Project Management
Prof. Raghunandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No # 3
Lecture No # 14
Application of Utility Theory in Project Management-I

Welcome back this is the fourteenth lecture for the project management (()) (00:20) we have already completed two weeks five lectures in each week and this is the last but one lecture for the third week the fourteenth one. So as we were discussing just in the last lecture in the thirteenth one ended with one or two minute introduction about utility and analysis. And how it will have a sense about decision tree in try to understand. So let us start with full figure and full force about utility analysis.

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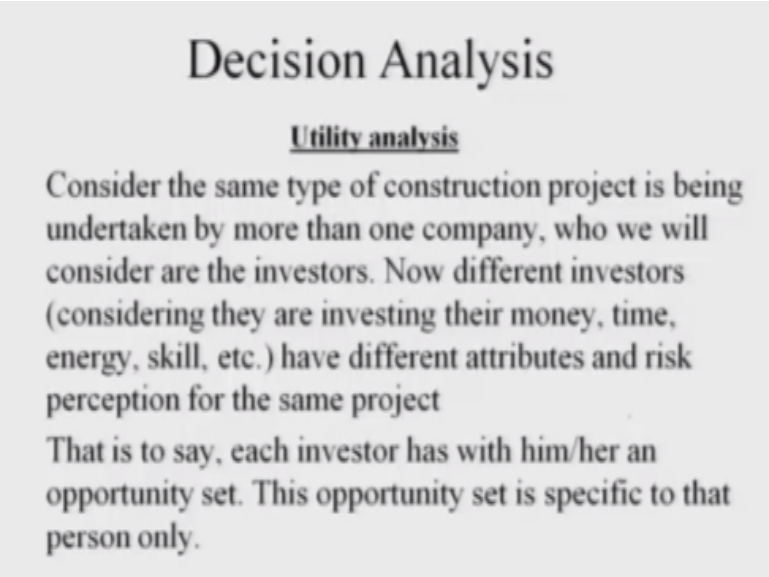

Consider the same type of construction project is being undertaken now you are coming into the (()) (00:58) of again project management though I am trying to give you the different examples but this concepts can be utilized in in project management and features also. So now by more than one company you will consider who are consider in this projects and investors. Now different investors consider investing the money, time, energy scale everything and different attributes risk perceptive for the same project.

So that is to say the project the each investor as in within him her an opportunity said such that based on opportunity said the particular group of persons or the individual persons or the investor who is trying to build up that project will make a decision. So again considering the continuing the discussion consider the shop floor manager has two different machines mark A and B both doing the same time operation consider it is a grinding machine both them do the grinding work or consider it a lathe machine they do the lathe work.

Considering they are say for example the CNC machine they do different type of jobs the outcomes from two different machines are given how let us see this.

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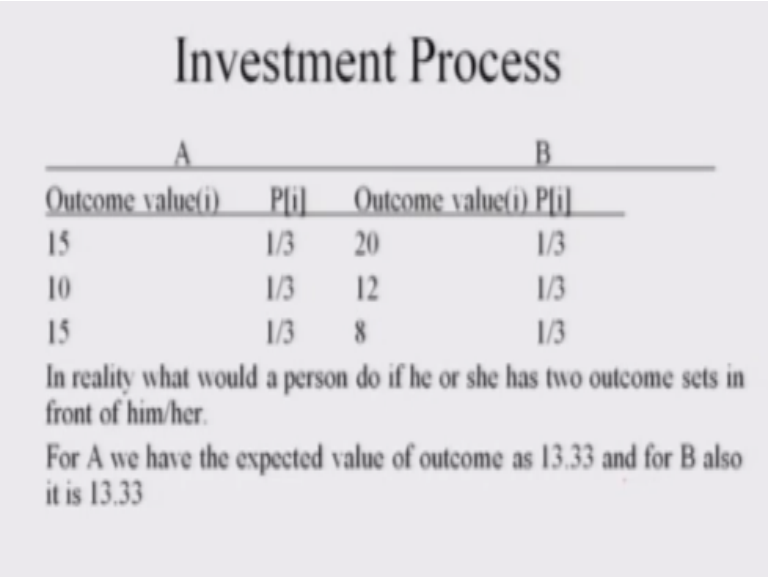

So in the investment process even though the heading means investment process I am considering him from point of view of investor or the project manager. So now the machines are A B and if you see the first column which are the outcomes the second column is the PI and the corresponding probability then you have the outcomes for in the third column. So I am just using the word outcome, outcome basically means the value sense not to rupees, not dollars not yen's it may something else I will come to that.

The last column is is related to again probability so the first two columns are specific to the machine A and the third and the fourth column are specific to machine B. In reality what would

be the person do if he or she has to outcome sets in front of him or her. So if I consider that outcomes values as given fifteen then and fifteen for machine A and twenty twelve eight for machine B which technically means the values of the output the so called utility of the output for machine A is fifteen with the probability of one by three.

