Quantitative Finance

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Module – 01
Lecture – 05

Till now we are discussed the case of, trying to find out the efficient frontier, if you remember considering only n stocks, all of them in risky; that means, they have some standard deviations and some returns. our main question in the next step would be, what if, we have one risk free interest rate also, which means basically very simply; the bank interested, or if you remember the T bill T 91 day bills.

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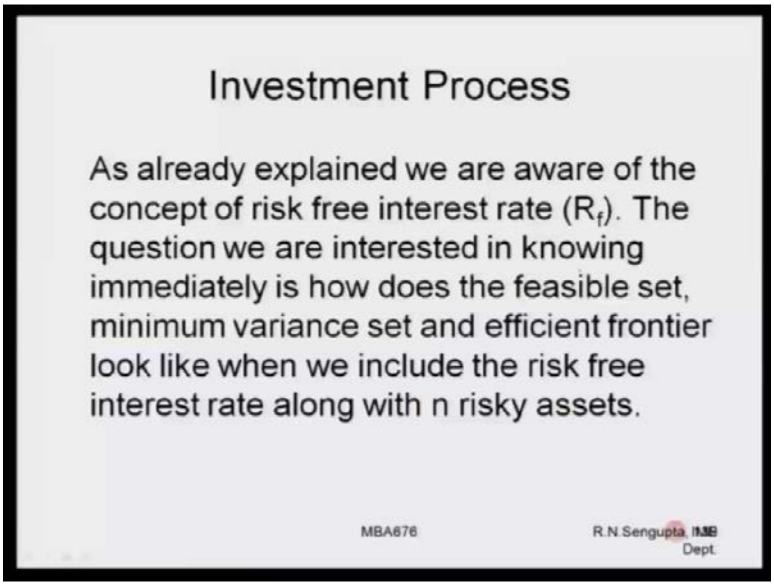

So, as already explained we are aware of the concept of risk free interest rate. The funny question is, now of more important is to us, is that how does the feasible set look like; does it change, does it change, does it have the same shape and size. Considering that we have now have a new portfolio where, the n risk is assets are already there plus the nth plus 1 is being added. So, now, we are basically n plus 1 different type of securities which are there, where n of them are risky, and nth plus 1 is risk free interest rate.

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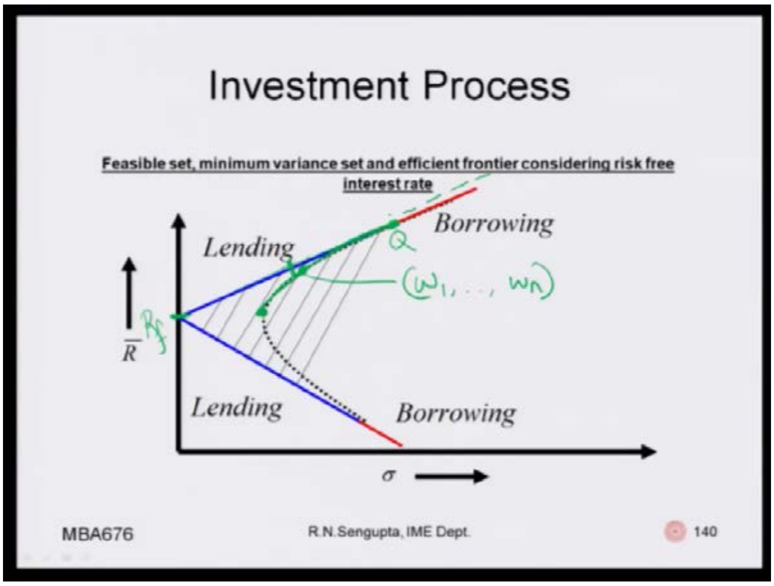

So, now, consider the diagram, and in little bit detail. So, if you see the dotted lines which you have drawn. do not confuse that dotted line with the short selling case. If you see the dotted lines that was the feasible set, which we have initially explained, and to make things easier. Let us consider that the actual feasible set, basically resulted in the frontier which was this. So, this is the minimum variants point, this is the maximum one which we had, which means corresponding to the case where we had lambda zero mu one mu one lambda zero which we have already discussed. And any extension would basically be for this green one would be the short selling case.

Now let us bring for our first instant the nth plus 1 which is risk free interest rate. So; obviously, to remain that it has no value of risk; that means, that point would be, along the y access which is the return. now what we do with that. Now let us pause and see, that how we can explain that qualitatively. Initially, we had this; say for example, arbitrary any point, which is a combination of n risky assets, where rates are this.

