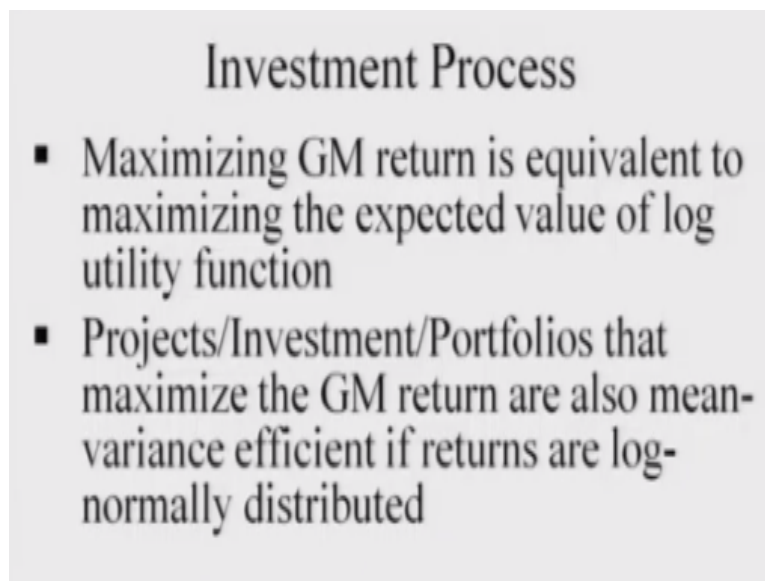


Project Management
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Module No #4
Lecture No #18
Other Criteria Used For Project Selection

Welcome back to all my students and people who are taking this course I am sure everybody is fine and enjoying this course of project management. So this is the eighteenth lecture and as i discussed in the last few minutes for the seventeenth one and I will try to wrap up the utility analysis and the concept in the eighteenth one and then again start of using these concept which i just would be wrapping up in the utility analysis in the area of project management.

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Investment Process

- Maximizing GM return is equivalent to maximizing the expected value of log utility function
- Projects/Investment/Portfolios that maximize the GM return are also mean-variance efficient if returns are log-normally distributed

As we had been continuing we had been discussing the concept of a expected value of a decision what is the fare gamble how people are risk average, risk loving, risk neutral then we consider certainty value then we tried to use the general concept. How you can find out the utility function of human being? Then you we went into mean variance concept or it make sense considering the utility function is quadratic and it is relevance to normal distribution.

Then we went into geometric mean concept and we will continue that we had other concept like safety first principle we consider that as we progress with this eighteenth lecture. Maximizing the i will just mention few important points without going to the theory just for

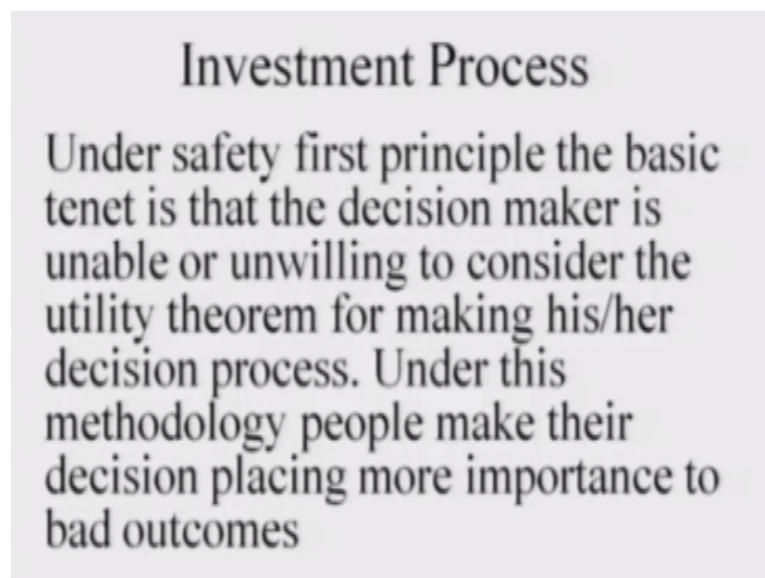
the interest of the people who are taking this course. Maximizing geometric return is equivalent to maximizing the expected value of log utility function.

So for mean variance and quadratic utility function there it make sense while for geometric return and the log utility function they make sense that means one to one correspondence is there between these two concepts separately. Project investment portfolios is that maximize the geometric mean return now also mean variance efficient so this is i will just discuss in two or three bullet points also mean variance efficient if returns are log normally distributed.

