Project Management
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Module No # 3
Lecture No # 16
Application of Utility Theory in Project Management-III

Welcome back and hope everybody is enjoying this course in project management in the fifteenth lecture which was the last lecture for the third week. So I just discussed with a simple excel sheets am drawing the graphs for the quadratic utility function as a I mentioned in the last three or four seconds or one minute from class of fifteenth one I did mention that I will go in the same way for the other three utility function which are exponential then the logarithmic and the power function.

So let us go to the example for the logarithmic one so logarithmic utility function is LNR W and remember W is always positive.

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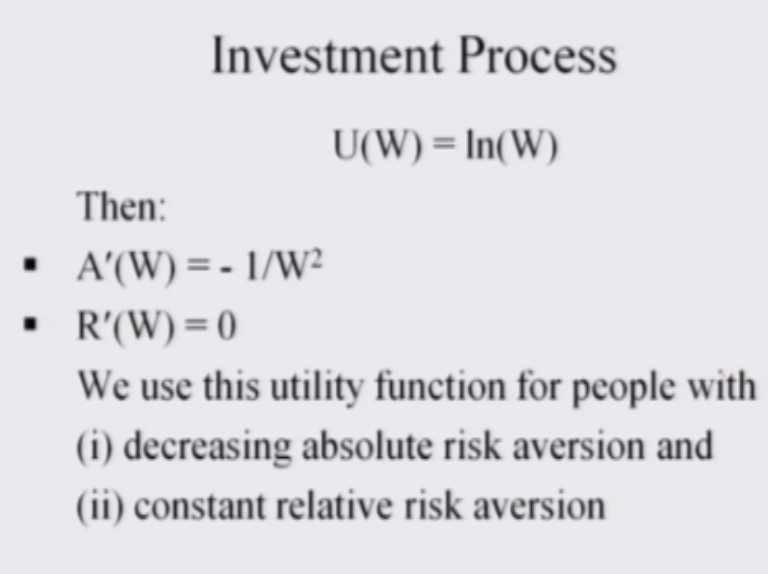

So if I differentiate that and then you use that double differentiation concepts then go to A value and find out prime the value comes out to be for A prime is minus one by W square. So W square always positive so the sign would be always be negative for A prime so with that immediately

come to the first point which is mentioned which means it has got a decreasing absolute risk aversion property.

So if I again differentiate that and use the concept of if I use the characteristic of R and R prime the R prime value comes out to be zero which means it has got a constant risk aversion property and then you can go back to the slide and refer that what it means from the point of view of taking the risk and not taking the risk or being indifferent to the risk.

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So in this example again is thought out example or r or am again taken the value of W in the first column this value are increasing the quantum of one one two three four and with the values are given.

This LNW which is utility function values are again marked so if I find out LN1, LN2 the values are there. I have purposefully skilled U prime which is D LNW of DW which will be U prime and the fourth column which is also missing here. I have done it purposefully so that if you put this values in (()) (02:35) that is UW prime and once you find out the U double prime immediately all the calculation is followed you will use again find out the value of A. A prime R and R prime remembering the fact that the calculations from A to R is simply multiplying the A value with W.

So minus into minus one is one actually go into this last value ten into minus zero point one would be one. So these values which I have written down for R, R considering that I am basically taking two places of decimal person that does not matter. So I have the R values and then the R prime values so if you concentrate on the A prime value which is the fourth column here in this slide which is the one sixty seventh slide and if you look at the R prime value which is the last common in this slide column.

So this R prime values are always zero which means it has got the constant relative risk aversion property and I have just mentioned in the last slide. So it will make sense that what I was trying to prove using simple derivative actually comes out when I us a hypothetical example so it makes a sense that what I was talking from the mathematical point of you also make sense in the practical sense. So these practical values can be changed using any W wealth and any utility function which in the case LN you can do the calculations.