Let us consider the rates as they are given, and this is the risk free interest rate. So, what we want to achieve is, now our overall set of feasible region you will expand in the sense. Now you have the nth plus 1. So, if you consider simplicity, what you will do is that. If you draw a straight line; the blue one, blue and the red one combine together. What is the significant of blue, and what is the significance of red I will come to that within two minutes. If you consider and draw tangent, with respect to the initial efficient front a which you have, then it would mean, that any combination of two different portfolios. I am using the word portfolio in very general sense. the first portfolio be the combination of all this n risky assets, with rates w 1 to w n, and nth plus 1 is the risk free interest rate. So, if we combine them, any combination when there is no short selling. what is short selling in this case, let us discuss. There is no short selling between the risk free interest rate and the n number of such risk assets would be the straight line, which is tangent to this point, starting from r f, and; obviously, if you look at the bottom figure, this is again a triangle with the blue and the red extended.

So, any region inside the hash to one which we have the black line hash one, is basically feasible region. And again if you apply the property of non satiation, and basically the property of risk aversion, you will get this straight line which is, where I am drawing my the curve, till this point we have no short selling, and if you basically continue, the red one is basically short selling is there. Now instantly you will realize, that very initial case of n risk assets had a efficient frontier which was curve. Now the efficient frontier, is just a straight line, with two points which are important for us. Point one is the risk free interest rate, and point two, we already know, because if you remember when you solve the m v p which is the minimum variants point of the maximum point, were taken that Lagrangian 1 lambda 0 mu 1 mu 1 lambda 0. So, with using that you have already found the maximum point.

So, join those maximum point, and the risk free interest rate and immediately you get the efficient frontier for the new case, where n number of risk asset plus nth 1 is the risk free interest rate. Now, if you are at point this. let us mention this as q, and let us mention this as r f. So, if you have portfolio says that it is in q, it means that you have invested all your amount of money in n number of risk assets and zero amount of money is there in the bank. If you are at r f, it means that we have invested all you money in the bank, kept in the bank, and you have not invested any amount of money in the risk assets, n number of risk assets, but now if you basically extend to the red one, it means that you are borrowing, but borrowing what. You are borrowing from the bank; that means, you are going short selling on the bank, and trying to invest that extra amount of money in those risk assets.

If you are in between, say for example, you are at mid point, it means that you invested 50 percent of your money in risk free interest rate, and 50 percent of the money in q. So, now, our life is much simple. In initial case you would have curve which is the efficient frontier with extension for short selling being there. And in this case we have a straight line, with the blue one being. There is no short selling which is basically lending. lending means that I am basically investing some money in this risk free interest rate, and some money in the n number of risk assets. And the red one will basically means borrowing that I am short selling from the bank, getting some money from the bank, and basically trying to invest in the stock market, which are basically n number of risk assets.

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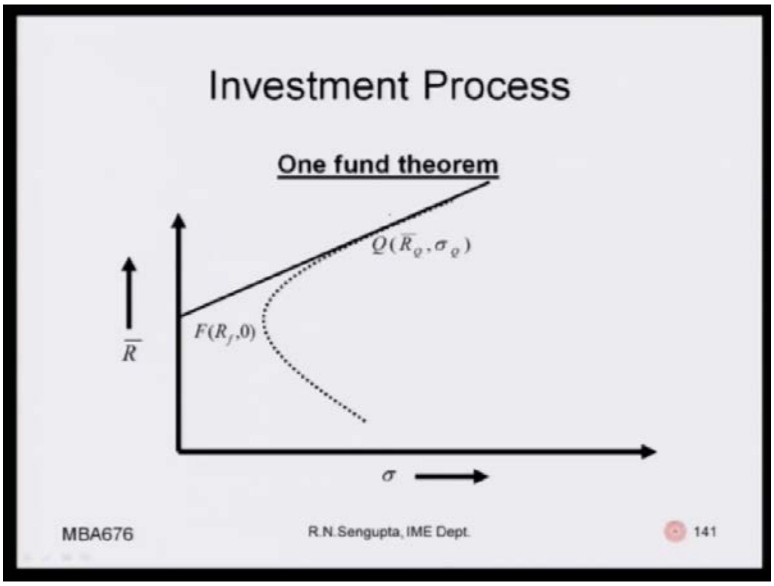

So, this is the q point which we have an r f or f point which we have already discussed. Now what is a one front theorem. In the two front theorem, we had two stocks or concepts of portfolios combine them would give you all the set of portfolios which an investor can have. In the one front theorem what you need. we do not need to fund, because one is already known to us, there is risk free interest rate, and another is the q point which we have already found out, combine them and any particular investor would always be along as straight line, in order to get the best benefit depending on his or her risk profile and the return profile.