So in case returns are for the prices or the stocks prices or the investments or log normally distributed then the mean variance efficient if the returns are considered then the using the concept the ranking or the best policy which we get for any project, any investment, any decision also come are to be true if you are considering the concept of geometric mean return.

So I am just mentioning is as important points not to be dead in the theoretical frame work but more from the practical point of view so try to understand. Now we come to the next concept which were the safety first principle under safety first principle.

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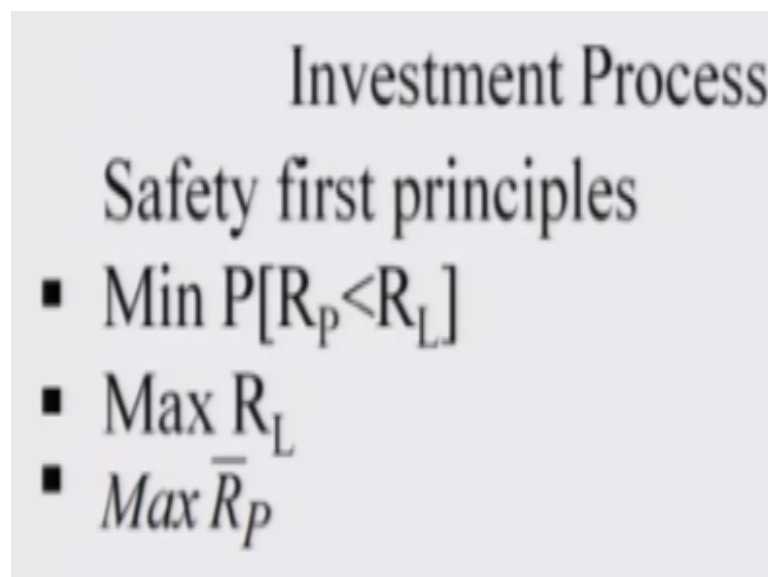
The basic tenet is that the decision is unable or unwilling to consider the utility theorem or the concepts to make his or her decision process. Under this methodology people make the decision placing more importance to bad outcomes that means we always consider people are

negatively inclined and they want to base their situation based on the fact that the negative outcomes are coming out to be more true.

So we are always playing it is very safe to loss of more importance and if profit is there you are happy but not to that extent in the relative scale that if you make a loss you are much more sad. So ten rupees profit if you make with respect to that a loss of ten rupees which you make the consequence for your decision is more efficient by the negative movement of the prices that is minus ten the loss.

Who will generally we will consider that in a very simple the theoretical frame work and the quantity model so that what we mentioned is basically the concept of safety the first principle.

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The slide is titled "Investment Process" and "Safety first principles". It lists three bullet points: "Min P[R_p<R_L]", "Max R_L", and "Max $\bar{R}_p$ ".

Investment Process

Safety first principles

- Min $P[R_p < R_L]$
- Max R_L
- Max $\bar{R}_p$

So safety first principle as define formulations so i will just write down this formulations and you will understand that how they make sense. The first one is that you tried to minimize P is the probability tried to minimize the probability that the value of the so called port full of the projects which we have less than some RL.

RL is basically some fixed value which we have set for yourself consider you are making a decision, you are making an investment, you are making a project, you are applying a project and invest some money. There you will consider that the total amount of the project should definitely be as now as say for example twenty percent not less than that. So you will try to

minimize the such chances that the return of the project does not follow below that overall percentage which you have set for yourself.

So as these are non deterministic processes the returns are changing obviously they would be if you stimulate them or see them or basically have a look at how things are happening there would be chances or there would be some instances whether the value of the so called portfolio is which you have is less than R_L .

But you want to minimize such occurrences other one is basically you want to maximize R_L that means the return based on which you try to analyse your portfolio so higher it is. That means if i consider the real line onto my left are my negative values on to right are the positive values.

So i want to push it as high as possible onto the right and another one is basically trying to maximize the average return of the portfolio considering that the combinations or the portfolio or the project can be change according to the variance you are trying to invest. Say for example there may be instances you want to invest ten percent of your total money in activity one or project ones and similarly based on that you take a decision.