Similarly for machine A for the second value whatever the value is now the value is I am seeing what is coming out am not seeing what is going in like to give you a very simple example. Even though that would make sense later on if it is continue the discussion of utility consider there are two human beings one is a rich one an another is a poor person. So the net worth of hundred rupees note two person one is rich and the net value of the hundred rupee note two person score has to different implication.

So the hundred rupees only by say for example for the first person he or she would buy only a chocolate a set of cool drinks which is worthwhile to him or her but while for the second person who is poor he would basically try to spend it or trying to buy some rice, some pulses, some vegetables and milk or ghee whatever it is. So net worth of using an hundred rupees for these two different persons. So the values which are there in fifteen ten fifteen twenty twelve eight are those values what is the net worth not the hundred rupees what we are trying to utilize.

So the corresponding probabilities are given that for fifteen one third ten one third fifteen one third similarly for the second machine or second decision it is twenty one third twelve one third eight one third. Now the question I am arise is that why did not combine fifteen one third and fifteen one third.

What you are seeing in front of you fifteen fifteen are the net output but the equation based on which we are trying to find out may be such that the overall amount input or the monetary sense for these to fifteen may be different if the f of X value what you are trying to get for these two equations which are three in front of you are the same.

So in reality what would the person do if he or she two outcomes in front of him or her so for A the net value would be found out multiplying fifty into one third plus ten into one third plus

fifteen into one third. You take one third outside is the common it will be fifteen plus fifteen thirty thirty divided by plus ten which is forty forty divided by three would give you the value of thirteen point three three similarly if you do it for B again it is twenty plus twelve plus eight which is again forty divided by three because one by three is constant hence the values are thirteen point three three for both the cases which means that if you are trying to find out from the point of view of the so called picture which is give on front of you.

You as a person would so called indifferent between decision A and decision B or project A and project B. Now let us pause you for one minute the outcome value which I have given in the first column and which are given in a third column may be specific to a person. Say for example if I am trying to analyze the same problem from person two for the same type of inputs like consider this.

The inputs for fifteen ten fifteen for the project A and twenty twelve A for project B say for example it is let me give you some values in the algebraic sense not certain values not ten twenty thirty forty not for that. But just to make you understand if you input X1 you get fifteen rupees if you put X1 twenty for A and B if you put X2 you get ten if you out X2 you get twelve.

If you put X3 you get fifteen if you put X3 you get eight now these fifteen ten fifteen twenty twelve eight are the first person. If a second person comes if you are try to analyze the problem in the same way where it is X3 for project A and project B the outcome need not be fifteen ten fifteen twenty twelve eight because that person will try to analyze this problem from this perspective for which the function value or so called utility functions.

I am mentioning the word utility function for the first time may be different so in that perspective the problem would be framed accordingly.

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Investment Process

A		B	
Outcome value(i)	P[i]	Outcome value(i)	P[i]
15	½	20	1/3
10	¼	12	1/3
15	¼	8	1/3

Now for A we have the expected value of outcome as 13.75 and for B it is still 13.33.

Now let us change the probabilities so the probabilities are now for fifteen ten fifteen now fifty percent twenty five percent twenty five percent and outcome for the concept related to the decision B that means the same so in this case A would have the expected value thirteen point seven five an d B would have a expected value of thirteen point three three. So I is the person who has in front of him this set of values where the probabilities has change and now it is thirteen point seven five for A so hence I will take decision A.

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Investment Process

Outcome	Team X	Team Y
Wins	40	45
Draws	20	5
Losses	10	20

Case I		Case II	
Outcome	Points	Outcome	Points
Win	2	Win	5
Draw	1	Draw	1
Lose	0	Lose	0

Let us extend this to a second type of analysis very simple analysis where there is a football match or cricket match whatever you think you are interested in. So the outcomes are given as win draw or loss and I am trying to analyze the position of two different teams. Let us name

them as X and Y so now due to some reason one of the the organization of that that match or this tournament had mention that the outcomes would be point accordingly where a win will fetch to or draw or won or zero.

While the other organization due to some confusion said the win would fetch one draw win would fetch five sorry my mistake a draw would fetch one and lose would basically a fetch zero. So in that case the point points earned are different and the wins and draws are different. So if I try to basically bring that in the picture for case one and case two and try to compare team X and team Y based on case one and case two the situation is like this.