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So, there it is a single fund q of risky assets as that any efficient font can we constructed as a combination on the fund; which is q and the risk free interest rate which is r or f, and we combine them which is a straight line.

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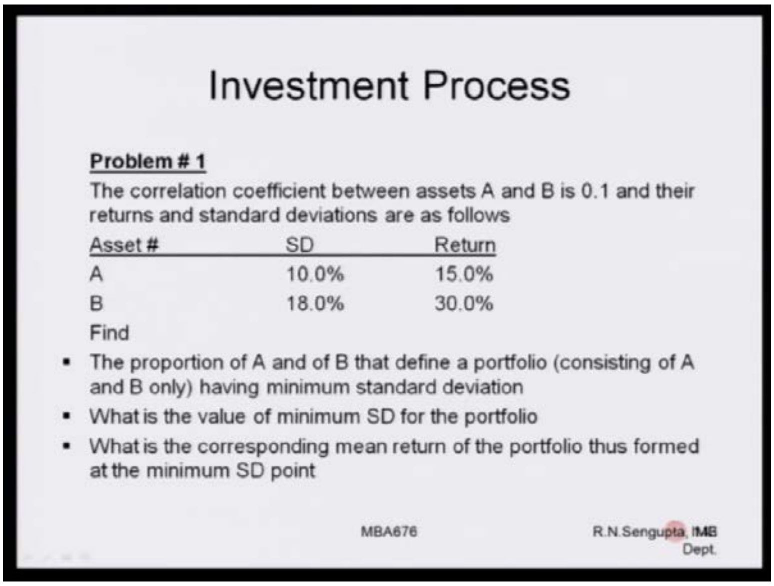

So, for the benefit; obviously, will have different type of assignments, but I would request the students to read this problem; the correlation coefficient existing within two stocks a and b is given as point one, and their risk and return profiles are given, the standard divisions return I given. The proportions of a and b says that the define a portfolio, consisting of a and b, having the minimum variants point would can be found out, using the concept of Lagrangian. what is the value. Once we find out the weights w

1 and w 2 which is w n w v. We can multiplied by the corresponding return, find out the, return of the portfolio, and use the same formula for the variances, you can find out the variants of the portfolio.

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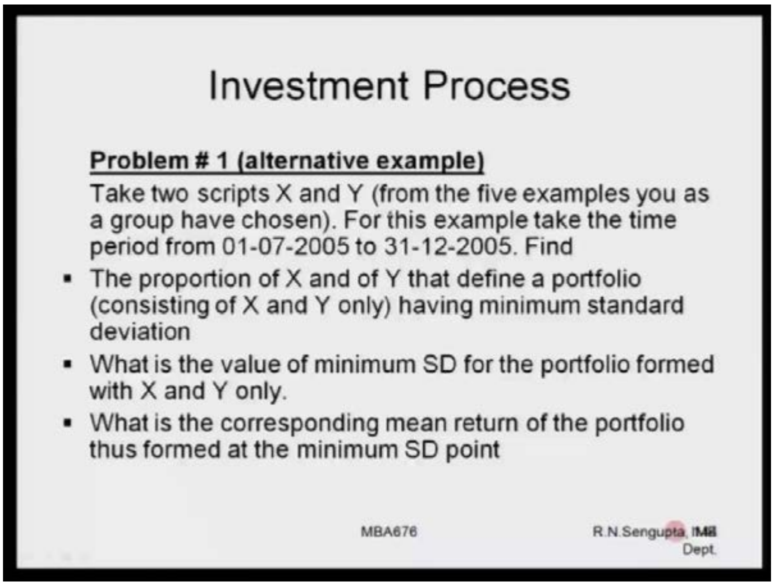

So, these are for the benefits if you read it, rather than discussing each and every problem. We will basically discuss the concept and basically, I would be open to all the question which are asked by the students. And we can definitely answer each of them, depending on what the actual queries are there with the students .

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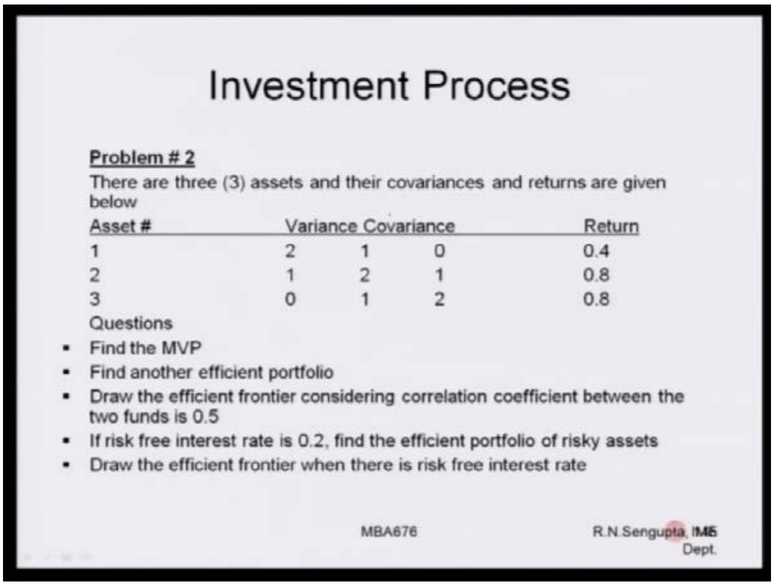