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Investment Process			
If returns are normally distributed then the optimal portfolio would be the one where R_L was the maximum number of SD away from the mean			
Let us consider an example for $\text{Min } P[R_p < R_L]$. Remember we consider the returns are normally distributed and the suffix P denotes the portfolio while R_L means a fixed level of return (5).			
	A	B	C
R_p	10	14	17
σ_p	5	4	8
Diff from 5%	$-1 * \sigma_A$	$-2.25 * \sigma_B$	$-1.5 * \sigma_C$

So if returns are normally distributed then the optimal portfolio for the project would be the one where R_L one was the maximum number of times standard deviation away from the mean. So how many number of times it is away either onto the right or the left or the standard deviation which is sigma is of more importance to us. So let us consider an example we try to

minimize the RP value which is the portfolio for the project being less than equal to less than some RL value.

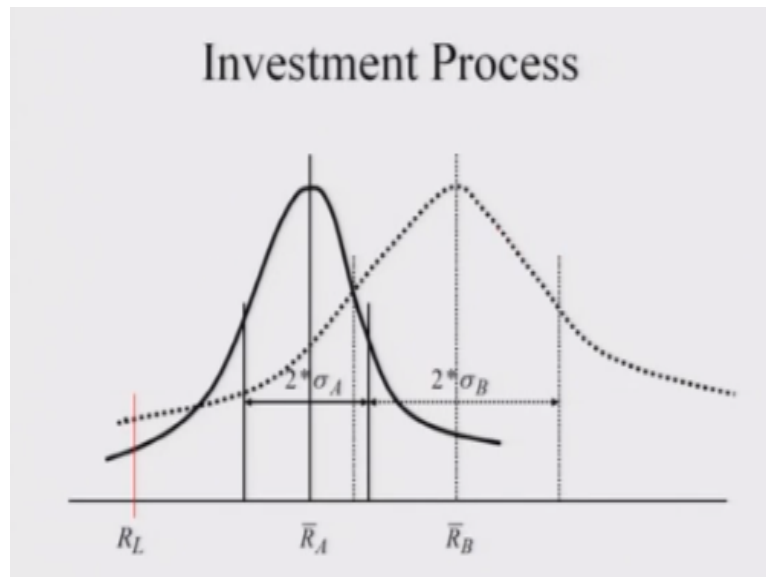
Remember that we consider the returns are normally distributed that means we are considering the concept of utility function being quadratic and the suffix P as i mentioned denotes the portfolio while RL means the fixed return depending on or else is scenario. So in our case consider it is five. So for instances A, B, C are the instances of different example the RP values for A.

So technically it is R P's comma A is given as ten percent RP comma B is given as fourteen percent similarly for CD is given as RP comma C as seventeen percent. The standard deviations for A, B, C are given as five four eight and if you had find out the differences from five percent which is the RL value they are given as it is basically minus one sigma A that means onto the left minus two point two five sigma B onto the left and minus one point five sigma C onto the left.

So basically if you want to minimize then you find out that what is the minimum value because you want to minimize them minimum probabilities such that onto the left now remember here there may be some instance where you need to normalize. So why I am using that concept is that as i have been talking many of you may have been thinking that well the returns and standard deviations are given is in it right. The question would definitely be asked by all of you with is in it right.

If we normalize them and bringing the concept of standard normal distribution in the picture hence ranking is much better. So that can also be done we will consider that later on so in this case the differences as I am again mentioning from five percent which was RL was given based on that you take a decision because as you made motive A is basically to minimize according to the safety first principle concept so there were three concept under safety first principle we are taking the first one.

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So now if i draw the diagram it will make sense so consider normal distribution had mentioned and on along the X axis the returns are being plotted. So you are basically $\bar{R}_A$ $\bar{R}_B$ there would be $\bar{R}_C$ and such set of portfolio for the projects and what i have drawn is basically the return distribution for the dark black one is basically for A. So this is the average value of A and this is the standard deviation.

So this is two sigma means I am going plus sigma onto the right minus sigma onto the left considering the sign means moving to the left to the negative. Again this the average value of B this dotted one is for the return distribution for the portfolio of the project for B and this again if i move from the average value onto the right this is the quantum of movement and if i move to the red this is the quantum movement.

Similarly if i have C i can draw the graph so what is now important is to note that what is R_L . Consider R_L i have fixed there which is the red one so i want to find out that what is the probability of such occurrences of the difference being less than R_L . So if i consider the black normal distribution which is for A then actually what I am interested to find out is first this probability which I am marking is the yellow one and if i basically go for the B.

What I am interested to find out is this one so it will continue till infinity exponential. So i want to minimize the overall area coverage that means i want to basically minimize the probability in the very simple sense. So less the probability better more the probability on the negative sense onto the left is definitely or not a decision to be taken.