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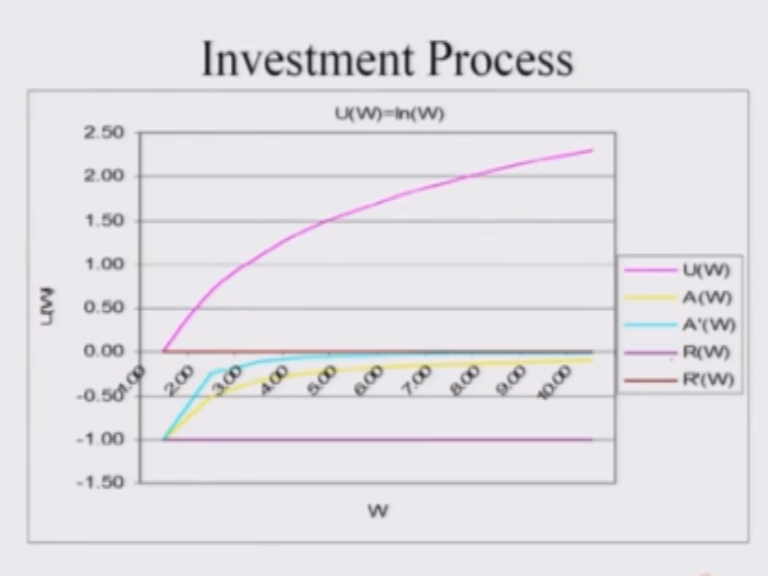

So based on that if I draw the curve you have the utility function LNW which is the point color and yellow one the light greenish blue one is for A and A prime and correspondingly unfortunately you do not have the R prime which is zero which is the X axis. So R and R prime values are given.

Let us go the third example which is the utility function which is exponential in characteristics so this A value the parameter value is technically known to you. If you find out A prime and R prime it comes out to be zero and A so if it is zero then we immediately is got constant absolute risk aversion property. And if it is A constant a parameter for the utility function it has got an increasing guilt risk aversion property because AV consider as positive incase it is negative obviously depending on the example technically is always positive.

So you can again do the differentiation to find out A, A prime, R and R prime again the same example extending it for this utility function values are given in the first column of W, second column is UW third and fourth column was are skipped for your usage only will be calculation which is U prime and U double prime. Based on the fact you find put the third column which is shown in this slide which is one seventieth slide which is the A then we have the A prime. So if you see A prime all zero as just mentioned in the last slide.

Similarly I find out R, R is the multiplication of R into W and then I find out R prime which is again a constant so the value of A so you can find see that whatever we have discussed and using the simple mathematics also comes out in this simple example. So it is constant absolute case and positive absolute risk relative risk property.

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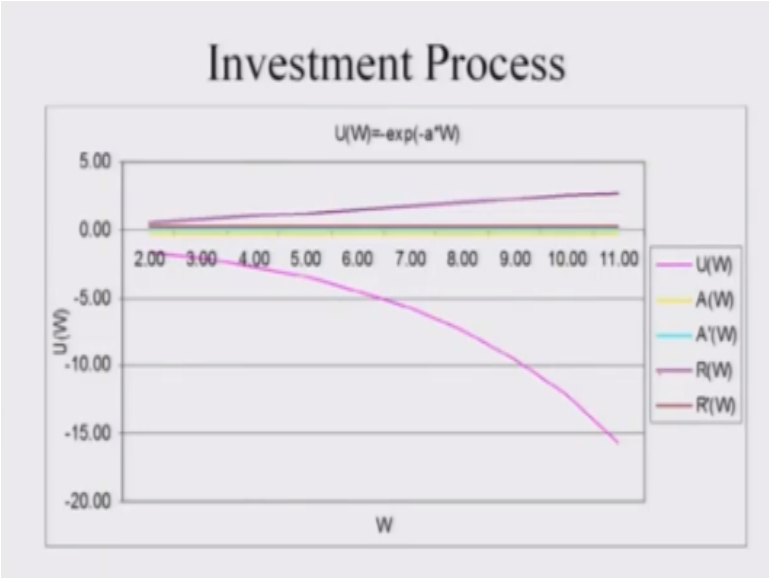

The exponential utility function hence again it is drawn the pink one and following the same color combinations. Pink one is U A is yellow then greenish blue is the A prime correspondingly then we have two different colors is difficult to differentiate in you one is brown which is R prime and one is a little bit violet combined brown which is for R and if you the curve you get this. So use excel sheet to find it out.