In problem two, again the same thing. now you have three assets; variances covariance’s are given. This is a very theoretical cases, just to understand us that the cases. So, if you see the variance covariance is given, returns are given. again use the same concept of Lagrangian, find other minimum variance point, use that concept of Lagrangian, find other one point of q join r f and q get the efficient frontier for the second case, where n number of risk assets and nth plus 1 is risk free, and also you can get the efficient frontier for the case, when we have only n number of risky assets.

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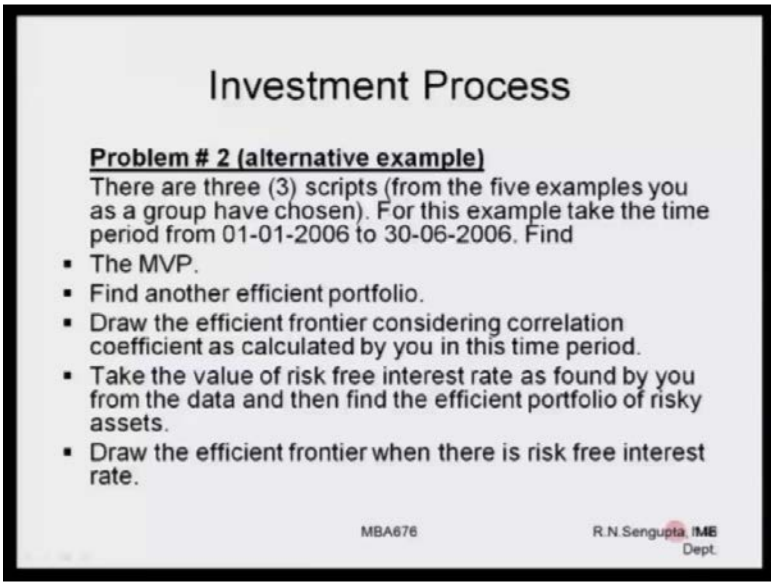

So, there are three scripts again same problem, consider that from the actual stock market, get them these values, and you can basically solve the problem accordingly.

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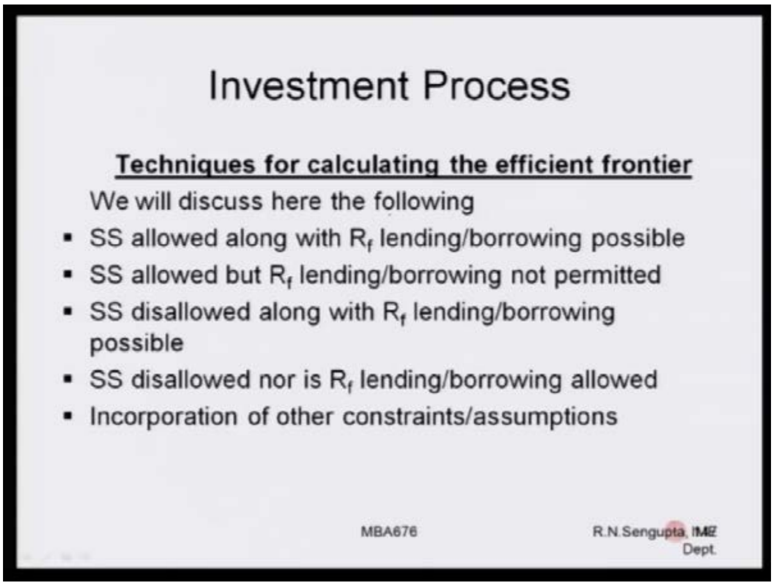

Now our question is, how do you find out the efficient frontier for these cases. s s allowed, s s not being allowed, and with different combinations of risk free interest rate being there or not there. So, with this small introduction, we will try to basically take a pause, and in the next class we will basically start with the techniques of how you calculate the, efficient frontier, given the data and given the set of information, that we have been already been able to solve the two efficient frontier cases, where n number of risk assets are there, and n number of risk assets plus the nth 1 being risk free interest rate, based on that we have already drawn the efficient frontiers.

Thank you.