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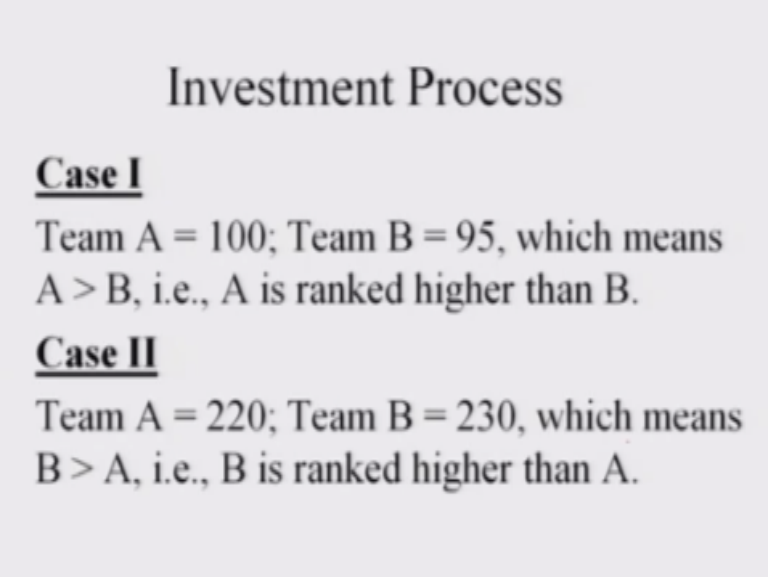

In case one team A as hundred so how do I get hundred I multiply the probabilities and the points B as ninety five which means that A as a ranking higher than B hence A would win. Suddenly the organization took organizer to come huffing and puffing and he or she say no no the points it means different based on case two. If I do the point system as given then in that case team A as two twenty and team B as two thirty hence again the trophy would now be given to the other team which is B.

Which is definitely not expected what that is the confusion which may create if you have different type of points system for the same type of concept which you are trying to analyze.

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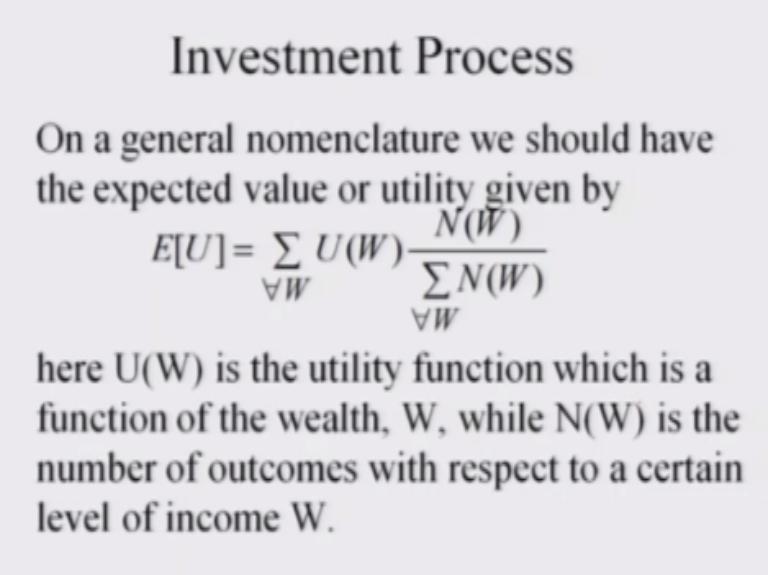

So on a general framework have been mentioning that try to find out the submission of the probability is multiplied by the values and such equations. So generally the concept of utility and value of expected value of the utility is given by this. So if you see UW first let me clarify it means a utility for the net investment W.

W is in rupees or dollars that would always be positive u suffix W is the utility which is the net out effect which is happening by investing W and the second term which is a ratio of with end of N W in the numerator submission of N W in the denominator is basically needs the probability. So the probabilities which you say one third one third one third or in the other problem for the machine it was half one forth one forth and so on for other examples that means in the dominator of the total number of outcomes and in the numerator we have that particular outcome for which the utility is given as UW.

So as W changes UW changes SW changes you will have different number of outcomes corresponding to that you will find out different probabilities. So technically the equation which you have on which right hand side the term which is the last which N W divided by submission of NW is basically the probability distribution corresponding to the occurrences of any particular decision for different output. Output means different outcomes which are happening here UW is the utility which is a function of the wealth while NW is the number of outcomes with respect to the certain level of income.

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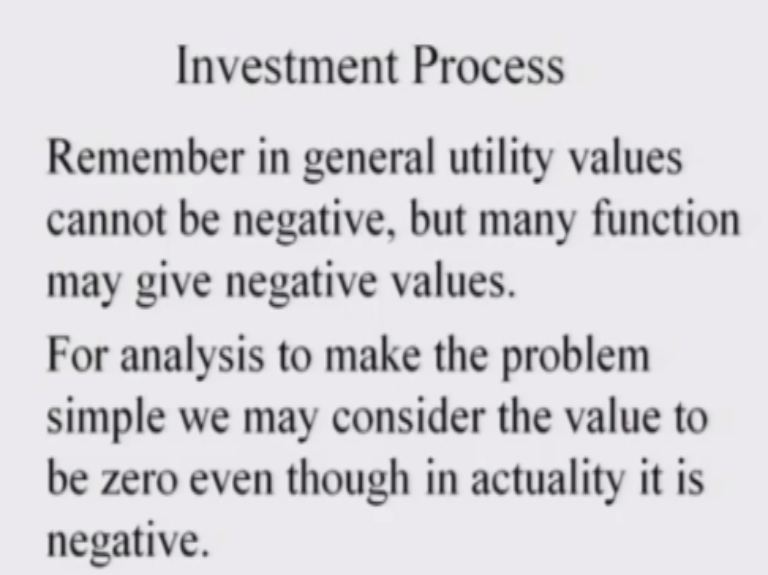