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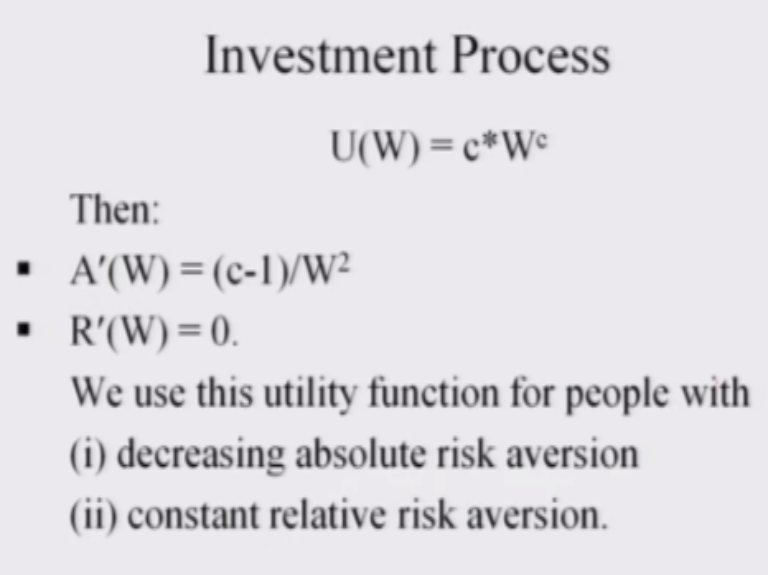

The last example is basically is power function so if you use again the simple concept of mathematics or derivative A prime is given as C-1 divided by W square again W square is positive. And as such W is positive to C – 1 would if you remember I did mention the C value is less than one and not equal to zero. So this will give you a property of prime R prime again you are finding out zero which means it will give for R prime is constant till risk aversion property and for A prime it will be decreasing absolutely risk aversion property.

Continuing in the same sequence of exponent ion I draw W R or write down values of W and the first column U W which is the utility function of the second column. Third and fourth are skipped do it yourself U prime and U W prime based on that you find out A, A prime, R, R prime and have a look A prime is negative because C – 1 would be negative. Because is less than one it is negative and R prime is zero it is constant negative risk aversion property conserving the last column and A prime would give you that it is (()) (08:16) it is increasing absolute risk aversion property.

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I draw the graphs so I have U then A, A prime R and R prime so if you see R prime it was basically a constant value so this is as it is shown and the and the last curve were wavering my finger this is the one so this is basically a line for R prime.

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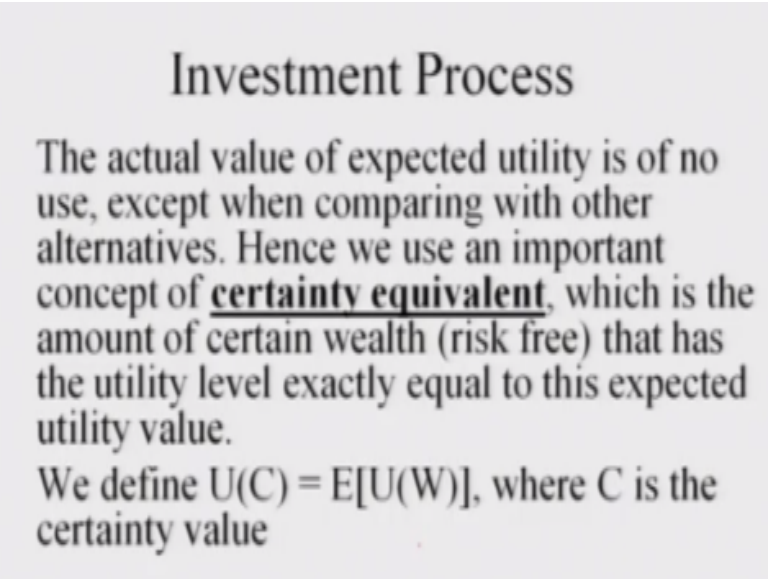

So an now I concept of certainty equivalent has some notion which we have already discussed is basically the concept used in the in the in point of view of investment from the point of view of of finance which is exactly going to mathematic concept of finding the expected value but the reason why we are discussing is that certainty value would basically give a very good notion that how you can make a decision according to what is the value of certainty value.

And based on that we will also make a decision whether the person does love risk is in different to risk and is basically hater or risk. So actual value of expected value of utility has no use to or not consequence to us expect when comparing with other alternative. So you have different alternatives if you remember he decision U problem whereto you want to drill or you want to basically come into the market a new moped you made a decision between different arms based on the expected value concept.

And I did also example where we are considering the decision tree problem one and decision tree problem two specifically for decision tree problem one I did also mention that in case the expected value are same then it go into the concept of variance. And it also I also mentioned that in case if you want to make a comparison between risk and return expected value of risk. You will try to basically take ratio of return to risk rank them from highest to lowest and take the one which is highest in that.