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Investment Process sf

In order to determine how many SDs, R_L lies below the mean we calculate R_L minus the mean return divided by the SD. Thus we have

$$\min\left(\frac{R_L - \bar{R}_P}{\sigma_P}\right) = \max\left(\frac{\bar{R}_P - R_L}{\sigma_P}\right)$$

This is equivalent to $\max\left(\frac{\bar{R}_P - R_F}{\sigma_P}\right)$

In order to determine how many standard deviations R_L lies below the mean we consider R_L minus the mean divided by the standard deviation that means the question which i did asked to all of you in the last slide whether it should be normalize is actually true. So if i want to basically find out the normalize mean. So what i do is very simply this i find out that what is basically the difference of R_L value with respect to the portfolio of the project divided by the sigma P which is for that project.

Or else you can maximize so minimizing the R_L minus R_P is also equal to maximizing the difference. So this is equivalent to the formula which is basically given here now if you see what I am trying to do I am trying to basically bring assemblage or conceptual lineage between the R_L value and the R_F value is the risk to interest rate.

So in one of the problems or in the problems we have already been discussing R_F which was small R because the return was calculated by I suffix one minus I suffix zero divided by I zero. So here capital R is calculated according to the formula which i have written on twice. So I am sure it is clear to you all.

So what i want to find out is basically difference between R_P and R_L or R_P , R_F and normalize by the standard deviation and if I am able to minimize this so it also means I am able to maximize and get the same answer. So it is basically simple concept of using the standard normal distribution in order to solve the problem.

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Investment Process

Even though for our example we have simplified our assumption by considering only normal distribution, but this would hold for any distributions having first and second moments.

Even though for our example we have simplified assumption by considering only the normal distribution but this was a hold to any distribution having the first and the second moment. Now there i will just pause and give you the concept like just give in a qualitative sense in the concept of different inequality which are used means statistics and i will just discuss one of them.

How it make sense in order to bring those type of inequalities in the decision making project for a portfolio for a decision for a project or a set of projects whatever that is so first let us consider the chebyshev inequality.

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Investment Process

According to Tchebychev (Chebyshev) inequality for any random variable X, such that $E(X)$ and $V(X)$ exists, then

$$P\left(\left|\frac{X - E(X)}{\sqrt{V(X)}}\right| > t\right) \leq \frac{1}{t^2} \Leftrightarrow P\left(\left|\frac{R_P - \bar{R}_P}{\sigma_P}\right| > K\right) \leq \frac{1}{K^2}$$

So consider this inequality is true with these assumptions there is a random variable X such that the expected value on the variance of X exists it does not mention anything about the

distribution. We know again this RP is probability so the difference between X minus the expected value divide by the standard deviation of the square root of the variance being greater than some T value.

T some fixed value which you have set for yourself is equivalent to one by T is less than equal to one by T square. So if to go to statistics you will understand the overall implication of chebyshev inequality. So what you are doing it in this problem is that you are trying to basically convert the chebyshev inequality in this concept of a portfolio of decision making process.

Where rather than basically take T we take the value of K and try to find out the concept here. So if you note down here what you are doing is that in the initial case we considered R_L as the cut off or for R_F as an equalance to R_L . So now R_F as a equalance to R_L sorry and here what we are doing is that we are trying to find out the difference between R_T for the portfolio with its mean value and then normalize considering the standard deviation is used as the denominator to divide it and make it on a normalize scale.

So if i consider that i consider the mode of that i basically take R_F minus R_P bar and divided by standard deviation if it is greater than T with some probability and that should be greater than less than equal to one by K square. So if i have that and this is k value is given i can find out is that using Chebyshev inequality i can also trying to different type of projects accordingly.

So this concepts of trying to normalize them using the standard normal distribution on using the concept of Chebyshev inequality. Where the first movement and the second movement exists without considering anything of the distributions they can be utilized in a very big way and very conceptually. So as that we get good interesting results for the case of portfolio after the projects.

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Investment Process

As we are interested in lower limit hence we simplify it and have

$$P\left(\frac{R_P - \bar{R}_P}{\sigma_P} > K\right) \leq \frac{1}{K^2}$$

$$P\left(\frac{R_P - \bar{R}_P}{\sigma_P} > \frac{R_L - \bar{R}_P}{\sigma_P}\right) \leq \frac{\sigma_P^2}{(R_L - \bar{R}_P)^2}$$

So as we are interested into consider in the continuation of Chebyshev inequality as we are interested in lower limits. Hence, we simplify it and we have this formula. So what I am doing is that I am trying to basically normalize them. So if i consider this R_P minus $\bar{R}_P$. So I am considering this K value as putting it like this so this is a normalize value. So R_L minus R_P whatever we have consider.