Remember utility functions cannot be negative but in any functions may take negative values will try to skip that report for negative values. Now if you remember let me pause your one minute go back to the drilling problem one of the dealing problem in the value of coming out to be negative if you remember that. So in that case we could also have taken a value of zero and proceeded with your calculation.

But the last lucky part was that if there is no drill the one arm was zero and in this case where it was negative part it was also zero. So in case it is zero in both the cases whether you take the negative sense or the zero value for the net outcome. For analysis will make the problem simple we will consider the value to be zero even though in actuality it it is negative.

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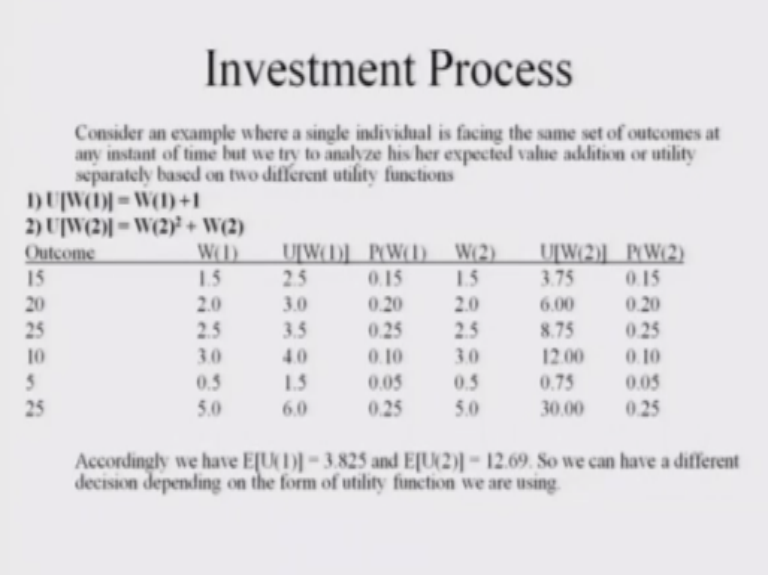

Let us consider so called third explanation or difference trying to explain the concept. Consider an example whereas single individual facial the same sort of outcomes at any instance of time but if you try to analyze his or her respected value which is based on utility function. But let us for the time being and this is for the first time which we are trying to do that for explanation. We consider different explanation example later on for the utility function are different.

In bullet point one utility function is simply a linear equation W and this bracket one means the for the equation plus one. And the second bullet point is a quadratic one that is W2 square 2 in the bracket square plus you have the outcomes again. Now outcomes are so called the numbers based on this not the outcome values they are the outcome values. So they are in the outer column then in the second and the third and the fourth it is corresponding to the utility one values of the investment W that means one rupee two rupee three rupee and so and so hence so forth.

While UW1 is given in the third column corresponding the W values how you find the utilities and the forth column W is the probabilities corresponding to the value which you have. So if you find out the total sum here in this case this one I will just highlight in the sense just mark it. So this is hundred so if it is divided by fifty fifteen by hundred which is point one five twenty divided by hundred become is point two.

Similarly for the last one twenty five divided by hundred point two five if I go to the W and the UW value one point five when I use the first equation which one point five plus one which is two point five. Similarly if I go to two point five two point five plus one is three point five so this utility values are given for the W. Similarly if i go W2 which is the forth the fifth and the sixth column these values are given for the for the investment these are same just for simplicity.

These UW's are the values found out using utility function two so in for the first case it would be what one point five square ply W2 that is two point two five plus one point five it would basically have three point five seven. Similarly if I found out for two two square for four plus to six so I find out all the values so the probability is remain the same. Accordingly we find out the expected value how to find out the expected value it will be this into this plus this into this till the last term this into this sum them up.

Similarly for utility two it is three point seven five equal to point one five continue doing this till the last term which is thirty into point two five add them up. I have the expected values three point eight two second one is twelve point six nine so we have different decision based on the fact that somebody is using utility function one or utility function two.

