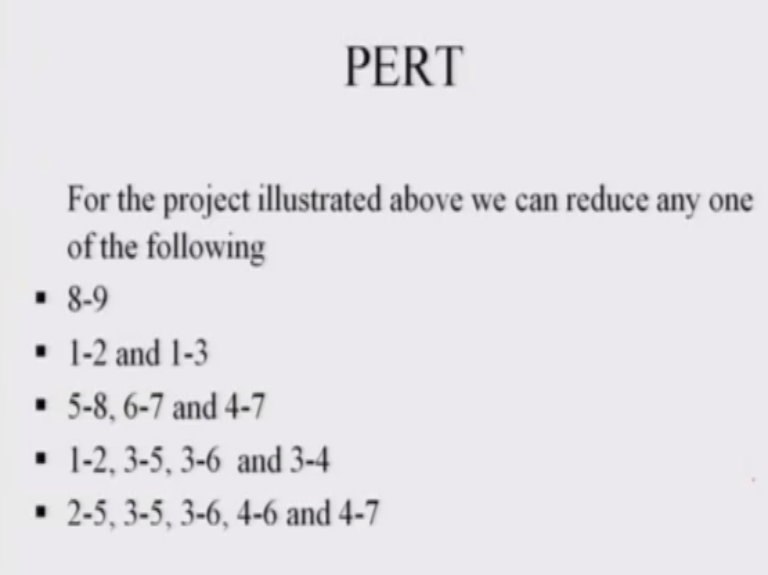

For the project illustrated above we can reduce any one of the following so if I consider the activities you can either gives you eight or nine which is the last one or you can do any combination between one and two and one three or say for example the forth bullet point one three five three six and three four. So any combination can be done so I am just giving the same is such. But when you are trying to reduce the number of days for any activities that activity would basically mean that you are trying to utilize some resources.

If that resources have been utilized then the total cost component are the increase in the total set of activities for the whole project should be considered in such a way for each and every of this set of activities which are there in front of you in slide number two sixty three would be such that the total cost implication of deduction in the cost is highest. But reduction in the cost you may be asking that if I crash the job obvious you would be increase in the cost so what I mean is that.

As you are decreasing the number of days there would be increase in the cost but I will consider that set of activities for which the reduction is highest in the sense that it has the negative impact on the total cost structure for the project. So reduction means I am basically looking at the loss in the negative sense which is positive.

So how you analyze the problem would depend that what is the main focus is it time, is it scheduling, is it cost structure whatever it is but for time being we are only considering the time component are important because we need to find out the critical path or the critical time.

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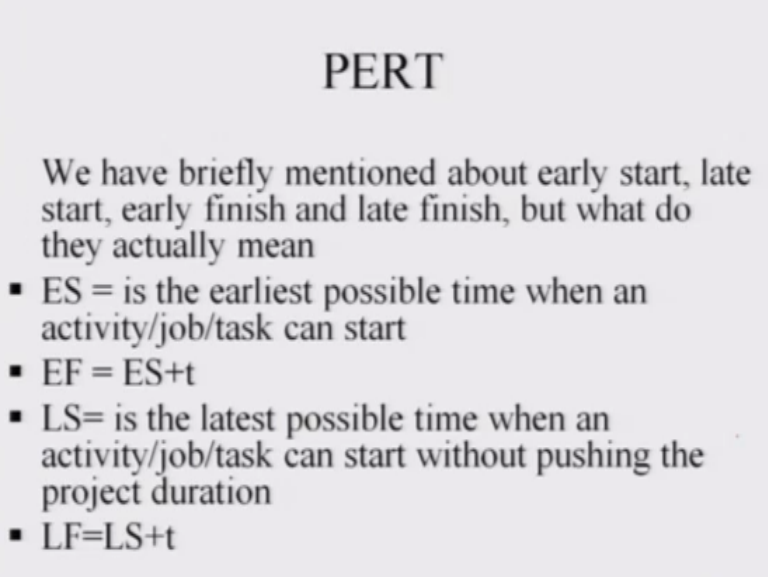

We have briefly mentioned about early start late start, early finish and late finish but what to do what do the actually mean. So early start means is the early possible time when an activity job can start and early finish would be the early start plus the duration of the day, duration of the job. So it was D suffix AD, D suffix B for the activities so it is basically T given her. Late start is the latest possible time when an activity job can start without pushing the project duration over and above the critical time duration.

So the late finish would be the late start plus the duration of the job which we have in the hand which is T.