Or if you reverse the ratio which is basically risk to return then you rank them from lowest to highest and the one take on which is lowest value. So the certainty equivalent value which is again the amount of certain wealth which is the risk free concept is basically the utility value of C which is a constant value which will give you the same expected value as coming out from a gamble.

Gamble means a decision so if you see the left hand side of education you have UC so C is the certainty value for which the concept of problem certainty would be one. If you see the certainty event like if you are tossing the coin as I mentioned with very simple example that sholey coin which is there which as heads on both sides in that case the probability is one. Hence it is multiplied by one such that it becomes UC1.

On the right hand side you have for a gamble for a decision or for a project or a investment you have to summation of different utilities multiplied by their corresponding probability. You find out that ten amps you multiply by the corresponding utility with the probability and then find out

the if it is equivalent to you see than what is the value of C. Such that you can see what is the actual certainty equivalent for the decision.

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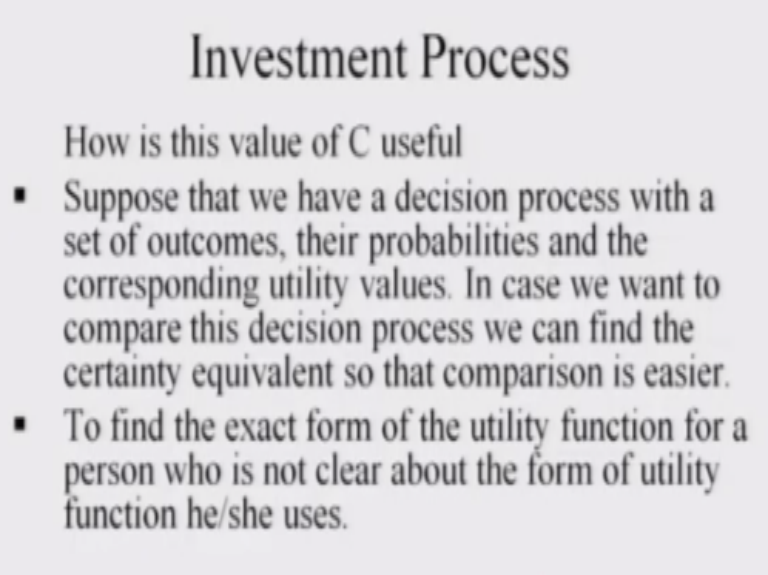

So how is the value C is useful suppose if we have a decision process with a set of outcomes their probabilities and the corresponding utility values are given are known to you. In case you want to compare this decision process we can find at the certain equivalent. So that come the become easier so without going to the rig model of detail calculation we have the value of C then you can immediately rank which decision is better and which is definitely not good.

To find the exact from the utility function with the certain equivalent function which is useful I will come to that with the simple thought out experiment and try to explain it from the point of investment in the project.

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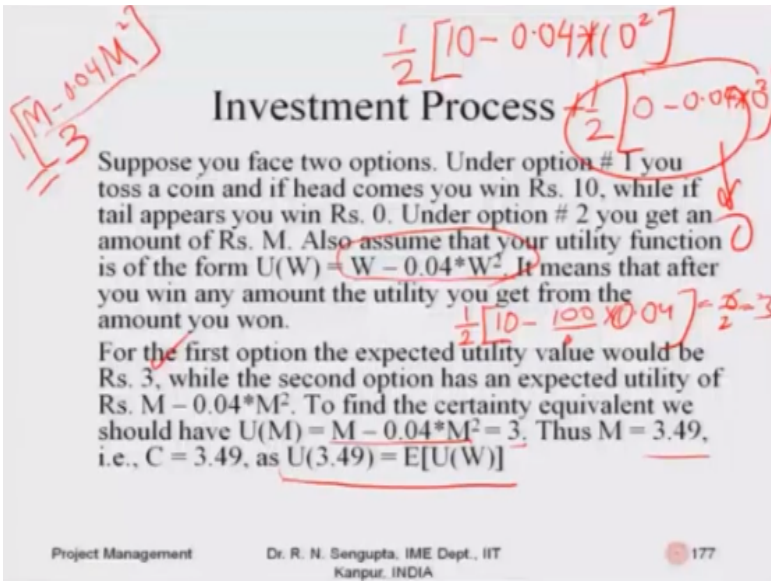

So let us consider with a very simple example suppose if its two option so under option one you toss a coin if a head comes which is a unbiased one if the head comes you win rupees ten while if a tail appears we you will win zero under option two you get an amount W M sorry. Now consider the utility function is quadratic in nature and the utility function is given like this W minus point zero four into W square so it is quadratic utility function.