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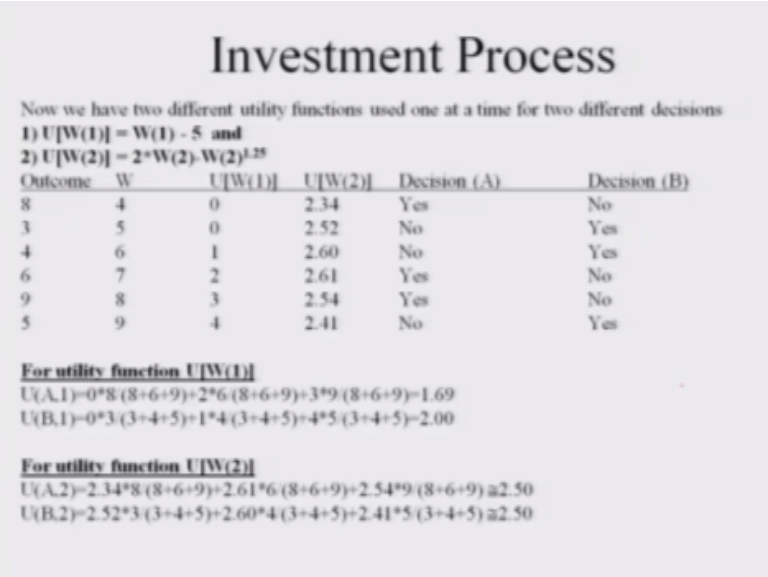

Continuing with a different ideas for the utility functions again two different utility functions first is again linear W1 minus five second is basically a non-linear function not quadratic the

outcomes are given the wealth W is given and utility is given. So now and the utility for the first one and the second one is given this is the third and the fourth column not what is to interesting note is that the last two column is basically mentioned decision A and decision B which means that whether the outcomes are specific to decision A or specific to decision B.

It can be a possible that for one of or to the projects that if I am trying to basically build up a project in in the in the state of Assam I may take some tax benefits. So that would basically be coming to the picture so how I do that from the outcome perspective I will consider that later. So for the decision A the first one is A next to is no then the next two after the note again two yes yes and one no.

For decision B is no, yes, yes, no, no, yes if try to find out the utility concept using A for utility function one sorry let me import it not A one for decision A and B it becomes like this if you see it if utility function is basically zero why?. If I put four, four minus five is minus one will consider is negative being zero. So input is zero if it is five, five minus five is zero so based on that we find out this calculation.

Now if I do it for utility two it is basically two into four minus four to the power one point two five we I find out this value. Now for decision A yes and no's are given how do I utilize? Consider the first term this is the utility value which is coming from here zero and why this term aided divided by eight plus six plus nine because out of the outcomes which see yes this is yes this is yes and this is yes.

That mean eight divided by the total number to which is eight plus six plus nine similarly for the second one which is yes two utility multiplied by six divided by eight plus six plus nine. For the last which is yes is three and what is the corresponding probability of that is nine divided by eight plus six plus nine it comes to one plus six nine. Similarly if I do for B which for this function for the decision sorry for the utility one so it will be yes here so is zero into three and divided by what?

So what are the yes A's one, two, three this is three, four and five so here with four, five if I continue doing it will be one into four that means yes one into four divided by three plus four plus five and the last one is four into five by three plus four plus five. One point six nine two where utility two again I do the same calculations if I find out two point five two point five. So what is interesting to note?

Is that again type of problem which we did for the football game or the cricket game match so if I try to use utility function one then I would definitely rate decision B to B to B decision A. But incase if I am trying to utilize it utility function two then you see both the values of two point five that means in difference based on the utility I am using. So I am I would basically be there A or B.

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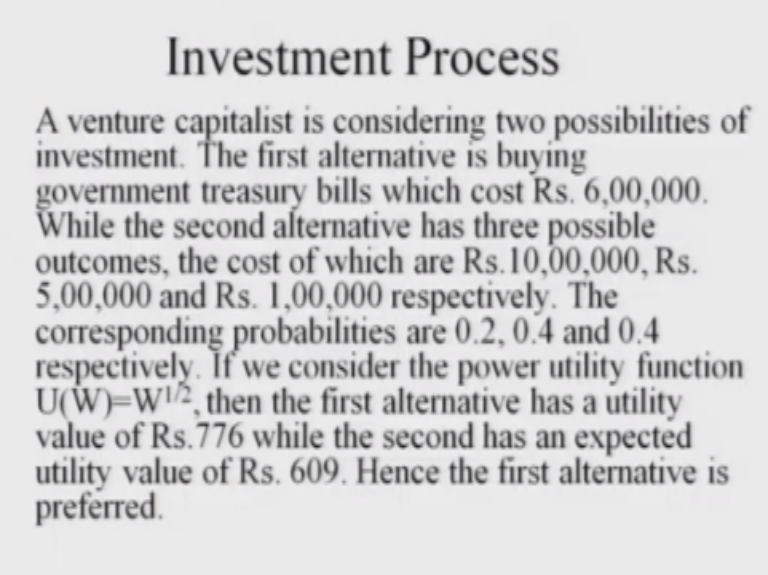

So consider is very simple example the venture capital is considering two possibilities of investment consider they are projects. The first alternative is government treasury bills which cost buying is also project if I mention project in a very big sense and example. So which cost six lakhs while the second alternative has three possible outcome which are costing ten lakhs five lakhs one lakhs is corresponding probability twenty percent forty percent forty percent consider the utility is quadratic.