So R_P or R_F whatever it is so I am normalize R_P value is respect with mean value and standard deviation i have normalize R_L value with respect to the mean of the portfolio of the project with it is the portfolio standard deviation. So this value comes out to the less than equal to sigma square P for that particular portfolio of project divided by the differences. So if you see the formula this is exactly what we needed.

Where all the standard deviations and the return of the portfolio is about the projects are given or risk to interest given whatever is there or R_L is given as the case may be ands then we find out this differences of the probabilities and find out the values and rank them or from the highest to the lowest or the lowest to the highest and then take the according values as that we are able to get the best so called portfolio of projects considering the safety first principle.

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Investment Process

The right hand side of the inequality is exactly equal to the decision process # 1 under safety first principle we have considered previously

$$P\left(\frac{R_P - \bar{R}_P}{\sigma_P} < \frac{R_L - \bar{R}_P}{\sigma_P}\right) \leq \frac{\sigma_P^2}{(R_L - \bar{R}_P)^2}$$

Handwritten notes: $P(R_P < R_L)$ and $P\left[\frac{R_P - \bar{R}_P}{\sigma_P} \leq \frac{R_L - \bar{R}_P}{\sigma_P}\right]$

The right hand side of the inequality is exactly equal to the decision process number one under safety first principle because we have considered previously this let me again go back here. So in this project is exactly this less than equal to sign or greater than equal to sign it does not make sense because it just one minus or without one minus and we basically solve the problem.

So what we have here is the probability so what we can do is again if you find out this PR is the probability R_P minus $\bar{R}_P$ by standard deviation less than equal to or less than whatever it is R_L or R_F as it is and you can find it using the standard normal table and solve it accordingly. I will solve one or two in very simple problems later on.

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Investment Process

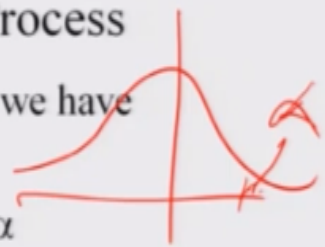
For the second criterion we have

$\max R_L$

such that $P(R_P < R_L) \leq \alpha$

We are given α (say 0.05), then we should have

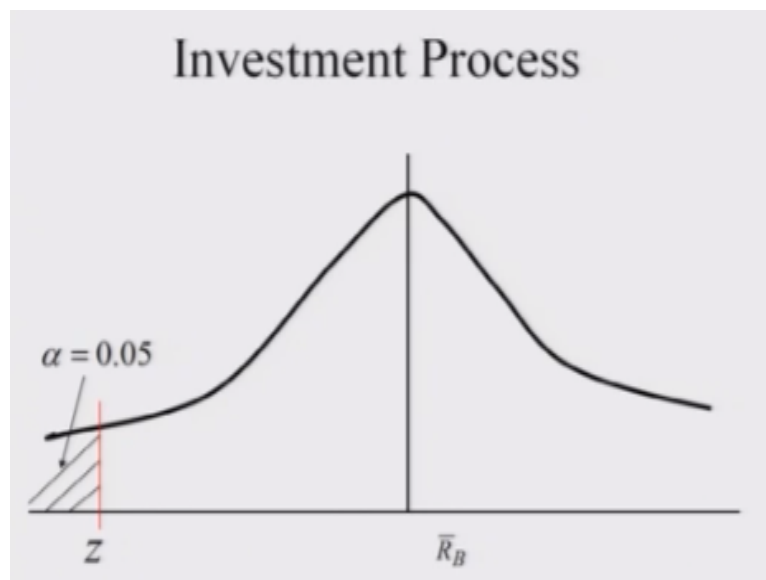
$$P\left(\frac{R_P - \bar{R}_P}{\sigma_P} < \frac{R_L - \bar{R}_P}{\sigma_P}\right) \leq 0.05$$



For the second criterion we are basically maximize RL so first one was the probability finding out the difference between two values less than that such that we have under the condition the probability of RP being or less than equal to RL is sum alpha. So alpha value is given we want to find out the maximum value of RL such that the probability value is already made.