It means the after you win an amount the utility which you get by that investment is exactly based on the file which utility function is as given. Now in the first option which is the head comes you win ten. Ten is the amount of money which you win it does not mean that utility which is net worth for you and for the outcome two of the tail appears you do not win any value it is zero.

So zero is he value which you win you have to basically find it out from the concept of utility what is the net value which accrues to you considering either the net comes out or zero comes out or whatever value depending on the example which is there. For the first option the expected utility would come out to be three so if I want to find out the expected utility it would be based on the fact that how do I find out the utility.

For the first one probability is half of the head so it will be W is ten. Ten minus zero point zero four into ten square so this is one arm plus at L comes and what is the outcome you are having W

is zero minus zero point zero four into zero square. So this value basically is zero so you calculate ten minus zero point zero four into hundred and then you divide by two and the expected value comes out to be two.

So let me calculate just give me one minute first option could be three and second option let us think let me calculate this is ten minus hundred divided by zero point into zero point zero four. So outcome ten minus four is six, six divided by two this right so the first option expected value comes out to be three. Now this is the gamble, gamble means there is the two outcomes not consider the second which is the certainty value.

Now if you remember just in the last slide which was the one seventy six this slide I mention that for the certain equivalent consider arbitrarily value of C need to find out. So if I want to find out what is the overall expected value of the outcome based on the value C you will calculate it accordingly as I am showing now.

So you actual expected value for that situation is probability is one because it is a output of the probability is one for this it is a certainty value. It will be what symbol of using I am using the value of WM, M minus zero point zero four into M square. So this is the expected value of certainty value this will be equated to three because I have to find out what is the M value three is the value which I have calculated.

From this you have a quadratic equation solve it find out the M value so while so let me continue well the second option is the expected utility and as given this one. To find out the certainty value we equate this value so this equation is equal to three from this we find out three point four nine which actually means that if in this example there was a gamble which game me ten rupees and another one zero rupees we probably half and hour and on the side of the table consider there are gamble or the certainty one.

A value of three rupees forty nine paise was kept on that table then in that case when I when I as a person as utility function as given as in this example I would different in the sense my certainty value with this symbol as certainty on which is there are equal only at the case there is give me

three rupees forty nine one. So what actually means is when I find out the expected value of three point four nine we should exactly come to be the value of three.

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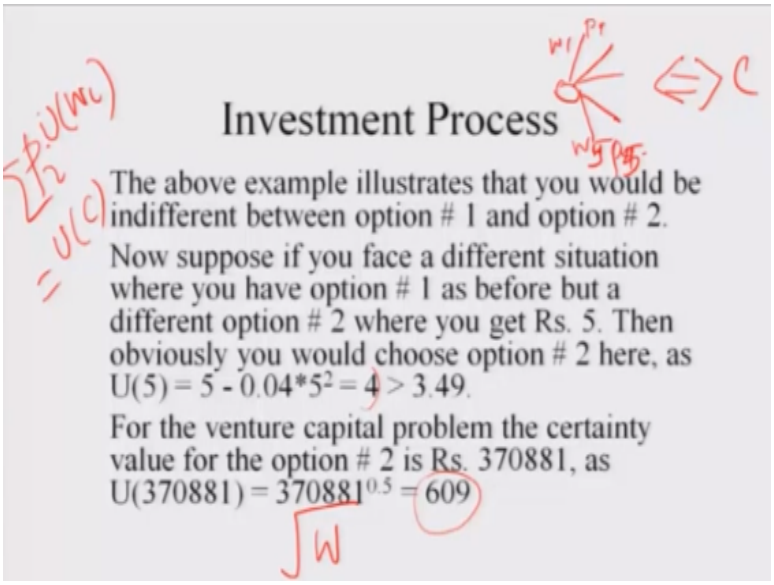

So above example is you would be different between option one and option two it can be many such options to compare point one. Point two for the gamble which I mentioned if it is project or investment it can have different arms in this sense that in the first just last example. It was ten rupees one with probability half zero rupees one with the probability half so this arms let me so let me first erase it. So it had arms like this so they were giving me investments of W1 P1 till 1, 2, 3, 4 this is W4, P4. So this is W2 T2 then you have W3, P3 so this will be five sorry.