So in the first example alternative utility value of seven seventy six so why it is seven seventy six government treasury bills do not have any risk which means probability is one. So if I want to find out expected value two B square root of six lakhs multiplied by one which would give you seven seventy six you can find it out. Again I am mentioning the first decision would have the expected value as square root of six lakhs why square root because W to the power of half and why multiplied by one because it is a treasury bill which is a probability of one.

If I come to the second decision what are the outcomes there are three outcomes twenty percent forty percent forty percent what are the values are monetary ten lakhs five lakhs one lakh what are the corresponding utility? It is square root of ten lakh in the first case square root of five lakhs in the second case an square root of one lakh in the third case. So the expected value would be square root of ten lakh multiplied by point two square root of five lakh multiplied by point four square root of one lakh multiplied by point four.

So do the calculations yourself and find out the value comes out to be six zero nine so hence the first alternative is to be chosen now let us mention few things if you change the utility make it say for example W2 the power half one forth or W square or W cube or W to the power two third whatever it is. You will find out the corresponding utility function are changing hence the expected value of the utility would also change. Say for example it is W square so in that case W square it will be square of this value multiplied by one for the first decision.

For the second decision it is sum of three terms what are the terms? The first value of W square multiplied by twenty percent of point two five lakh which is the W value for the second outcome in that decision square to forty percent and third one is one lakh square into fifty percent so you can find it accordingly and compare them.

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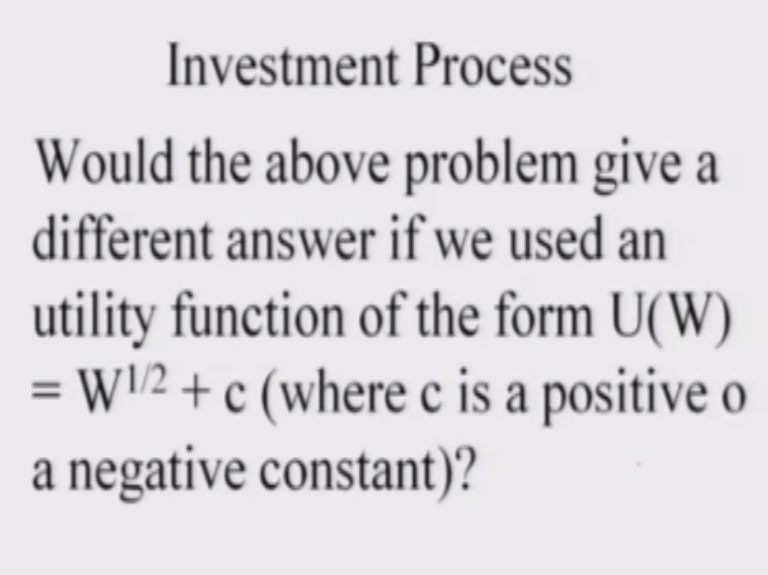

So for the other problem question definitely comes is that if you are used to utility function which is the value of W2 power half plus or constant C value would it change the overall outcome? Answer is no because W is there which is fixed means in the sense the functional form is fixed then we will have basically the value of C being coming into the picture would have basically no affect in the calculations accordingly.

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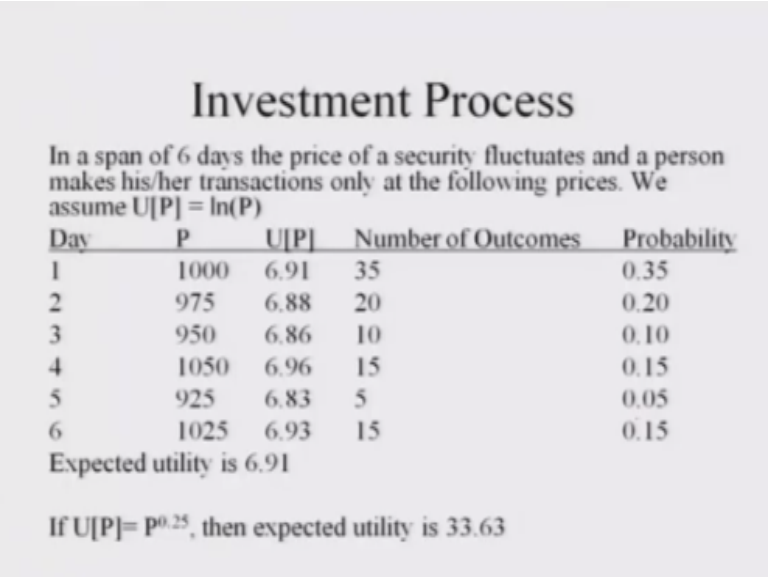

So now consider a security investment the price of the security fluctuations is such a way that it is given by log of the prices. So log of the prices gives you the total utility number of days is given on a simulation science price have been say for example one thousand for how many number of outcome for out of this hundred days this is thirty five price nineteen to twenty five

for how many days five days and the corresponding utility is based on the fact that you find out log of one thousand six point nine one log of nine fifties is six point eight nine six point eight six and so an henceforth.