So say for example we have the value of alpha as from the table as zero point zero five which is five percent then we have this value again using the standard normal distribution. This is the value of alpha and if you have the distribution as given this mean value this alpha value is given then you can find out accordingly that what is the second concept of safety first principle. How it can be utilised to find the rank of the project of the portfolio?

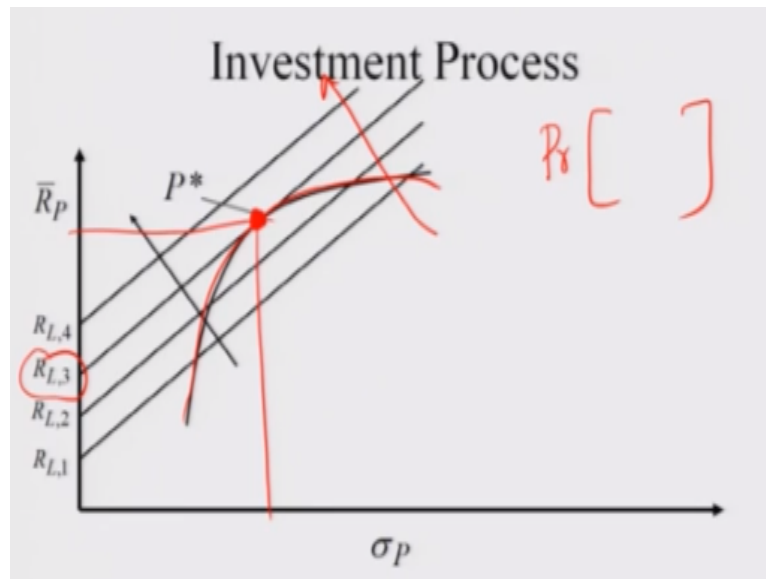
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So again this value this alpha being onto the left or to the right does not make any difference here I am just trying to give you a feel how this problem can be solved. So if you see the distribution which is the standard normal now because in have converted into the standard normal this $\bar{R}_B$ is the average value. So in the standard on normal case it will be zero but I am just trying to denote it using $\bar{R}_B$ for the understanding of the students.

So alpha value is given so alpha value being given that i want to minimize this given this i want to basically pull up RL as far as to the right. So basically better it is if it is onto the right. So i will basically rank them from the highest to the lowest and to choose the basic value as given.

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So in the other case if you consider the values of R_P with respect to σ_P so if you see the Y axis and the X axis it makes sense in the way that the concept of risk return framework from the mean variance concept is being used here. So consider the parallel lines which are inclined and they have the values of $R_{L,4}$ $R_{L,3}$ $R_{L,2}$ $R_{L,1}$.

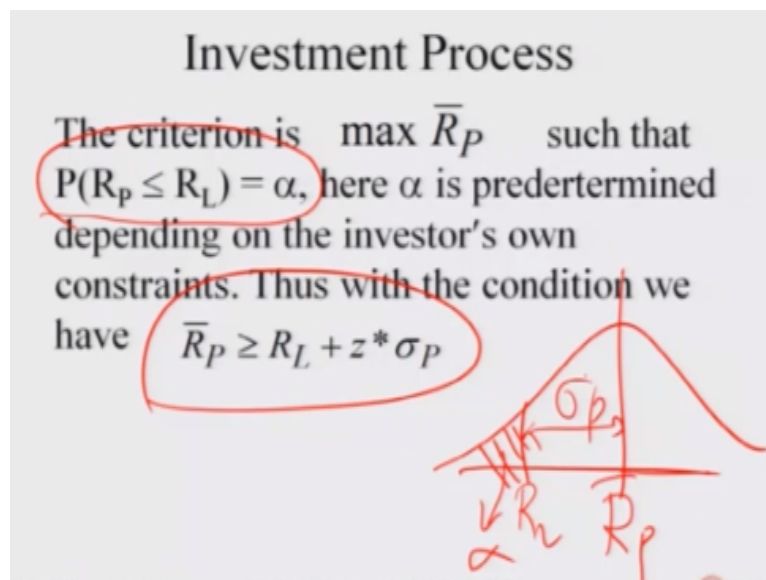
So these are different values are R_L so what i want to do is that consider the efficient frontier is this line which are the curve which is given and i want to find out that set of project portfolio which basically gives you the maximum return so how do i do that so this R_L value i want to maximize. So as per the concept of safety first principle i want to basically maximize a certain values with respect to the R_L because R_P are the return of the projects.