This will be W4, P4, W5, P5 so what I need to do in order to find out its equivalent with the certainty value would be 3I 1 to 5 multiplied by UWI this is the values which I have submit up and equal to the U value of C. SO C is given somewhere here from that I find out the certainty value. So there can be different arms also what I am saying so if you face a different situation also as option 1 as before 10 and 0 with probability half and half.

But different option C where you get 5 then obviously you will choose 2 because 3.49 gives you an utility is exactly three but five would give an high utility which is more than 3. So hence if I want to find out what is the utility for that value of five it comes out to be 4 so I will be no tempted to take the certainty decision which is there. That 5 is not the certainty value 5 is the

outcome based on which I with my utility function as given would be more inclined to take the certain event.

But if I want to find out the certainty value which means the gamble and the situation which is 3.49 which I am indifferent. For the venture capital problem to remember there was a government bond and there were three options are ten lakhs, five lakhs and those examples with probabilities of forty forty and twenty percent. For the venture capital problem the certainty value for the option two comes out to be 370881 this is three lakhs seventy thousand eight hundred and eighty one as the overall utility of that certainty value comes out to be six hundred and nine.

If you remember the six hundred and nine was the expected value so based on the fact that utility was quadratic. My certainty value would come out to be three lakhs seventy thousand eight hundred and eighty one so if that amount is kept so that means I would be indifferent between the gamble and the certainty decision which is there in the other side of the certainty. So any value greater than three lakhs seventy thousand eight hundred and eighty one would be definitely mean I would be better half taking that that in the month of probabilistic outcome.

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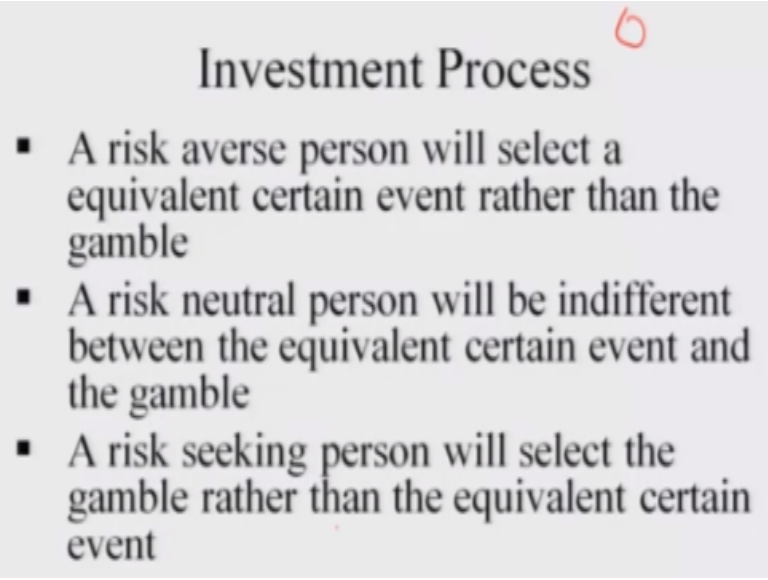

So continuing with the discussions the risk aversion person will select an equivalent certain event rather than the gamble. Risk neutral person will be indifferent between the certain equivalent on the gamble and the risk seeking person will select the gamble rather than the certainty equivalent.

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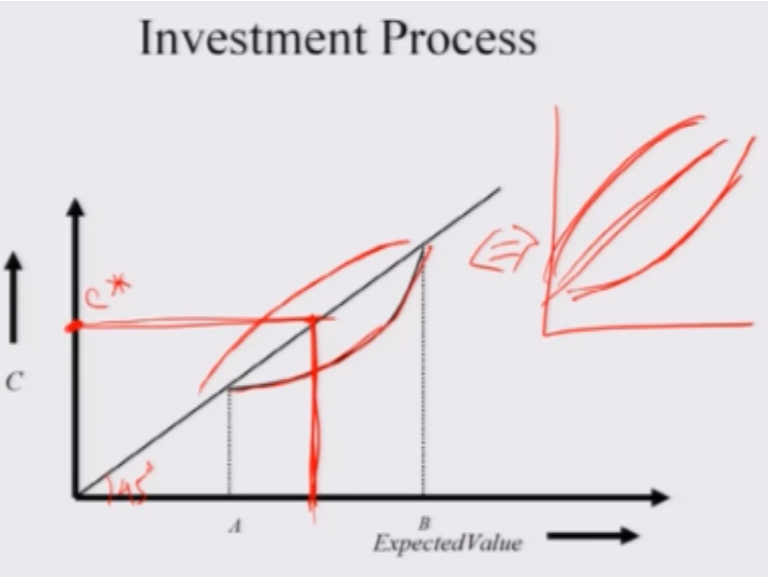