On which are the prices given on second column and the utility in the third column outcomes are given find out the probability find out the expected value as given in the formula. Utility multiply the probability sum them up in the value six point nine one. If the utility function is P to the power point two five corresponding utility concept to be the value concept would be thirty three point six three.

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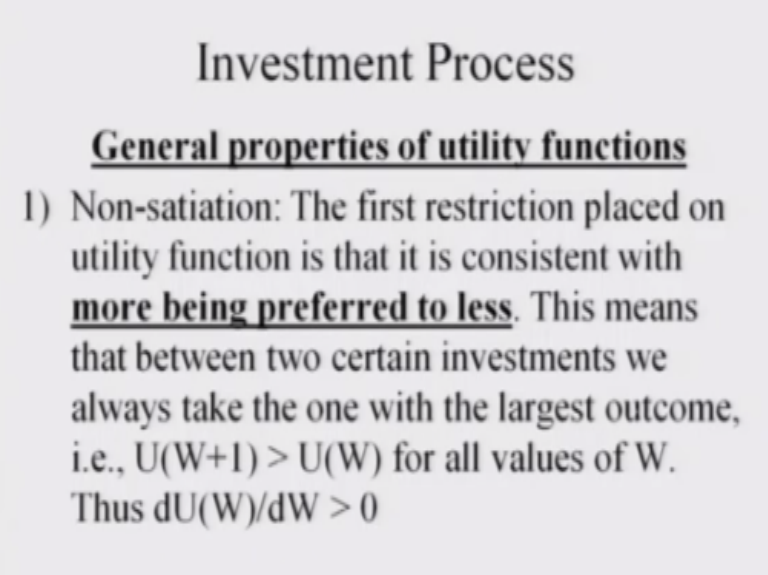

Now if basically mention three properties of utility so the first concept is more you get more you want which is the concept of enunciation which may not be true in actuation. So if I basically try to give an example why it is not in the actual sense let us consider very simple example which may not make much sense from the theoretical point of view of project management but it will be definitely give you an idea. Say for example one percent like sweets and early in the morning I give the person he or she is very hungry I gave him or her three sweets.

The person is very happy it sit up its really satisfied so again I asked this person he or she would try to have more sweets person says yes I give him or her the forth the fifth the sixth and each time the person relishes and takes it now consider after the tenth one the persons actual appetite

has become zero that means he or she total fill cannot take any more sweet and it has become very sweetie for the for that person and the person stomach is full.

So in that case when I offer him or her the eleven sweets the person would try to definitely no even though likes sweets which means the overall benefits the person had trying to consume the sweets the benefit can be taste buds for getting the person was very hungry and such examples. So for the eleventh one no which means the utility value the net worth the person gaining by consuming each extra unit of sweets trying to invest is now scrolling out to be coming out to be negative which we would not consider for enunciation.

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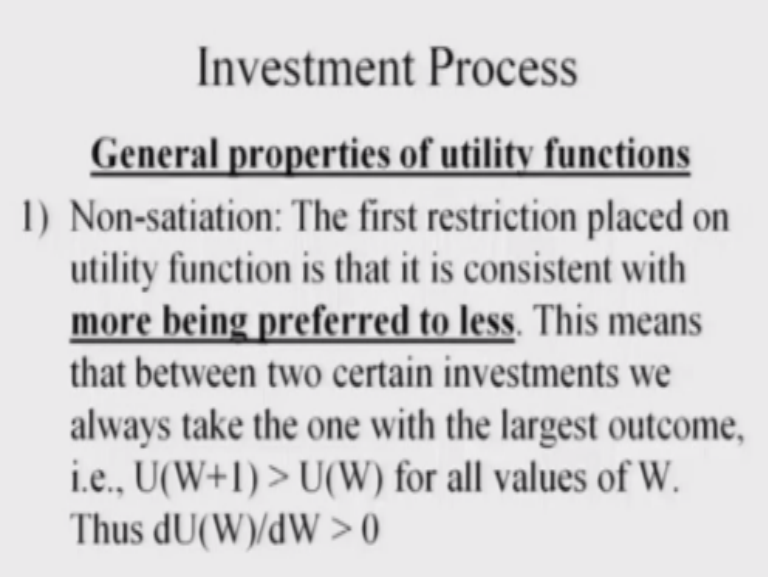

Which gives the definition that the first restriction placed on the utility function is that more is always preferred to less it means that two certain investment will always take the case. Where the utility function for W plus one be positive and greater with respect to UW which means first derivative is zero.