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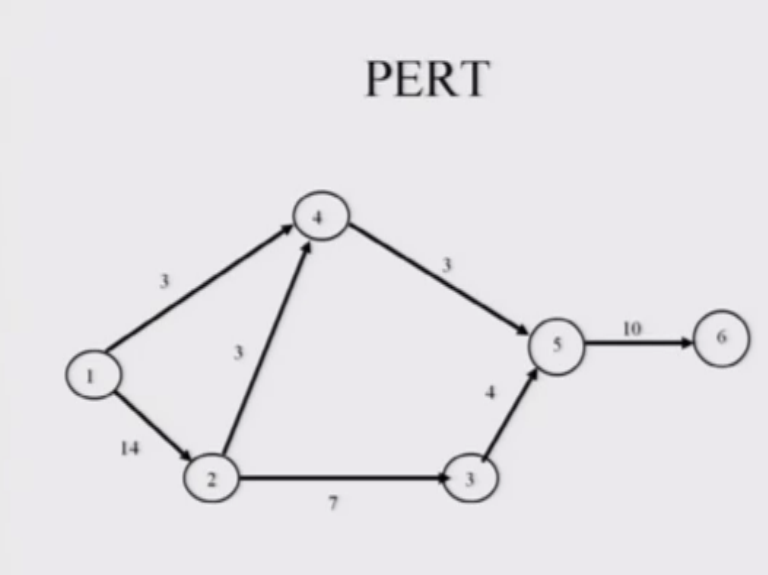

So in this diagram which is simply one which have basically one to two which is the duration is fourteen. One to three is three two to two three is three hence so and hence so forth. So this durations are given though relation would basically mean that if I early start of one is say for example zero so the time when two two can basically start this job two to four is earliest would be fourteen days has to be completed then only basically two can start.

Then it would be start coming fourteen if I am going from left to right the forward pass method I will do my calculations accordingly. If I am using the backward pass method I will basically start from six then six is the end and basically to our calculations of particular from the right to the left.

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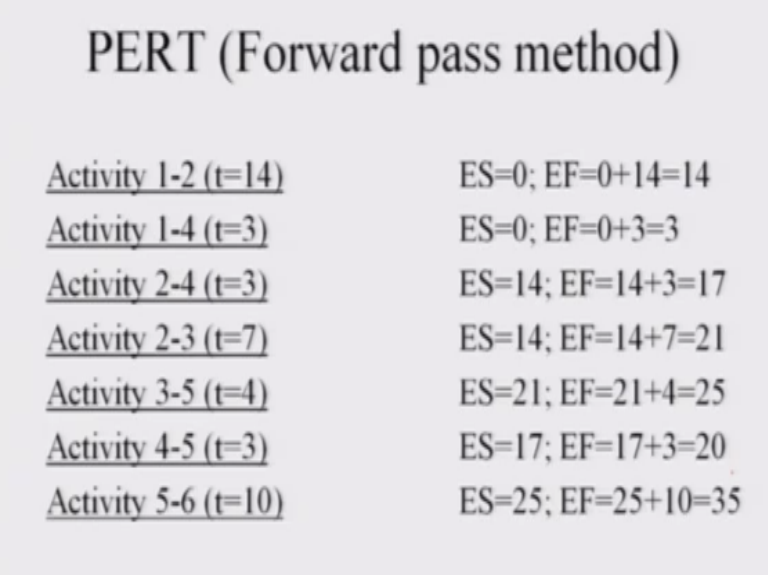

So the forward pass method calculation is being shown as simply as possible so activities are given the first column of the left column and T is the time duration which you just saw in the last time which you is the two sixty fifth slide. The durations are given fourteen three three seven four three ten now let us translate using the forward pass method. If I consider activity one two earliest it can start at zero th time so early finish can be what if it takes fourteen days.

So it will early finish zero when the clocks starts ticking start fourteen number of days so early finish would be fourteen days. If I consider one to four if the early start is zero early finish is basically zero where it can start process the number of days is three. Now let us come to two four and two three collectively now see if you consider two four two three the early start for the activities two two four two two three would not be three because it will be four. Because the jobs if you see that diagram can only start after the end of the fourteenth day.

So the early start for two four and two three is not three but it will be fourteen so if it is fourteen then the early finish then the two four will be fourteen plus three which is seventeen and two two three would be seven is twenty one. Now then if I go basically activity three and five so activity three and five can basically only start after three ends which is the twenty first day twenty first comes here.

And the total number of durations which is there for three five is four days so twenty one plus four is twenty five. If I go to basically activity four five so now let me go back where for four is there. Four is there for three and four is there for seventeen also so that means one to four early finish is three days and two to four early finish is basically seventeen days. Now if I use that earliest it can start is only after seventeen days is over.

So for activity four five it is seventeen here when it can early start number of days is three so the total number of early finish it can seventeen plus three which is twenty. Now if I come to five six I will find out where if five is ending five is ending either on the twenty fifth or the twentieth. So what we all consider the earliest start is not twenty but twenty five because it cannot start until and unless the overall activity three and five is finished.

Because the number of difference is five, five would consider later on how I will consider that so five can fifth, fifth and sixth can be job activity can start after twenty five days are over the total number of duration is ten. Hence twenty five plus ten is thirty five so what you are trying to do is that as you go from the forward pass method you check then maximum duration put that and add the number of days and proceed.

So in this way you can complete the whole forward pass method write the values for the diagram which is shown or whatever the diagram which is there in front of you and once that is done your first step of the job is the forward step method is over. So with this I will end the twenty first lecture starting the twenty secondth lecture I will basically consider the concept of backward pass method and show that how it can be utilized to calculate the slack. The critical path and any delays which are there in the overall project which is there in front of us. Thank you have a nice day