Now in a test give you a third out experiment how we can find out the character so I explain few minutes here. Considering the graph which is there in front of me so their on the Y axis U this C values are the certainty values which are 10 and on the X axis the so called outcomes of the gamble as given. So which is AB BA1 whatever it is but here let us consider that A and B are given.

Now let us for the time being draw a forty five degrees line so this is the forty five degrees line which you have and let us try to analyze the problem. Consider I know there is a gamble with probabilities are known to me consider half and half it can be P1, P2 but I am consider there are two outcome but the sum of the probabilities are one. Consider is unbiased coin it is an half and half and I have A and B as a amount whatever the A and B amount is.

Now I know that the actual expected value of that gamble is A into half pus B into half which would be the midpoint here so I am trying my level best to use my drawing skills. So this is the middle value and I go horizontally on to the left you consider this is sum C star now I place the

gamble and keep a value of C star here. Now if you see this straight line it means that actually the utility function is linear because if I give ten rupees my utility is ten rupees because UW = W.

So that is why it is linear line now coming back to the condition of the example I have the gamble the values are given A and B probability half and half on the left hand side and on the right had side there is a certainty there is no probability and I have kept a C star value which C star is known to me. Obviously when I ask the person place this two examples and I ask him or her what decision would that person take? C star would also know to the person because he or she is that similarly A and B is given.

Now consider the person says that he or she is indifferent if that person is indifferent it means that the certainty value for that decision for that person is C star. Technically it would mean that person is indifferent hence the point as I marked is right. But consider there are two different scenarios person says that he or she would definitely takes C star or not the gamble which means the certainty value for the case for the gamble value is less than C star for that person.

Because if it is equivalent certainty value would have given the decision which was had made that person indifferent between that value which is not C star and the gamble but as C star is more hence the person is more inclined to take the certainty actual event. On the other case the person says no I would not take the value of C star I would take the value of the gamble then there would mean the person has a value for that particular gamble which means the person is inclined to take the risk.

So now with this two scenario's slowly let us start changing the values of the problem let us take change the A and B and I mark it on the X axis. Similarly as I change A and B C star value would also change so I keep asking the person which decision will he or she take would it be C star which are changing C star one C star two and corresponding D values which are calculated based on the expected value and A B value are changing.

The person would basically give me an answer such that I will be in a position to draw the so called utility function on and empirical sense for that person. And if this lines are plotted points

are plotted it can either be a set of points below the straight line or it can be set of points above the straight line and that would come out as you keep doing that third out of this experiment provided that person is giving the rational answer and this this comes out to be true and you.

This is a very simple thought out experiment you can basically fine tune it much better way to get the example. Now if you see this graph the straight line the line which is above and the curve which is below now let us make a one to one correspondence to what you have just done. The color scheme is not there so would not able to immediate understand if you remember the blue one and the green one and the red one.

So in these cases it was increasing it was decreasing and it is constant. If you try to bring a one to one correspondence here the actual property of that person whether that person love risk whether the person hate risk or indifferent risk. Now if the person loves, hates or it different to risk you know that as per the concept of economics or say for so called of utility concept but the utility can be of four types generally quadratic, exponential, logarithmic and power.

And we also know what are the properties of R, R bar, A, A bar based on that you immediately classify that person as being a risk averse person or a risk hatred person or risk indifferent person such that using a simple example of the gamble and the certainty value which is there in front of you. And you late in front of that person you can find out what is the characteristics and based on that you can judge the investment concept or the concept of money being utilized and amount of money being utilized for person for different projects for making a particular decision.

So with that I will end this class and which is the sixteenth in number and start with seventeenth one depending on the different concept of utility and slowly try to cover utility function as fast as possible and then again utilize this concept of utilities in the project management since thank you very much for our attention have a nice day thank you.