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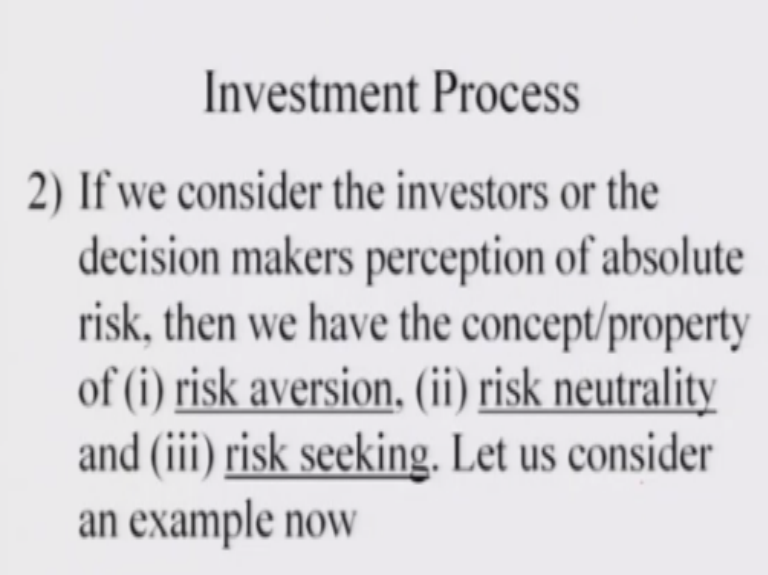

The second one is that we consider the investor or the decision maker perception absolute based on prefix one is concept of risk aversion next is the concept of risk neutrality and third is the concept of risk seeking. So let us consider that with an example.

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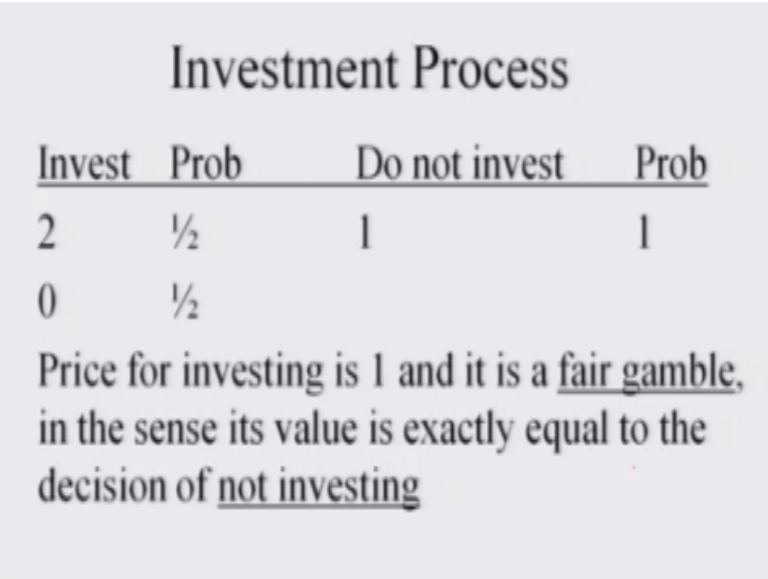

So consider the person i am trying to basically play a game with you and there are two outcome there is the coin with the actual Indian unbiased five rupees coin and the probabilities are half and consider you invest something and the outcomes are you get two outcome two value which you get two with probability half and another you do not get anything and this is half. So that means you invest some amount of money and the net outcome is two with probability half.

And you invest some amount of money the net value is zero with probability half in the other case you consider the sholay coin am sure all of the participants must have seen the famous Amithabachan Film Sholay were the coin basically have the heads on both sides. So consider you toss that coin and invest you do which give the value of one probability one. So if you consider these two examples with means the expected value in both the cases are one. One into one is one two into half zero into half is one.

Now consider this is the fair gamble let us now slowly change the stick in case or the coins given the same but let us consider the overall value which you are getting is not two not one or not zero. Zero name is zero but this value becomes twenty thousand the value becomes ten thousand. So in this cases expected value remains the same because twenty thousand into half plus zero into half one side and one ten thousand into one on the other side.

What is the interesting thing to note is that as I keep increasing the stakes at some point of time there I will find out person in the initial case was definitely mewing for this case at some other stage some other value person would be indifferent between these two decisions and in the third case the person as I increase the straight the person would be switch on to the deterministic sense.

Which means that the concept of risk aversion risk liking risk in different concept would be based on the fact what is utility function what is the amount of money which is invested and how the situation is being handled. With this I will close the fourteenth lecture and continue with the discussion of utility analysis with more examples to come and it relevant to the project management have a nice day thank you.