So consider R_L and R_B return value here so what i do is that i start pushing it parallelly. So this is the arrow basically it moves parallel the movement the tangent it from the certain point or the overall efficient portfolio. That value would give me the best combination of R_P and σ_P such that this R_L is the maximum value based on which I am able to attend that particular set of project portfolios.

So it is basically schematic way of trying to make you all understand obviously there are optimization problems based on which we can use it .So what we do is that fix the efficient frontier and start increasing the value of R_L such that it is a tangent at certain point considered is P^* . So at the P^* i find out $\bar{R}_P$ and σ_P and also find out the value of R_L three in this case.

Such that this RL three is that value of RL such that is basically follows the concept of safety first principle and gives you the best results such that I am able to get the portfolio of the projects which has the criteria which I am based on trying to work which is the safety first principle.

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The criteria for the last one is maximize $\bar{R}_P$ such that again this such that condition remains the same. So this R_P being less than equal to R_L as i have considered in the graph here in the last to last graph this was alpha is exactly the same. So this value is basically $\bar{R}_P$ whatever the portfolio or the project is and what i need to do is that i would need to basically push this mean value as far as possible to the right.

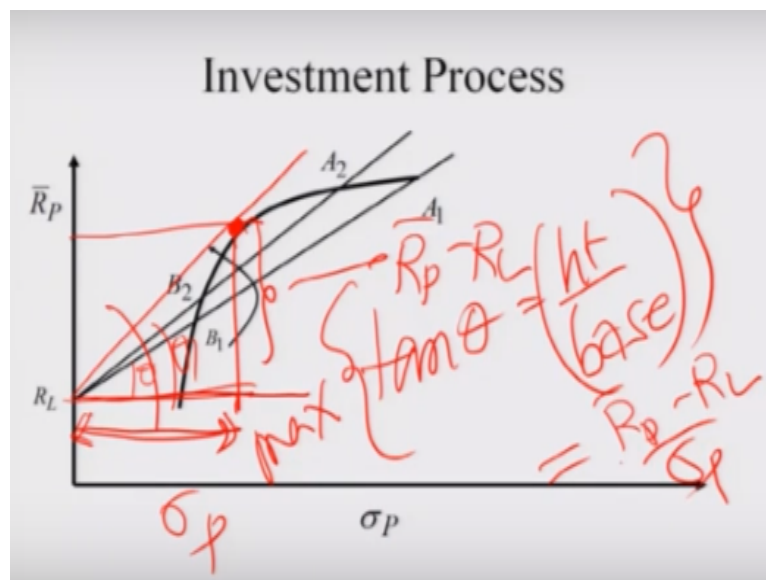
Provided this probability remains the same as alpha. So is this alpha value is the level of risk I am able to sustain for my overall project. So here alpha is determined depending on the invest of own constrains or for the project thus from the condition you will basically have this. So this means that R_P is pushed here. So R_L which is basically here is some standard deviation here this standard deviation.

If it is one it means that it is last minus one onto the right and the left it is four standard deviation it means it is plus two onto the right and minus two to the left. So this Z value would give me how many values of standard deviation that value of R_L is onto the left of R_P such that we are able to take the decision accordingly. So if I am able to maximize R_P then that means i will try to push R_P as far as to the right.

So the value of Z which we see here so R_P I am trying to increase σ_P value is basically standard deviation which is given and we will consider that is fixed because if you keep changing σ_P also then it become a by objective from problem. So i am only trying to basically maximize one objective which is $\bar{R}_P$ based on the fact this is alpha. So as it increases the difference increases. I want to find out that value of Z which is maximum and take my decision accordingly for the safety first principle which is the third criteria.

So what we did was basically be considered the concept of safety first principle from the concept of maximizing the probability or minimizing the probability that means how far it is onto the left. Then i basically consider that how you can basically stress on R_L only and then we want to in the concept of chebyshev inequality considering the distributions are not normal. And in the last case we consider that we try to push R_P which is the portfolio of the project as far as to the right and based on that we can basically finish our work accordingly.

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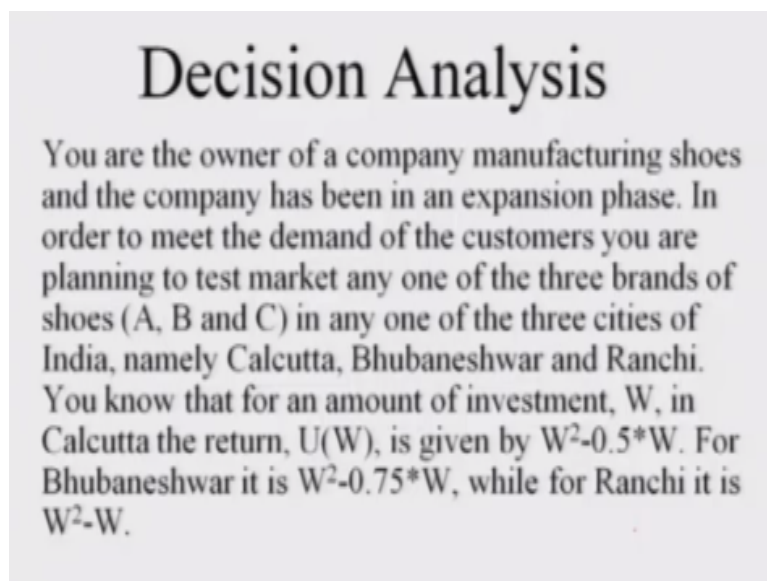


So again continuing this R value so what here is that in this case in the last to last graph we were moving R_L parallel. So it becomes a tangent at a particular P star value in this case what you are trying to do you are trying to increase the tan of this activity. So consider this so this is theta this is theta one so what we are trying to do we are trying to just see the pen marker I am making so I am trying to basically move it anticlockwise till it is basically tangent to this point.

So i have not been able to draw it till it is tangent so what i try to do is that the tan of this angle would basically be the height divided by the base and i tried to maximize that. So if i see how the value comes this value would be RP this value is given as RL or RF.

So this difference is given by RP minus RL and this base is given by sigma P. So if i put in the formula this value is RP bar minus RL by sigma P so sigma RL and RP are in the formulation such that trying to basically increase the tan of the diagram also basically the same concept in the different way trying to basically maximize and used the concept of safety first principle.

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A slide titled "Decision Analysis" with a light purple background. The text describes a company owner planning to test market three shoe brands (A, B, and C) in three Indian cities (Calcutta, Bhubaneswar, and Ranchi). It provides quadratic utility functions for each city based on investment W.

Decision Analysis

You are the owner of a company manufacturing shoes and the company has been in an expansion phase. In order to meet the demand of the customers you are planning to test market any one of the three brands of shoes (A, B and C) in any one of the three cities of India, namely Calcutta, Bhubaneswar and Ranchi. You know that for an amount of investment, W , in Calcutta the return, $U(W)$, is given by $W^2 - 0.5W$. For Bhubaneswar it is $W^2 - 0.75W$, while for Ranchi it is $W^2 - W$.

So you are the owner of this company manufacturing shoes i will take extra another one or two minutes and try to wrap up decision analysis you are the owner of the company manufacturing shoes and the company has been in expansion phase and you have three cities in India namely Calcutta bhubaneswar and ranchi you know that from an amount of investment W utility function is quadratic is bhubaneswar it is other quadratic function for ranchi it is another quadratic utility function.

If you try to utilise that concept you will be using the quadratic utility functions with different parameters parameters are minus zero point five minus zero point seven five and minus one for three different cities and then basically multiply those utility with the corresponding probabilities and try to find out the best equivalence for the expected value.

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Decision Analysis

The proportions of the total investment, where the total amount of investment is Rs.5,00,000, for the test marketing phase for brands A, B and C in the three cities would be (i) 0.4, 0.4, 0.2 in Calcutta (ii) 0.3, 0.3, 0.4 in Bhubaneshwar and (iii) 0.2, 0.4, 0.5 in Ranchi.

The probabilities, which you guess from historical data, of outcomes for brand A, B and C in the three cities are (i) 0.1, 0.2, 0.7, (ii) 0.5, 0.4, 0.1 and (iii) $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$ respectively

So the proportions were total investments are given ABC and the cities are forty percent forty percent twenty percent the arms in say for example Calcutta in bhubaneshwar is thirty thirty forty in ranchi it is twenty fourth fifty the probability are the outcomes from the historical data for A, B, C it is three cities are given ten twenty seventy. Second is fifty forty ten and third is one third based on that we can do the calculations accordingly.

With this i will end this lecture for the utility analysis and then start of the concept of the project management again and then slowly see how the concept of utility analysis and project management can be utilised. So i will also do some problems later on for the safety first principle. So i will just request the students to have patience you will definitely do this problems as we proceed with the class thank you very much.